Homework #6
(Due October 24 at the beginning of the class.)

Numerical Method Series #1: Gauss Elimination Method

NOTE: All Mathcad programs should be written on the basis of “ORIGIN \equiv 1”

Goal:
2. Understanding the programming logic for algorithm development in the Gauss Elimination method and getting more familiar with advanced control structures such as nested “for”-loops and other control structures: Problem 2.
3. Expand our understanding of the programming logic in the Gauss Elimination method to solving a case of multi-column RHS: Problem 3.

Problem 1. Solve the following systems of linear algebraic equations using the Gauss Elimination method by hand.

(A) \[
\begin{bmatrix}
3 & 5 & 2 \\
2 & 3 & -1 \\
1 & -2 & -3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= \begin{bmatrix}
8 \\
1 \\
-1
\end{bmatrix}
\]

(B) \[
\begin{bmatrix}
3 & 5 & 2 \\
2 & 3 & -1 \\
1 & -2 & -3
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
y_3
\end{bmatrix}
= \begin{bmatrix}
-3 \\
2 \\
10
\end{bmatrix}
\]

Note: the coefficient matrix on the left hand side in both (A) and (B) is identical.

(C) \[
\begin{bmatrix}
5 & -1 & 0 & 2 \\
-1 & 4 & -2 & 1 \\
0 & -2 & 3 & -1 \\
2 & 1 & -1 & 4
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
= \begin{bmatrix}
1 \\
-3 \\
2 \\
3
\end{bmatrix}
\]

Instruction for Problem 1:
1. You work must illustrate a step-by-step process of hand solution.
2. You work must be neatly written on engineering paper.
3. You must validate your hand calculation using Mathcad. For example, use a built-in inverse matrix operator to solve for the unknowns such that

\[ A \cdot x = b \]

\[ A^{-1} \cdot A \cdot x = A^{-1} \cdot b \]

\[ I \cdot x = A^{-1} \cdot b \]

\[ \therefore x = A^{-1} \cdot b \]

where “A” represents the coefficient matrix, “x” represents the unknowns, “I” represents the identity matrix, and “b” represents the right hand side vector.

If any of the above instructions is not followed, your solution for Problem 1 will be returned with zero grades.

**Problem 2.**

Write two separate MathCAD functions that perform the two parts of Gauss Elimination: (A) the first function written for Forward Elimination and (B) the second one written for Back Substitution.

**Instruction** for Problem 2:

1. You must follow the homework template format as posted at the web page (www.ce.ufl.edu/~jchun/CGN3421/downloads.html)
2. For the input section, use the matrices given in (B) and (C) of Problem 1.
3. Use your hand solution for Problem 1 as the verification section.
4. Do not use the built-in summation \( \sum \) operator in your program.

If any of the above instructions is not followed, your solution for Problem 2 will be returned with zero grades.

**Problem 3.**

1. Comparing the solution process in Part (A) of Problem 1 to that in Part (B) of Problem 1, identify a similar pattern in the gauss elimination process.

Note: the coefficient matrix on the left hand side in both (A) and (B) is identical.

2. Describe how you would like to modify (or re-write) those two functions that have been developed in Problem 2 in order to accommodate a multiple column
RHS (e.g., RHS becomes a matrix) in detail. In other words, state how modification can be made in your functions (that were developed in Problem 2) in order to solve a system of linear equations that has the multiple column RHS shown as below:

\[
\begin{bmatrix}
3 & 5 & 2 \\
2 & 3 & -1 \\
1 & -2 & -3 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
\end{bmatrix}
=
\begin{bmatrix}
8 & -3 \\
1 & 2 \\
-1 & 10 \\
\end{bmatrix}
\]

**Instruction** for Problem 3

1. You work **must** be neatly written on engineering paper.
2. Your work **must** illustrate sufficiently logical thought process that can be used as a pseudocode to implement into a Mathcad program.

If any of the above instructions is not followed, your solution for Problem 3 will be returned with zero grades.

**Problem 4 only for CE or ME major.**

A lightweight structure constructed of triangular elements may be capable of supporting large weights. To analyze the forces on such a structure, known as a **truss**, a system of linear equations describing the equilibrium of the horizontal and vertical forces on each node (or joint) of the truss must be solved.

We assume that **unknown** forces in each of the members of the truss \((F_{12}, F_{13}, \text{and } F_{23})\) are acting to pull the structure together. \(V_1\) and \(V_2\) are **unknown** vertical forces supporting the structure (at node 1 and node 2, respectively), \(H_1\) is an **unknown** horizontal bracing force at node 1, and \(W\) is a known force, representing the weight of the structure. We define forces to be positive if they act to the right, or in an upward direction. If a computed quantity is negative, it indicates that the force acts in a downward direction.
The system of equations that represents the conditions for vertical and horizontal equilibrium at the three nodes is given as:

**Node 1**

\[ V_1 + F_{13} \sin(\alpha) = 0 \]

\[ H_1 + F_{12} + F_{13} \cos(\alpha) = 0 \]

**Node 2**

\[ V_2 + F_{23} \sin(\beta) = 0 \]

\[ -F_{12} - F_{23} \cos(\beta) = 0 \]

**Node 3**

\[ -F_{13} \sin(\alpha) - F_{23} \sin(\beta) = W \]

\[ -F_{13} \cos(\alpha) + F_{23} \cos(\beta) = 0 \]

where \( F_{12} \) represents a member force between Node #1 and Node #2, etc.

We can solve this problem by using the Gauss Elimination method. In other words, Solve the system using your MathCAD functions that were developed in Problem 2.

**Instruction** for Problem 4:

1. You **must** follow the homework template format as posted at the web page ([www.ce.ufl.edu/~jchun/CGN3421/downloads.html](http://www.ce.ufl.edu/~jchun/CGN3421/downloads.html))
2. For the input section, create matrices that contain the information regarding the equilibrium equations.
3. You must validate your solution using the principle of force equilibrium at one of the joints (at least).

If any of the above instructions is not followed, your solution for Problem 4 will be returned with zero grades.

Problem 4 only for ABE major.

Shown in the figure, five reactors are linked by pipes. As we perform a mass balance for a conservative substance, i.e., mass of the material does not increase or decrease due to chemical transformation, the rate of transfer of chemicals through each pipe is equal to a flow rate (Q, with units of cubic meters per minute) multiplied by the concentration of the reactor from which the flow originates (C, with units of milligrams per cubic meter).
If the system is at a steady state, the transfer into each reactor will balance the transfer out. The mass balance equations for the reactors are given (See below)

For Reactor 1,

\[ 5(10) + Q_{31}C_3 = Q_{12}C_1 + Q_{15}C_1 \]

or, substituting the values for flow given in the figure

\[ 6C_1 - C_3 = 50 \]

Similar equations can be developed for the other reactors:

\[ -3C_1 + 3C_2 = 0 \]
\[ -C_2 + 9C_3 = 160 \]
\[ -C_2 - 8C_3 + 11C_4 - 2C_5 = 0 \]
\[ -3C_1 - C_2 + 4C_5 = 0 \]

We can solve the unknowns (i.e., C's) by using the Gauss Elimination method. In other words, Solve the five simultaneous linear algebraic equations for their concentrations using your MathCAD functions that were developed in Problem 2.

**Instruction** for Problem 4:

1. You **must** follow the homework template format as posted at the web page [www.ce.ufl.edu/~jchun/CGN3421/downloads.html](http://www.ce.ufl.edu/~jchun/CGN3421/downloads.html)
2. For the input section, create matrices that contain the information regarding the mass balance equations.
3. You don’t have to use units in your solution.
4. You **must** validate your solution by verifying a total mass balance of the system.

If any of the above instructions is not followed, your solution for Problem 4 will be returned with zero grades.