Lecture 7

Scope and Anaphora
Scope and Anaphora

Today

We will discuss ways to express scope ambiguities related to

• Quantifiers
• Negation
• Wh-words (questions words like who, which, what, …)
Scope and Anaphora

A few simple examples first
to illustrate the nature of problems
Scope and Anaphora

*Not everyone lived in castles during the Middle Ages.*

*Many people think of the Middle Ages as a romantic time when gallant knights rescued lovely damsels in distress and everyone lived in castles. (…)*
Scope and Anaphora

Not everyone lived in castles during the Middle Ages.
Many people think of the Middle Ages as a romantic time when gallant knights rescued lovely damsels in distress and everyone lived in castles. (…)

Now, compare this with

Some people did not live in castles during the Middle Ages.
Many people think of the Middle Ages as a romantic time when gallant knights rescued lovely damsels in distress and everyone lived in castles. (…)

Rhetorically speaking, the second version with the altered title is much less effective, although truth-conditionally it is equivalent to the first version with the original title.
Scope and Anaphora

(i) Everybody needs someone to lean on.
Scope and Anaphora

(i) Everybody needs someone to lean on.
(ii) There is someone needed by everybody to lean on.
Scope and Anaphora

(i) Everybody needs someone to lean on.
(ii) There is someone needed by everybody to lean on.
(iii) Someone needs everybody to lean on.
Scope and Anaphora

(i) *Everyone did not know for sure Saddam had WMD.*  active sentence
(ii) *That Saddam had WMD wasn’t known by everyone.*  passive sentence

• For most native speakers of English, (i) is ambiguous and has the following two readings:

  (a) No one knew for sure that Saddam had WMD

  (b) Some people knew for sure and some people did not know for sure …

• (ii) is taken not to be ambiguous, it can probably be only understood as having only the reading (b).
Scope and Anaphora

So - how do we represent such scope ambiguities?
Scope and Anaphora

Quantified DPs may occur in the subject and in the object position, and we may have more than one quantified NP/DP in one clause:

a. No student read every book.  subject
b. Mary ate every apple.  object
Scope and Anaphora

Sentences with more than one quantifier may be ambiguous:

Every scientist admires a/some/one film star.

(i) \( \forall x[\text{scientist}(x) \rightarrow \exists y[\text{film star}(y) \wedge \text{admire}(x,y)]] \)
    For every scientist \( x \), for some film star \( y \), \( x \) admires \( y \)

(ii) \( \exists y[\text{film star}(y) \wedge \forall x[\text{scientist}(x) \rightarrow \text{admire}(x,y)]] \)
    For some film star \( y \), for every scientist \( x \), \( x \) admires \( y \)
Scope and Anaphora

Every scientist admires a/some/one film star.

(i) \( \forall x[\text{scientist}(x) \rightarrow \exists y[\text{film star}(y) \land \text{admire}(x,y)]] \)  \( \forall > \exists \)

For every scientist \( x \), for some film star \( y \), \( x \) admires \( y \)

The sentence is true if and only if every scientist admires some film star or another, with possibly a different film star admired by each different scientist. Model 1:
Scope and Anaphora

Every scientist admires a/some/one film star.

(i) \( \forall x[\text{scientist}(x) \rightarrow \exists y[\text{film star}(y) \land \text{admire}(x,y)]] \quad \forall > \exists \)

For every scientist \( x \), for some film star \( y \), \( x \) admires \( y \)

Another model which makes the above sentence true under the interpretation of (i):

Model 2:

Albert \( \rightarrow \) Brigitte

Paul \( \rightarrow \) Angelina

David \( \rightarrow \) Liz
Scope and Anaphora

*Every scientist admires a/some/one film star.*

(ii) \[ \exists y [\text{film star}(y) \land \forall x [\text{scientist}(x) \rightarrow \text{admire}(x,y)]] \quad \exists > \forall \]

For some film star y, for every scientist x, x admires y

The sentence is true if and only if there is one particular film star who is universally admired by all the different scientists. Model 1:
Scope and Anaphora

Every scientist admires a/some/one film star.

(ii) \( \exists y [\text{film star}(y) \land \forall x [\text{scientist}(x) \rightarrow \text{admire}(x, y)]] \quad \exists > \forall \\
For some film star y, for every scientist x, x admires y

Another Model 2 which makes (ii) true:
Scope and Anaphora

What we have seen so far:

• The order of the quantifiers in a sentence of a natural language like English does not necessarily reflect their scope with respect to each other.

• Such sentences exhibit scope ambiguities with respect to their quantifiers.
Scope and Anaphora

Every scientist admires a/some/one film star.

(i) \( \forall x[\text{scientist}(x) \rightarrow \exists y[\text{film star}(y) \land \text{admire}(x,y)]] \)
For every scientist x, for some film star y, x admires y

(ii) \( \exists y[\text{film star}(y) \land \forall x[\text{scientist}(x) \rightarrow \text{admire}(x,y)]] \)
For some film star y, for every scientist x, x admires y

• The order of the quantifiers in a logical formula reflects their scope with respect to each other.

• Predicate logic formulas never display any scope ambiguities.
Scope and Anaphora

We say that in a formula like

\[ \forall x[\text{scientist}(x) \rightarrow \exists y[\text{film star}(y) \land \text{admire}(x,y)]] \]

For every scientist x, for some film star y, x admires y

- “\( \forall x \) (the universal quantifier) takes wide scope with respect to \( \exists y \) (the existential quantifier).”

- “\( \exists y \) (the existential quantifier) takes narrow scope with respect to \( \forall x \) (the universal quantifier).”

- “\( \exists y \) (the existential quantifier) is in the scope of \( \forall x \) (the universal quantifier).”

- Also abbreviated as: \( \forall > \exists \) or \( \exists < \forall \)
Scope and Anaphora

We find such scopal ambiguities also with respect to other operators, like *negation*:

*All that glitters isn’t gold.*

What matters here is NOT the order of two quantifiers with respect to each other, but the order of a negation with respect to a quantifier.
Scope and Anaphora

All that glitters isn’t gold.

(i) $\neg \forall x [\text{glitters}(x) \rightarrow \text{gold}(x)]$
It is not the case, for all $x$ that glitters, $x$ is gold
Not everything that glitters is gold, some is and some is not.

(ii) $\forall x [\text{glitters}(x) \rightarrow \neg \text{gold}(x)]$
For all $x$ that glitters, it is not the case, $x$ is gold
Whatever glitters is not gold.
Nothing that glitters is gold.
Scope and Anaphora

(i)  *Everyone did not know for sure Saddam had WMD.*  \(\text{active sentence}\)

For most native speakers of English, (i) is ambiguous and has the following two readings:

(a)  \(\forall x \neg \text{Know}(x, \text{Saddam-has-WMD})\)

\(=\)  No one knew for sure that Saddam had WMD

(b)  \(\neg \forall x \text{Know}(x, \text{Saddam-has-WMD})\)

\(=\)  Some people knew for sure and some people did not know for sure …

(ii)  *That Saddam had WMD wasn’t known by everyone.*  \(\text{passive sentence}\)

can probably be only understood as having the reading (b) above.

- **Problem:** If the passive transformation is thought of as a meaning-preserving transformation, (i) and (ii), which are related through the passive transformation, are problematic, because they don’t have the same meaning (see already Chomsky 1957).
Scope and Anaphora

• Some natural languages are like predicate logic in that the surface order of the quantifiers reflects their semantic scope.

• Example: Mandarin Chinese (e.g., May 1985, Aoun & Li 1993)
Scope and Anaphora

• In languages like English, the scope of scope-bearing words and phrases is less tied to the surface syntactic structure.

• In general, scope ambiguities arise when it is not possible to determine which quantifier has scope over another one from the surface syntax.
Scope and Anaphora

Someone loved everybody.
Scope and Anaphora

Someone loved everybody.

(i) $\exists x \forall y \text{ [Love}(x,y)]$
Scope and Anaphora

Someone loved everybody.

(i)  $\exists x \forall y \ [\text{Love}(x,y)]$

(ii) $\forall y \exists x \ [\text{Love}(x,y)]$
Scope and Anaphora

No one has read an email I sent.
Scope and Anaphora

No one has read an email I sent.

(i) \( \neg \exists x \exists y [\text{Email-I-send}(y) \land \text{Read}(x,y)] \)

It’s not the case that there is at least one person who read an email I sent. No one read an email I sent.
Scope and Anaphora

No one has read an email I sent.

(i) \(\neg \exists x \exists y[\text{Email-I-send}(y) \land \text{Read}(x,y)]\)

It’s not the case that there is at least one person who read an email I sent. No one read an email I sent.

(ii) \(\exists y \neg \exists x[\text{Email-I-send}(y) \land \text{Read}(x,y)]\)

One particular email I sent was read by nobody.

(But other emails I sent were read.)
Scope and Anaphora

John didn’t find a mistake. (Karttunen 1976)

(i) It is not the case that John found a mistake.
\[ \neg \exists x [\text{Mistake}(x) \land \text{Miss}(\text{John},x)] \quad \text{Neg > } \exists \]
= John found no mistake at all.
non-specific reading of a mistake
One situation which makes this sentence true under the wide scope reading of \text{Neg} is a world in which there was no mistake, hence no mistake was found.

(ii) There was a particular mistake that John didn’t find.
\[ \exists x [\text{Mistake}(x) \land \neg \text{Miss}(\text{John},x)] \quad \exists > \text{Neg} \]
specific reading of a mistake
This interpretation allows for the possibility that John found some other mistakes.
Scope and Anaphora

John didn’t find a mistake. (Karttunen 1976)

(i) \(\neg \exists x [\text{Mistake}(x) \land \text{Miss}(\text{John},x)]\)  \(\text{Neg} \quad \exists\)

= John found no mistake at all.

A continuation of with … it was very easy to miss is not possible (indicated by ‘#’) under this interpretation:

I.e., John didn’t find a mistake. #It was very easy to miss.

(ii) There was a particular mistake that John didn’t find.

\(\exists x [\text{Mistake}(x) \land \neg \text{Miss}(\text{John},x)]\)  \(\exists \quad \text{Neg}\)

A continuation of with … it was very easy to miss is possible under this interpretation:

I.e., John didn’t find a mistake [we knew about]. It was very easy to miss.
Scope and Anaphora

Someone loved everybody.

(i) $\exists x \forall y \ [\text{Love}(x,y)]$
(ii) $\forall y \exists x \ [\text{Love}(x,y)]$

• (i) entails (ii), because
  whenever there is one specific person that loves everybody, then
  everybody is loved by someone.

• BUT (ii) does not entail (i). If everybody is loved by someone, it does
  not need to be the SAME person that loves everybody.
Scope and Anaphora

• However, this does not mean that we cannot treat scope ambiguities in terms of entailment relations between the relevant readings
Scope and Anaphora

No one has read an email I sent.

(i) \( \neg \exists x \exists y [\text{Email-I-send}(y) \land \text{Read}(x,y)] \)
It’s not the case that there is at least one person who read an email I sent. Nobody read an email I sent.

(ii) \( \exists y \neg \exists x [\text{Email-I-send}(y) \land \text{Read}(x,y)] \)
One particular email I sent was read by nobody.
(But other emails I sent were read.)

(i) does not entail (ii)
(ii) does not entail (i)
Scope and Anaphora

• **Laws of quantifier (in)dependence**

  In doubly quantified propositions, if both quantifiers are universal or both are existential, their linear order in the proposition is irrelevant.

  • $\forall x \forall y [\phi(x,y)] \iff \forall y \forall x [\phi(x,y)]$
    Everybody loved everybody $\iff$ Everybody was loved by everybody.

  • $\exists x \exists y [\phi(x,y)] \iff \exists y \exists x [\phi(x,y)]$
    Somebody loved someone $\iff$ Somebody was loved by someone.
Scope and Anaphora

• Laws of quantifier (in)dependence

Reversing the order of existential and universal quantifiers produces a non-equivalent statement.

• $\exists x \forall y [\phi(x,y)] \Rightarrow \forall y \exists x [\phi(x,y)]$
  Somebody loved everybody $\Rightarrow$ Everybody was loved by someone.
Scope and Anaphora

• **Laws of Quantifier Negation**
  There are a number of equivalences based on the set-theoretic semantics of predicate logic.

• Take for instance a statement of the form
  \( \exists x \neg \phi(x) \) (Example: *Somebody did not pass the test.*)
  It asserts that there is at least one individual \( e \) in the universe of discourse which makes \( \neg \phi(x) \) true.
  That means that \( e \) makes \( \phi(x) \) false.
  That, in turn, means that \( \forall x \phi(x) \) is false,
  which means that \( \neg \forall x \phi(x) \) is true.
Scope and Anaphora

- **Laws of Quantifier Negation**
  
  The same reasoning can be applied in the reverse direction:
  
  If \( \neg \forall x \phi(x) \) (Not everyone passed the test) is true, then
  
  \( \forall x \phi(x) \) (Everyone passed the test) is false, which is
  
  when there is at least one individual \( e \) that makes \( \phi(x) \) false and that
  
  makes \( \neg \phi(x) \) true.
  
  There is at least one individual \( e \) that makes \( \neg \phi(x) \) true is expressed as:
  
  \[ \exists x \neg \phi(x) \]

  The result of this reasoning is the first quantifier law:

  - **Law 1:** \( \neg \forall x \phi(x) \iff \exists x \neg \phi(x) \)

  Not everyone passed the test \( \iff \) Someone did not pass the test.
Scope and Anaphora

• Laws of Quantifier Negation

Because of the Law of Double Negation, i.e., $\neg \neg \phi \iff \phi$, Law 1 could also be written in the following equivalent forms:

Law 1': $\forall x \phi(x) \iff \neg \exists x \neg \phi(x)$
Law 1’’': $\neg \forall x \neg \phi(x) \iff \exists x \phi(x)$
Law 1’’’': $\forall x \neg \phi(x) \iff \neg \exists x \phi(x)$

Example for Law 1’’’': Everyone did not pass the test (i.e., everyone failed the test) $\iff$ No one passed the test.

• A consequence of Law 1 and the Law of Double Negation is that either quantifier could be eliminated from predicate logic in favor of the other and the result would be an equivalent system.
Scope and Anaphora

- Scope ambiguities arise with
  - Quantifiers
  - Negation

as we have seen, and they also arise with
  - Interrogative NPs, i.e., *wh*-expressions like *who*, *which*, *what*, *where*, etc.
Scope and Anaphora

• **Scope in questions**

Which woman does every man love?

• Three kinds of answers

(i) *Brigitte*    an individual answer
(ii) *Paul loves Brigitte, Albert loves Liz, David loves Liz, Paul loves Angelina, ...*  a pair list answer (Groenendijk & Stokhof 1984)
(iii) *his wife*  a functional answer: *wife(x)*

• These three kinds of answers are related to a scope ambiguity between which woman and every man
Scope and Anaphora

Which woman does every man love?

scope ambiguity

(i) Which woman is such that every man loves her? Wh > ∀
(ii) For every man, which woman does he love? ∀ > Wh
Scope and Anaphora

Which woman does every man love?

Take the interpretation (i):
Which woman is such that every man loves her? \( \text{Wh} > \forall \)

The appropriate answer to our question under the interpretation (i) is the first type of answer, exemplified by *Brigitte*, supposing a situation like:
Scope and Anaphora

Which woman does every man love?

Take the interpretation (ii):
For every man, which woman does he love? \( \forall > \text{Wh} \)
The appropriate answer is a list of pairs. We have: \(<\text{albert, brigitte}>, <\text{paul, brigitte}>, <\text{paul, angelina}>, <\text{david, liz}>\)
Scope and Anaphora

Question: *Which woman does every man love?*
Answer: *His wife.*

  e.g., <albert, elsa>, <paul, robin>, …

- The answer is such that there is **one function, wife(\(x\)),** but for every man \(x\) the outcome will be a different unique woman

- A function attributes a unique value to the argument it is applied to. I.e., a function denotes a set of pairs relating the argument and its value.

- This means that **the pair list answer** and the **functional answer** are related: A list of pairs corresponds to the extension of a function, because a function is a special kind of a two-place relation.
Scope and Anaphora

Though related, the pair list answer and the functional answer must be kept apart. Reason: Not all quantifiers allow both pair-list and functional answers.

Question: Which woman does no man love?
Possible answers: (i) Mother Theresa.
(ii) #Albert loves Elsa, Paul loves Robin, ...
(iii) His wife.

• Negative quantifiers like no man take narrow scope with respect to the wh-expressions. I.e., the above question is interpreted as Which woman is such that no man loves her? Wh > no man. This triggers an individual answer like (i).

• Negative quantifiers like no man cannot take wide scope reading with respect to the wh-expressions. It makes no sense, semantically and pragmatically to ask: For no man x, which woman does x love? Hence, *∀ > Wh.

Consequently, the pair list answer is blocked.
Scope and Anaphora

- Co-indexing
- Co-referentiality
- Binding

(Recall Lecture 2)
Scope and Anaphora

PRONOUNS

- deictic
- non-deictic / anaphoric

I am glad he is gone. John entered the room. He took off his coat.

- PRONOUNS have a deictic interpretation if their interpretation depends on the context of an utterance.
- PRONOUNS have an anaphoric interpretation if their interpretation depends on the linguistic context.
Scope and Anaphora

I am glad the bastard is gone.  
↓
definite description  
↓

I am glad Rufus is gone.  
↓
proper name  
↓

• Definite descriptions and proper names behave like deictic pronouns in so far as they may refer directly to some individual in the discourse.
• Definite descriptions and proper names are referential, because they pick out individuals in the domain of discourse.
Scope and Anaphora

- Generally, a given expression (e.g., pronoun) is used anaphorically when it “picks up its reference” from another phrase in the linguistic context (sentence context or discourse context).

\[
\text{John entered the room.} \quad \downarrow \quad \text{He took off his coat.} \\
\text{antecedent NP} \quad \quad \text{anaphoric pronoun}
\]

- Anaphorically interpreted NPs (e.g., pronouns like she, her, himself, his) are said to be \textbf{coreferential with} or \textbf{referentially dependent on} their antecedent NPs.

- \textbf{COINDEXING}

\[
\text{[John]}_i \text{ entered the room.} \quad \text{[He]}_i \text{ took off his coat.} \\
\text{antecedent NP} \quad \quad \text{anaphoric pronoun}
\]
Scope and Anaphora

(1) Every time I see your brother$_i$, I feel like choking the bastard$_i$. CO-REFERENTIALITY

(2) Every time I see your brother$_j$, I feel like choking the bastard$_j$
Scope and Anaphora

• Anaphorically interpreted pronouns which are dependent for their interpretation on proper names or other referential expressions are translated in terms of the same **individual constant** as their antecedent proper name or other referential expression:

(i) Milly likes herself.
Like\((m, m)\)

(ii) If Milly\(_i\) likes Fred\(_j\), she\(_i\) will invite him\(_j\).
Like \((m, f) \rightarrow \text{Invite}(m, f)\)

(iii) Milly\(_i\) and Fred\(_j\) like each other\(_i,j\).
Like\((m,j) \land \text{Like}(f,m)\)

• This amounts to interpreting **coindexing** in the syntax as **coreferentiality** in the semantics.
Scope and Anaphora

Question: Can all examples of anaphoric pronouns be subsumed under one characterization?
Answer: No. Some pronouns do not refer to an individual at all.

Background:

Every dog barked.  Pluto barked.
Some truck were damaged.  The tallest man in the world won.

↓
quantified NP’s referential
non-referential

A quantified noun phrase like every dog and some truck in the above sentences cannot be represented by an individual in our domain of discourse unlike Pluto (proper name), the tallest man (definite description) or he in the above sentences.

Quantified noun phrases are non-referential.
Scope and Anaphora

**Question:** Can *all* examples of anaphoric pronouns be subsumed under one characterization?

**Answer:** No. Some pronouns do not refer to an individual at all.

Background:

- *Every dog barked.*  
  *Some truck were damaged.*  
  *Pluto barked.*  
  *The tallest man in the world won.*  
  *He sneezed.*

  - quantified NP’s
  - non-referential

  ↓

  referential

- *Pluto* (proper name), *the tallest man* (definite description) or *he* in the above sentences can be represented by an individual in our domain of discourse.
- A quantified noun phrase like *every dog* and *some truck* in the above sentences cannot be represented by an individual in our domain of discourse.
- **Quantified noun phrases are non-referential.**
Scope and Anaphora

- **Bound Variable Anaphora**
  
  *Every man* put a screen in front of him.  **binding of variables**

  quantified NP  
  antecedent  
  non-referential

  pronoun = bound variable  
  anaphor  
  non-referential

- Since *every man* is non-referential, the pronoun *him* must be non-referential, as well, because the pronoun cannot derive its reference from its antecedent.

- The pronoun *him* here does not refer to an individual any more than its antecedent *every man* does.

- Therefore, *not* all anaphoric pronouns can be treated as referential.
Scope and Anaphora

• Bound Variable Anaphora

\( \text{Every man}_i \text{ put a screen in front of him}_i. \) binding of variables

• Since the quantified NP is non-referential, the coindexing between the pronoun and the quantificational NP cannot be interpreted in terms of coreferentiality in this case. Instead, co-indexing is interpreted as binding of variables.

• The interpretation/value of the pronoun \textit{him} covaries with the interpretation/value of the variable introduced by the quantified subject NP \textit{every man}.

• If your ‘world’ (the universe of discourse) consists of Steven, Leopold and Rufus, then you may first pick Steven as the value for \textit{him} AND for the variable introduced by the quantified subject NP and check whether Steven put a screen in front of him. Then you do the same with Leopold and Rufus. The sentence is true just in case Steven, Leopold and Rufus each put a screen in front of him.
Scope and Anaphora

Everyone_{i} hates himself_{i}.
Scope and Anaphora

Everyone\(i\) hates himself\(i\).

\(\forall x \text{ Hate}(x,x)\)
Scope and Anaphora

Everyone $i$ thinks he $i$ is brilliant.
Scope and Anaphora

Everyone \emph{he} is brilliant.

∀x[x thinks that x is brilliant]

• Verbs like \emph{think} are relations between individuals and propositions

• Such relations cannot be represented in the first order predicate logic.

• For now, we simplify, and leave a part of the formula not analyzed.
Scope and Anaphora

Everyone$^i$ likes his$^i$ mother.
Scope and Anaphora

Everyone\textsubscript{i} likes his\textsubscript{i} mother.

\[\forall x \exists y [\text{Mother-of}(y,x) \land \forall z (\text{Mother-of}(z,x) \rightarrow [y=z]) \land \text{Like}(x,y)]\]

Uniqueness condition: It ensures that everyone has exactly one mother
Scope and Anaphora

• The following sentence that contains a universally quantified subject NP and the anaphoric pronoun in the subject position of the embedded clause.

\[
\forall x \left[ \left( \text{cat}(x) \land \exists y \left[ \text{mouse}(y) \land \text{catch}(x,y) \right] \right) \rightarrow \text{happy}(x) \right]
\]

• Here too, the anaphoric pronoun is in the scope of the universal quantifier in the main clause.
Scope and Anaphora

- **Prediction**: Anaphoric relations between pronouns and quantificational antecedents are not licensed across the sentence boundary (indicated by ‘#’ below), because the variable introduced by a pronoun will not be in the scope of the quantifier introduced by a quantificational antecedent.

- This prediction is borne out:
  
  ```
  Every cat, came in.  #She, wanted to be fed.
  ∀x[cat(x) → come-in(x)] ∧ want-to-be-fed(x)
  ```

  ‘#’: adding the second sentence to the first is odd.

- The pronoun _she_ is not in the scope of the universal quantifier, it introduces an individual variable that is free.

- Therefore, no bound variable interpretation is available, i.e., the pronoun cannot be interpreted as introducing an individual variable that is bound by the universal quantifier, and whose value co-varies with the individual variable introduced by the quantified subject NP in the first sentence.
Scope and Anaphora

• **Summary:**

The interpretation of anaphoric pronouns at the sentence and discourse level can be derived in a straightforward way from the semantic properties of their antecedents.
Scope and Anaphora

SUMMARY

• Co-reference
  implies reference. Two (or more) expressions (or occurrences of an expression) co-refer iff they refer to the same individual. It follows that if two expressions co-refer, then each of them refers to something.

• Binding
  In [Every man], put a screen in front of [him], neither every man nor him refer to anything. This is not an instance of co-reference, but rather an instance of variable binding (in this case, bound variable anaphora).

• Anaphor
  depends for its interpretation on some other element in the same sentence or in the wider discourse. The expression the anaphor is (referentially) dependent on is called the antecedent.
Scope and Anaphora

SUMMARY

uses of pronouns

non-referring

bound-variable

(a)

deictic

(b)

referring

non-deictic

co-referring

anaphoric

(c)

cataphoric

(d)

(a) Every man put a screen in front of him.
(b) I am glad he is gone.
(c) John entered the room. He took off his coat.
(d) If she calls, tell Joan I’ve gone to the movies.
Scope and Anaphora

When quantifiers are embedded in a formula, it is useful to move them up front to make it clear what their scope is.

Movement of quantifiers in conditionals is restricted by the following laws:

Laws of quantifier movement

\[(\phi \rightarrow \forall x \varphi(x)) \iff \forall x (\phi \rightarrow \varphi(x)), \text{ provided } x \text{ is not free in } \phi\]

\[(\phi \rightarrow \exists x \varphi(x)) \iff \exists x (\phi \rightarrow \varphi(x)), \text{ provided } x \text{ is not free in } \phi\]

\[(\forall x (\phi(x) \rightarrow \varphi) \iff \exists x (\phi(x) \rightarrow \varphi)) \text{ provided } x \text{ is not free in } \varphi\]

\[(\exists x (\phi(x) \rightarrow \varphi) \iff \forall x (\phi(x) \rightarrow \varphi)) \text{ provided } x \text{ is not free in } \varphi\]

- A universal quantifier in the antecedent of a conditional switches to an existential quantifier that takes wide scope over the conditional as a whole.
- An existential quantifier switches to a universal quantifier.
Scope and Anaphora

Laws of quantifier movement

If every problem set counts, Sue fails the exam. ⇔
There is an object such that if it is a problem set and it counts, Sue fails the exam.

\[(\forall x \phi(x) \rightarrow \varphi) \leftrightarrow \exists x (\phi(x) \rightarrow \varphi)]\)
Scope and Anaphora

Laws of quantifier movement

If a storm hits the coast, there is a lot of damage.
⇔
For every object, it is the case that if it is a storm that hits the coast, there is a lot of damage.

\((\exists x \phi(x) \rightarrow \varphi) \iff \forall x (\phi(x) \rightarrow \varphi))\)