Optional Homework: Simpson's Rule

Let \( \Delta x = x_i - x_{i-1} = x_{i+1} - x_i \).

Show that the area under the parabola passing through the points

\[
(x_{i-1}, f(x_{i-1})), \quad (x_i, f(x_i)), \quad (x_{i+1}, f(x_{i+1}))
\]

is equal to

\[
\frac{\Delta x}{3} \left( f(x_{i-1}) + 4f(x_i) + f(x_{i+1}) \right).
\]

Hints:

- don't assume that \((x_i, f(x_i))\) is the vertex of the parabola, this isn't necessarily true, it just looks that way in my drawing.

- try writing the equation for the parabola as \(p(x) = ax^2 + bx + c\). Then compute \(p(x_{i-1}), p(x_i), p(x_{i+1})\) and use this info to solve for \(a, b,\) and \(c\).

- if you get stuck, follow along with this youtube video and try to mimic her technique: \(http://www.youtube.com/watch?v=uc4xJsi99bk\)

Due by Wednesday May 28th, 2014