P1. Find a numerical approximation of

\[ \int_{-2}^{2} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx \]

to within 0.01 of the exact value. Show that you are this accurate using the appropriate error formula.

**Trapezoid Rule:**

\[ E_T = \frac{M \cdot 4^3}{12} = 0.01 \]

given: \( |f''(x)| \leq \frac{4e^{9}}{\pi} \Rightarrow n = \frac{\sqrt{\frac{2e^{9}}{\pi} \cdot 2e^{9}}}{4} = 44 \)

**Simpson's Rule:**

\[ E_S = \frac{M \cdot 45}{180} = 0.01 \]

given: \( |f^{(4)}(x)| \leq \frac{3}{\pi} \Rightarrow n = \frac{\sqrt{\frac{45}{180 \cdot 0.01 \cdot \sqrt{\frac{3}{\pi}}}}}{5} = 5.1 \]

Since Simpson's rule requires an even number of sub-intervals, we round up to the next highest even integer: \( n = 6 \).

Then, we have \( \Delta x = \frac{2 - (-2)}{6} = \frac{2}{3} \).

So the Simpson's rule approximation is as follows:

\[ \Delta x \left( f(x_0) + 4f(x_1) + 2f(x_2) + \cdots \right) \]

\[ = \frac{2}{9} \left( 0.05 + 2 \cdot 0.164 + 2 \cdot 0.319 + \cdots \right) \]

\[ = 0.954 \] (actual: 0.95545...)

P2. Approximate the area bounded by the curves \( f(x) = \sqrt{x^2 + 1} \) and \( g(x) = \frac{x}{\sqrt{x^2 + 1}} \) on the interval \([0, 2]\).

The area between curves is given by \( \int_{a}^{b} |f(x) - g(x)| \, dx \). (You may or may not recognize it with absolute value signs, but when we "find which function is bigger", this is what we're doing.) So, let \( h(x) = |f(x) - g(x)| \), find \( \int_{0}^{2} h(x) \, dx \).

I think that Simpson's rule on eight subintervals will give sufficient accuracy.

So the Simpson's rule approximation is as follows:

\[ \Delta x \left( |f(x_0) - g(x_0)| + 4 |f(x_1) - g(x_1)| + 2 |f(x_2) - g(x_2)| + \cdots \right) \]

\[ = \frac{1}{12} \left( 1 + 4 \cdot 0.91 + 2 \cdot 0.671 + 4 \cdot 0.35 + \cdots \right) \]

\[ = 1.3005 \] (actual: 1.30052...)