P1. (3pts.) Find the domain of $f(x) = \frac{1}{\sqrt{x}}$. Then compute $f(4) - f\left(\frac{1}{16}\right)$.

First, we find the domain. In the expression $1/\sqrt{x}$, we see two things: a denominator and an expression under a square root. We need the denominator to be non-zero, so we write

$$\sqrt{x} \neq 0$$

Squaring both sides, this gives us

$$x \neq 0$$

Now, any expression under a square root must be non-negative (i.e. greater than or equal to 0), and $x$ appears under a square root, so we have

$$x \geq 0$$

Combining $x \neq 0$ and $x \geq 0$ gives us $x > 0$, so we conclude that the domain of $f$ is

$$(0, \infty)$$

Next, we carry out the requested computation.

$$f(4) - f\left(\frac{1}{16}\right) = \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{\frac{1}{16}}}$$

$$= \frac{1}{2} - \frac{1}{\frac{1}{4}}$$

$$= \frac{1}{2} - 4$$

$$= \frac{1}{2} - \frac{8}{2}$$

$$= \frac{-7}{2}$$

For those of you having trouble evaluating $\frac{1}{\sqrt{\frac{1}{16}}}$, it may help you to rewrite $f(x) = x^{-1/2}$ (which we can do, because $\sqrt{x} = x^{1/2}$, so $1/\sqrt{x} = x^{-1/2}$). Then we can simply use exponent laws:

$$f(16^{-1}) = (16^{-1})^{-1/2} = 16^{1/2} = 4$$
P2. (3pts.) Take $P = (-1, 2)$ and $Q = (7, 6)$. Write the equation of a circle passing through the origin whose center is the midpoint of the line segment connecting $P$ and $Q$.

First let’s find the midpoint $m$ of $PQ$.

$$m = \left( \frac{P_x + Q_x}{2}, \frac{P_y + Q_y}{2} \right)$$

$$= \left( \frac{-1 + 7}{2}, \frac{2 + 6}{2} \right)$$

$$= \left( \frac{6}{2}, \frac{8}{2} \right)$$

$$= (3, 4)$$

Plugging the coordinates of $m$ into the standard form of the circle equation gives us

$$(x - 3)^2 + (y - 4)^2 = r^2$$

All that remains is finding $r$. Every point $(x, y)$ on the circle must satisfy the equation above, and we’re given in the problem that $(0, 0)$ is a point on the circle, so we can substitute the point in to solve for $r$.

$$(0 - 3)^2 + (0 - 4)^2 = r^2$$

$$9 + 16 = r^2$$

$$25 = r^2$$

$$5 = r$$

So our final answer is

$$(x - 3)^2 + (y - 4)^2 = 5^2$$

Many of you who got this problem wrong made the mistake of substituting the coordinates for $P$ or $Q$ instead of $(0, 0)$ to solve for $r$. Note that the question didn’t say the circle should pass through either $P$ or $Q$, only that it should pass through the origin. Be sure to read questions like this carefully.

P3. (3pts.) Find the distance between the point $(-2, 7)$ and the center of the circle $x^2 + 4x + y^2 - 6y - 3 = 0$. Then write the equation for the line passing through these two points.

We begin by converting its equation to standard form.

$$(x^2 + 4x) + (y^2 - 6y) - 3 = 0$$

$$(x^2 + 4x + 4) - 4 + (y^2 - 6y + 9) - 9 - 3 = 0$$

$$(x + 2)^2 - 4 + (y - 3)^2 - 9 - 3 = 0$$

$$(x + 2)^2 + (y - 3)^2 - 16 = 0$$

$$(x + 2)^2 + (y - 3)^2 = 4^2$$

The center of the circle is, therefore, the point $(-2, 3)$. We need the distance between this and $(-2, 7)$.
We pause here briefly to make an important observation: both points have the same the \(x\)-coordinate. Therefore, the distance is simply the difference between their \(y\)-coordinates. Alternatively, we could also notice that \((-2, 7)\) is a point on the circle (at the "top"), so the distance between it and the center is precisely the radius of the circle, which we have already calculated above. These are easy to see if you draw the circle after finding its equation, which I suggest you always do.

After making either of the above observations, it would be overkill to actually compute the distance using the distance formula, but here is how to do it in case you didn’t notice them.

\[
distance = \sqrt{(-2 - (-2))^2 + (7 - 3)^2}
\]
\[
= \sqrt{0^2 + (4)^2}
\]
\[
= \sqrt{4^2}
\]
\[
= 4
\]

Finally, the question asks for the equation of the line passing through these two points. Since \((-2, 7)\) and \((-2, 3)\) both have the same \(x\)-coordinate, the line passing through them is vertical. Vertical lines have the form \(x = \text{some constant}\), so the line is

\[
x = -2
\]

P4. (1pt.) What’s the best name for a pet tiger?

Azealia Banks.