Optimal Income Taxation

Indirect Taxes - (e.g., taxes on commodities); government only interacts with consumers through markets (consumers are anonymous)

Direct Taxes - (e.g., income tax); government treats consumers as individuals

Issues -
(1) Distribution of burden of public expenditure
(2) Do we want to actively transfer income to some individuals (pole tax in commodity tax section returned tax equally to all consumers)
(3) Factors in income tax schedules (e.g., vary with family size)
(4) What should pattern of marginal tax rate be

Utilitarians - addressed first two issues using inelastic labor supply; argued for different tax rates based on ability to pay (with inelastic labor supply, income measures ability to pay)

Elastic Labor Supply - income tax creates labor supply distortions (so utilitarian results not very useful); ability to pay is determinant of income but so is effort

Linear Income Tax - \[ T = \begin{cases} 0 & \text{if } y \leq \hat{y} \\ x\% (y - \hat{y}) & \text{if } y > \hat{y} \end{cases} \]

Modify for Min Consumption - \[ T = -a + x\% y \]

Real World - marginal tax rates vary with income

Lots o' Parameters - include break points, marginal tax rate at each break point, number of break points

Simple Say - fully non-linear income tax

Vickrey - first thought about optimal income tax problem, but couldn't solve it because choices depend on entire budget set (not convex)

Asymmetric Information Problem - Mirrlees insight; income is choice which depends on ability, effort and tax schedule; government only observes income

Ability - wage; fixed in short-run (i.e., ability and effort are not related); Mirrlees used continuum of abilities, but finite case is easier to understand

Effort - hours worked

Income - wage times hours

Revelation Principle - modeling strategy; have people report wage and assign them effort level and tax (i.e., income) based on wage (this doesn't make sense because government can't observe wage but it's just the model to derive the tax schedule for the real problem: people report incomes)

Implied Tax Schedule - if presented with tax schedule developed this way, people will make same choice of effort when they report income rather than wage

Incentive Compatibility - want to guarantee people will tell the truth

Brito & Oakland - solve problem with continuum of abilities using optimal control (easier than the way Mirrlees solved it)
Stiglitz - used finite types of ability (wages): $w_1$ & $w_2$

**Income** - $y_i = w_iL_i$ ($L_i$ = labor [effort])

**Ability** - government doesn't know an individual's type, but does know distribution of types; $N_i$, people of type $w_i$

**Consumer Utility** - $U^i(c_i, L_i)$ (usually assume it's concave);

$U_c > 0$ (consumption good) and $U_L < 0$ (labor bad)

**Budget Constraint** - $c_i = y_i - T(y_i)$

**Observable Utility** - $U^i(c_i, L_i) = U^i(c_i, y_i / w_i)$, but government doesn't observe labor (effort) or wage (ability) so embed the wage into the function (which isn't actually observed anyway) and use $V^i(c_i, y_i)$

**Single Crossing Assumption** - indifference curves cross at most once; at any point of intersection (low ability) $V^1$'s indifference curve is steeper than $V^2$'s

**Common Utility Function** - if $U(c, L)$ is identical for each consumer so only wage (ability) differences change the slope of the indifference curves (recall we could also write it as $U(c_i / y / w_i)$), then single crossing results from consumption of normal good; common utility function is not required for most of the results we'll see, but it helps for intuition

**Tax System** - defines relationship between consumption and earned income (see budget constraint above)

**Taxes Paid** - type $i$ consumer pays $y_i - c_i$

**Direct Mechanism** - based on wage; can't actually implement as a tax system, but uses revelation principle to derive tax schedule; individual reports ability (wage) and government assigns that person an income (hence effort) and tax (hence consumption)

**Gov't Goal** - want type $w_i$ to consume $c_i$ and earns $y_i$

**Choice** - person with $w_2$ has two options:

1. Report $w_2$ and get $V^2(c_2, y_2)$
2. Report $w_1$ and get $V^2(c_1, y_1)$

**Incentive Compatibility (IC)** - also called self-selection; to get person with $w_2$ to tell the truth, ensure: $V^2(c_2, y_2) \geq V^2(c_1, y_1)$ For person with $w_1$: $V^1(c_1, y_1) \geq V^1(c_2, y_2)$ (usually not binding)

**Pareto Problem** - this is how Stiglitz solved the problem

$$\max_{c_1, y_1, c_2, y_2} \quad \begin{array}{ll} & V^2(c_2, y_2) \\ \text{s.t.} & V^1(c_1, y_1) \geq \mu^1 \\ & N_1(y_1 - c_1) + N_2(y_2 - c_2) \geq \mu \\ & V^2(c_2, y_2) \geq V^2(c_1, y_1) \\ & V^1(c_1, y_1) \geq V^1(c_2, y_2) \end{array}$$

(Pareto constraint; type 1's utility) (Govt revenue requirement [budget constraint]) (IC for type 2) (IC for type 1)

**Binding Constraints** - Stiglitz assumed $\mu$ and $\gamma > 0$ (i.e., Pareto and revenue constraints are binding); then he considered cases for the IC constraints
Graphs - consider competitive equilibrium (no tax); both types are on the 45° line so \( MRS^i = -\frac{\partial V^i}{\partial c} / \frac{\partial V^i}{\partial y} = 1 \) (\( i = 1, 2 \)) at equilibrium points \( \therefore \) no distortion in labor supply

Lump Sum Tax - shifts consumer budget line down (less income); still have \( MRS^i = 1 \) so there’s no distortion in labor supply

Assumptions - from here on out we’ll assume \( N_1 = N_2 \) (same number of each type of consumer) and \( R = 0 \) so tax is purely redistributive... makes graphs easier to interpret because the amount of transfer (shift in one consumer’s budget line above the 45° line) has to be offset by lump sum tax (shift in other consumer’s budget line below the 45° line)

"Allocation" - government assigns income & tax... hence consumption so consider (\( y, c \)) being assigned for the direct mechanism... just using this to design the tax schedule

Back to Stiglitz’s three cases:

1. **Neither Bounds** - \( \lambda_1 = \lambda_2 = 0 \); self selection constraints are satisfied; each allocation lies below the other type’s indifference curve \( \therefore \) for small amounts of redistribution there is no distortion

Marginal Tax Rate - \( 1 - MRS^i \) at \( i \)’s bundle

Lump Sum Tax - in this case the income tax schedules look like a lump sum tax; each type either gets benefit \( A_1 \) or tax \( A_2 \), both at 0% marginal tax rate

Result - marginal tax rate will be zero if IC constraints don’t bind

Tax Schedule - requirements:
(a) has to go through \( 1^* \) and \( 2^* \)
(b) has to be tangent to indifference curves at \( 1^* \) and \( 2^* \)
(c) has to stay below the indifference curves through \( 1^* \) and \( 2^* \) everywhere else

Not Unique - as long as these three criteria are met, the tax schedule can be anything

(2) **High Ability Binds** - \( \lambda_1 = 0 \ & \ \lambda_2 > 0 \); in first graph below, both consumers prefer \( 1^* \) so self selection constraint for type 2 is violated; we can’t move type 1’s indifference curve because of Pareto constraints (already at \( \bar{u}_1 \) ) \( \therefore \) we have to slide type 1’s allocation along this indifference curve... that requires extra revenue so we have to...
move type 2 to a lower indifference curve; to maximize \( R \) (i.e., shift 2 down the minimum amount) we want to keep the type 2 allocation with \( MRS = 1 \) (zero marginal tax rate)

**Tax Schedule** - not differentiable; has to be tangent to \( \bar{u}_1 \) from the left and tangent to \( \hat{u}_2 \) from the right

**Result** - at final allocation for type 1: \( MRS^2 > MRS^1 > 1 \) (i.e., marginal tax rate for type 1 is > 0%; there is labor distortion and type 1 works less than he should); at allocation for type 2: \( MRS^2 = 1 \) (zero marginal tax rate for type 2)

**Source of Distortions** -

Type 1 - \( V^1(c_1, y_1) \geq \bar{u}_1 \) and \( V^2(c_2, y_2) \geq V^2(c_1, y_1) \)

Want \((c_1, y_1)\) big for Pareto const. but small for 2's IC constraint... conflict

Type 2 - \( \max_{c_1, y_1, c_2, y_2} V^2(c_2, y_2) \) and \( V^2(c_2, y_2) \geq V^2(c_1, y_1) \)

Want to make both bigger... no conflict

**Problem** - once 2's allocation is fixed, we have to move \((c_1, y_1)\) on or below \( \hat{u}_2 \).
\[ V^2(c_2, y_2) \geq V^2(c_1, y_1) \] defines a non-convex set

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(3) **Low Ability Binds** - \( \lambda_1 > 0 \) & \( \lambda_2 = 0 \); in first graph below there's no problem; all we did was switch who's paying the tax and who's receiving the transfer (normally we'd think of the high wage paying to support the low wage, but that's not required... just play along); in the second graph, both consumers prefer 2* so self selection constraint for type 1 is violated; we're not going to move type 1's indifference curve again, instead we'll lower type 2's indifference curve to the point where it intersects type 1's at the budget line (i.e., no change in the lump sum transfer)
Tax Schedule - as before it's not differentiable; has to be tangent to $\bar{u}_2$ from the left and tangent to $\hat{u}_2$ from the right (same as before)

Result - at final allocation for type 2: $\text{MRS}^1 > \text{MRS}^2 > 1$ (i.e., marginal tax rate for type 2 is <0%; there is labor distortion and type 2 works more than he should); at allocation for type 1: $\text{MRS}^1 = 1$ (zero marginal tax for type 1)

Source of Distortions -
\[ \max_{c_1, y_1, c_2, y_2} V^2(c_2, y_2) \text{ and } V^2(c_2, y_2) \geq V^2(c_1, y_1) \text{ and } V^2(c_2, y_2) \geq V^2(c_1, y_1) \]

Want to make $(c_2, y_2)$ "smaller" so type 1 doesn't like it, but do so at min utility cost to type 2 because objective requires $(c_2, y_2)$ "bigger"

Stiglitz Summary - 2 goods (consumption & labor) and 2 types (high and low ability)

Type 2 Self Selection - $V^2(c_2, y_2) \geq V^2(c_1, y_1)$ ... if binding, adjust $(c_1, y_1)$ so $\text{MRS}^1 < 1$ (distort labor supply for type 1)

Type 1 Self Selection - $V^1(c_1, y_1) \geq V^1(c_2, y_2)$ ... if binding, adjust $(c_2, y_2)$ so $\text{MRS}^2 > 1$ (distort labor supply for type 2)

No Distortion - for small amounts of redistribution

Single Crossing - assumed indifference curves have this property; used this to derive fact that at most 1 self-selection constraint binds, which then implicitly suggested type paying greatest total tax is not envied by anyone

Utility Possibilities Frontier - asymmetric information causes distortions the further away from competitive equilibrium; may even have truncated UPF so some values are not attainable; full information case (self selection constraints irrelevant) is the dotted line
Continuous Ability Distribution -
Sadka - proposed theorems:
- (Mirrlees) If tax schedule is optimal under additive social welfare function, then it is nondecreasing (i.e., the marginal rate is non-negative)
- Zero marginal tax rate for top income person... look at graph; if top income person is at $y_{\text{max}}$ with a positive marginal rate, it's possible to get him to produce more income ($\hat{y}$), make him better off (higher utility), and keep government revenue constant by giving him a zero marginal tax rate ($\text{MRS} = 1$)

Limited Applicability - only works for the top income earner
Diamond - derived optimal income tax with continuous ability distribution; had high marginal tax rates at low and high incomes (near 91%)

Why High Rates for Low Income - taking away benefits intended for low income people as their income rises is equivalent to high tax rate; giving a minimum consumption level to zero income individuals means tax schedule has to be flatter than 45° (i.e., positive marginal tax rates) to get the tax schedule below the 45° line so revenue is generated to pay for the benefits

Low Tax Rate Problem - based on distribution of incomes, as income rises from low income the number of people increases (just like it increases when income falls from the top earners); problem with low marginal tax rates is that the benefits would have to be given to more people (unaffordable) and those people probably don't need the benefit anyway

High Tax Rate Problem - with high marginal tax rate as in graph on left, expect people to pile up at zero income; Earned Income Tax Credit (EITC) tries to solve this by putting a kink in the tax schedule so people have to work to draw the benefits

Finite Case - recall "high ability binds" from bottom of p.3; redistributing form high to low ability results in zero marginal tax for high income and distorts low income (i.e., marginal tax rate > 0); this is common result for redistribution

Self-Selection - high marginal rates on low income in this case results from enforcing the self selection constraint on high income individuals; government can use other means to help with self-selection that may cause additional distortions (e.g., "embarrass" those who take advantage of welfare... recall food stamp vs. food ATM card discussion)

Negative Income Tax - supported by Milton Friedman; wanted to integrate welfare system into income tax (like EITC) to avoid stigma of collecting welfare

Problem - "stigma" is good for self-selection; usually a small utility cost to those who need welfare, but more severe for someone who doesn't need it

No Single Crossing - consider Stiglitz’s model (pp. 2-5) over multiple periods: Will there be the same tax schedule across periods?

\[ V^1(c_2(1), y_2(1)) + \rho V^2(c_2(2), y_2(2)) \] (assuming additive separability; person 2’s utility in first period plus the discounted utility from the second period)

There is no analog to single crossing in this case so BHSS drop the assumption (similar to dropping assumption of quasiconcavity of preferences to use revealed preferences and exploiting linearity of budget constraint)

No Envy of Highest Tax Payer - consider graph; 2 good model (so we can just use two bundles to represent the tax schedule); single crossing does not apply; here both self-selection constraints bind using bundles 1 and 2, but there is a possible Pareto improvement: since type a is indifferent between 1 and 2 let him be at bundle 2 (results in more tax revenue); now we can make type b better off with 1* (no revenue cost to government)... so without single crossing constraint we get Stiglitz’s original (implicit) result: person paying highest tax cannot be envied by anyone paying lower tax ("weakly envied" means indifference)

Multiple Types - m classes with \( N_i \) people in each class \( (i = 1, \ldots, m) \); n goods (let good one good be leisure [negative leisure is labor supply] .: we’re looking at optimal tax combining both income and commodity taxes)

Net Trades - observe net trades, \( x^i \in R^n \) or each class

Utility - \( U^i(x^i) \); assume it’s continuous and strictly monotonic (Note: we’re not assuming quasiconcavity [equivalent of single crossing])

Producer Prices - \( p \); assume they’re constant (not require for the results, but makes the math easier)

Tax Function - \( T(x) \); doesn’t need to be differentiable since we have distinct types (it won’t be differentiable if any self selection constraints bind; see "Tax Schedule" remark at top of p.4)

Constraints -

(1) Individual Budget Constraint - \( p \cdot x^i + T(x^i) = 0 \) (binding from monotonicity of \( U^i \))

Linear Taxes - for optimal commodity tax, we assumed \( T(x) \) was linear so the budget constraint was \( q \cdot x = p \cdot x + t \cdot x = 0 \) ... we are not assuming \( T(x) \) is linear for what we’re about to do

Allowable Tax Function - \( T(x) \) is "allowable" (feasible) for each \( i = 1, \ldots, m \) if

\[ \exists x^i \in X^i \text{ s.t. } p \cdot x^i + T(x^i) \leq 0 \] (i.e., there exists a net trade vector in the set of feasible trades for class \( i \) such that the budget constraint it satisfied)

(2) Government’s Revenue Requirement - \[ \sum_{i=1}^{m} N_i T(x^i) \geq G \]

Mirrlees’ Trick - think of \( x^i \) not \( T(x^i) \)... we’ll pick bundles for consumers using their individual budget constraints so we sub \( p \cdot x = -T(x^i) \) into the government’s revenue constraints:

\[ G + \sum_{i=1}^{m} N_i (p \cdot x^i) \leq 0 \]

(3) Self selection - \( U^i(x^i) \geq U^j(x^j) \) \( \forall \ i, j \)
Utility Vector - there are lots of self selection constraints and we can eliminate most of them by defining the utility vector \( \mathbf{W} = (\omega_1, \omega_2, \ldots, \omega_m) \) for the net trade vector \( \mathbf{x} = (x^1, x^2, \ldots, x^n) \) where \( U^i(x^i) = \omega_i \); now self selection becomes:
\[
U^i(x^j) \leq \omega_i \quad \forall \quad j \neq i
\]

(4) Pareto Efficiency - no other \( \hat{\mathbf{W}}, \hat{\mathbf{x}} \) such that \( \hat{\omega}_j \geq \omega_j \quad \forall \quad j \) and \( \hat{\omega}_i > \omega_i \) for some \( i \)

(5) Production Efficiency - for now assume government is on its budget constraint (i.e., production efficiency); we'll prove this later

Min Producer Cost - set up problem to minimize producer cost of net trades:
\[
\min_{\mathbf{x}} \quad \sum_{i=1}^{m} N_i x^i
\]
\[
s.t. \quad U^i(x^i) = \omega_i \quad \forall \quad i \quad \text{Pareto}
\]
\[
U^i(x^j) \leq \omega_j \quad \forall \quad i, j \neq i \quad \text{Self Selection}
\]

Decomposes - each term in objective uses 1 type's bundle; same with each constraint so we can decompose this into \( m \) problems
\[
\min_{\mathbf{x}} \quad \mathbf{p} \cdot N_k x^k
\]
\[
s.t. \quad U^k(x^k) = \omega_k \quad \text{Pareto}
\]
\[
U^j(x^k) \leq \omega_j \quad \forall \quad j \neq k \quad \text{Self Selection}
\]

Simple Solution - if non of the self selection constraints is binding, the solution "follows directly" (pick \( x^k \) such that \( U^k(x^k) = \omega_k \))

Proposition 1 - "at a Pareto efficient allocation, a group must strictly prefer its own bundle to that of a group which pays a larger total tax": if \( T^b > T^a \) then \( U^i(\hat{\mathbf{x}}^a) > U^i(\hat{\mathbf{x}}^b) \)

Math - if \( \hat{\mathbf{x}} = (\hat{x}^1, \ldots, \hat{x}^m) \) is constrained Pareto efficient and \( U^i(\hat{\mathbf{x}}^i) = U^i(\hat{\mathbf{x}}^j) \) then
\[
-T(\hat{\mathbf{x}}^i) = \mathbf{p} \cdot \hat{\mathbf{x}}^i \leq \mathbf{p} \cdot \hat{\mathbf{x}}^j = -T(\hat{\mathbf{x}}^j)
\]

English - if \( i \) envies \( j \)'s bundle, \( j \)'s bundle must cost at least as much \( i \)'s (i.e., \( \mathbf{p} \cdot \hat{\mathbf{x}}^i \leq \mathbf{p} \cdot \hat{\mathbf{x}}^j \)); from self selection constraints, "envies" means type \( i \) is indifferent to \( j \)'s bundle: \( U^i(\hat{\mathbf{x}}^i) = U^i(\hat{\mathbf{x}}^j) \)

Proof: if \( j \)'s bundle cost less than \( i \)'s, then giving type \( i \) \( j \)'s bundle would be a Pareto improvement (same utility and spend less money) so the original allocation wouldn't be optimal

Another Way - \( \mathbf{p} \cdot \hat{\mathbf{x}}^i \leq \mathbf{p} \cdot \hat{\mathbf{x}}^j \Rightarrow T(\hat{\mathbf{x}}^i) \geq T(\hat{\mathbf{x}}^j) \).: if type \( i \) is indifferent to type \( j \)'s bundle, type \( i \) must pay at least as much in taxes otherwise the government could increase revenue by moving type \( i \) to type \( j \)'s bundle; recall form individual budget constraint that if type \( i \) pays less for the bundle, he pays for in taxes
**Pooling** - two or more types get the same bundle; two causes
   (i) Someone else envies the original bundles; (ii) all types assigned that bundle are undistorted (proposition 3)

- Can make both better off and collect more revenue using \( x^a \) & \( x^b \)
- \( \therefore \) if optimal solution is \( x^a = x^b \), the better bundles violate self selection for other types

**Self Selection Cycles** - type \( a \) is indifferent to \( b \)'s bundle; \( b \) is indifferent to \( c \)'s bundle and \( c \) is indifferent to \( a \)'s bundle: \( U^a(x^a) = U^a(x^b) \), \( U^b(x^b) = U^b(x^c) \), \( U^c(x^c) = U^c(x^a) \)

**Proposition 2** - in a Pareto efficient allocation, there are no self selection cycles (they are broken by pooling types that are indifferent [deleting the bundle that raises less revenue])

**Proposition 4** - in a Pareto efficient allocation, the group paying largest total tax is undistorted

**Proposition 5** - Note: this is Len's interpretation (probably wrong): in a Pareto efficient allocation if type \( a \) is allocated \( x^a \) and is distorted to meet type \( b \)'s self selection constraint
1. \( \text{MRS}^b(x^a) < \text{MRS}^a(x^a) < 1 \) (MRT) if \( x^a_i > x^b_i \) (i.e., \( a \) has less income)
2. \( \text{MRT} 1 < \text{MRS}^c(x^a) < \text{MRS}^b(x^a) \) if \( x^a_i < x^b_i \) (i.e., \( a \) has more income)
where \( x^i_i < 0 \) is labor supply (negative leisure)

Same result at Stiglitz's original model (see summary on p.5)

**Randomization** - one type gets different bundles (choose among lotteries over bundles); this pops up in incentive problems

**Stiglitz** - 1982 paper was original to address this; used 50-50 lottery

**Reason** - self selection constraints define a nonconvex set (see p.4)

**Special Case** - bundles in lottery only vary in consumption (same income); example would be chance of getting audited after filing income taxes (income hasn't changed, but consumption can)

**General Case** - suppose \( V^b(c_b, y_b) = V^b(c_a, y_a) \) (type \( b \) is indifferent between \( b \)'s allocation or \( a \)'s allocation)

**Lottery** - anyone who says he's type \( a \) faces 2 bundles: \( (c_L, y_L) \) with probability \( \hat{p} \) or \( (c_H, y_H) \) with probability \( 1 - \hat{p} \); both points lie on type \( a \)'s indifference curve so \( V^a(c_a, y_a) = V^a(c_L, y_L) = V^a(c_H, y_H) \)

\( \therefore \) probability is irrelevant to type \( a \) (but it does matter to get expected taxes)

**Effect on Self Selection** - \( V^b(c_b, y_b) \geq \hat{p}V^b(c_L, y_L) + (1 - \hat{p})V^b(c_H, y_H) \)
if type \( b \) is risk averse, he'll report the truth to avoid the lottery

**When to Use** - FOCs don't reveal anything; "proof is tedious... goes on for several pages"

Define \( H^i(c_j, y_j) = \begin{bmatrix} V_{cc}^i & V_{cy}^i \\ V_{yc}^i & V_{yy}^i \end{bmatrix} \) (evaluated at \((c_j, y_j)\); where \( V_{ci}^j = \frac{\partial V^i}{\partial c} \) & \( V_{cy}^i = \frac{\partial^2 V^i}{\partial c \partial y} \)

Randomization is desirable if \( \frac{H^b(c_b, y_b)}{V^b \left( 1 - \text{MRS}^b \right)} - \frac{H^a(c_b, y_b)}{V^a \left( 1 - \text{MRS}^a \right)} \) is not negative semidefinite
English -
1) probabilities don't matter because we're evaluating at deterministic bundle $(c_j, y_j)$
2) this weird matrix is basically subtracting b's risk aversion from a's; by saying that the matrix is not negative semidefinite, we're saying b must be more risk averse than a
Note: risk aversion for $U(c)$ is $-U_{cc}/U_c$

Example - think of $U^a(x)$ and $U^b(x)$ and lottery of $\bar{x}$ and $x$; transfer of $\delta$ from b to a
and want type b to consumer $x^*$

- a's self selection: $pU^a(x+\delta) + (1-p)U^a(\bar{x}+\delta) > U^a(x^*-\delta)$
- b’s self selection: $U^b(x^* - \delta) > pU^b(x+\delta) + (1-p)U^b(\bar{x}+\delta)$

Is the lottery worth using? Yes, if $\frac{2U^a_x}{U^b_x} - \frac{U^a_y}{U^b_y} > 0$
(i.e., b is twice as risk averse as a)

(Hamilton didn’t really go into how to derive this)

Dynamic Taxation - Brito, Hamilton, Slutsky & Stiglitz; government can link periods so instead of using $T_i(y, \cdot)$ tax can be set up as $T_i(y, y_{i-1}, \ldots, y_1)$

No Commitment - government can change tax each period; supposed government uses self-selection constraints in first period to identify low and high income (ability) workers; after first period government will try to change the tax system to get as close to a lump sum tax on ability as possible

Roberts - (RESTud, 1984) looked at infinite horizon; result is people won’t separate in first period; worse outcome in terms of welfare than imposing self-selection constraints (can’t do any redistribution)

Too Strong? - if we embed overlapping generations (new generation entering labor force each period), government can’t tax high ability more or new generations will all pool at low ability

Full Commitment - government can charge different tax in each period, but once announced it can’t change it

Discount Factor - assume government, high ability and low ability types have same discount factor $\rho$

Weighted Utilitarian SWF - $\max_{c_i, y_i, c_j, y_j} \sum_{i=1}^{n} \rho^{i-1} [\alpha V^a(c^i_1, y^i_1) + (1-\alpha)V^b(c^i_1, y^i_1)]$

Self Selection - suppose government imposes large punishment for switching types, then we only have to worry about a single self selection constraint for each type:
$\sum_{i=1}^{n} \rho^{i-1} [V^i(c^i_1, y^i_1) - V^i(c^i_j, y^i_j)] \geq 0 \quad (i = a, b, \ j \neq i)$

Budget Constraint - $\sum_{i=1}^{n} \rho^{i-1} [c^a_i - y^a_i + c^b_i - y^b_i] \leq 0$

Stationary Solution - repeat one period outcome
Nonstationary Solutions - analogous to randomization with fixed probabilities determined by $\rho$
Taxing Commodities and Income

Expand consumption to vector: \( V^1(c_1, y_1) \) and \( V^2(c_2, y_2) \) where \( c_i = (c_{i1}, c_{i2}, \ldots, c_{in}) \)

Measure consumer prices in terms of producer prices so \( p_k = 1 \)

Pareto Problem - assuming we’re transferring from type 2 to type 1

\[
\begin{align*}
\text{max } & V^2 \\
\text{s.t. } & V^1 \geq \bar{u}_i \\
& V^2(c_2, y_2) \geq V^2(c_1, y_1) \\
& (\lambda_2) \text{ self selection constraint} \\
& N_1y_1 + N_2y_2 - \sum_{k=1}^{n}(N_1c_{1k} + N_2c_{2k}) \geq \bar{R} \\
& (\gamma) \text{ revenue constraint}
\end{align*}
\]

Weak Separability - \( U(c, L) \) is separable in \( c \) & \( L \)

\[
\frac{\partial^2 U^i}{\partial c_i \partial L_j} = 0 \quad \forall \quad i \quad \text{(types)} \quad \& \quad j \quad \text{(goods)} \Rightarrow \frac{\partial V^1}{\partial c_{ij}} = \frac{\partial V^2}{\partial c_{2j}}
\]

Type 2 - completely undistorted

Labor-Commodities - \( \frac{\partial V^2}{\partial c_{2j}} = \text{MRS}_{c,y} = 1 \) (no labor to commodity distortion)

Commodities - \( \frac{\partial V^2}{\partial c_{2j}} = \text{MRS}_{c,y} = 1 \) (no distortion among commodities)

Type 1 - undistorted among commodities, but will be distorted for labor

Commodities - \( \frac{\partial V^2}{\partial c_{2j}} = \text{MRS}_{c,y} = \frac{N_1\gamma + \lambda_2 \frac{\partial V^2}{\partial c_{1j}}}{N_1\gamma + \lambda_2 \frac{\partial V^2}{\partial c_{1k}}} = 1 \)

Atkinson & Stiglitz - (1972) same result for continuous case: don’t tax commodities if \( U \) is weakly separable

* this is opposite scenario as Cortilet-Hague

Deaton - (1979) don’t tax commodities with optimal linear income tax if

1. preferences are weakly separable in commodities and leisure
2. parallel, linear Engel curves for all goods (don’t need this for nonlinear income tax)

Christensen - optimal nonlinear income tax and linear taxes on commodities... used unique definition of substitutes and complements

Broadway & Keen - income tax with public goods; move away from Samuelson rule because public goods impact self selection:

\( V^2(c_2, y_2, g) \geq V^2(c_1, y_1, g) \)

Interactions - have to worry about interactions between \( y \) (income) and \( g \) (public good)

Example: spend money on bike trails increases MU of leisure so increasing \( g \) makes it more likely to report low ability (low income)

** Makes self selection constraint even more significant (i.e., more distortion)

Type Based Exclusions - for “public goods” that aren’t (i.e., aren’t non-excludable), self selection can be relaxed by using type based exclusions

Example: give away opera tickets to high ability types; idea is that low ability types who want opera tickets are really high ability misreporting