Commodity Tax Reform

Narrow Problem - when lump-sum taxes are infeasible, what set of commodity taxes raises a target level of revenue and leaves consumers as well off as possible

Fixed Revenue - fixed for now; later we'll reverse the problem and consider how much revenue can be raise for a given level of inefficiency (which we'll use to determine if a public good should be funded... have to account for inefficiency that results from tax)

More Generally - study optimal commodity taxation has other benefits:
(1) Direct vs. Indirect Taxation
   Direct Taxes - based on property, wealth, income, factor income
   Benefits - (i) can tax different people at different rates; (ii) factors are inelastically supplied while indirect taxes tend to cause consumption distortions
   Labor-Leisure - some argue labor supply isn't inelastic when factoring labor/leisure choice instead of just labor supply
   Indirect Taxes - based on business transactions (e.g., sales tax, excise tax, tariffs)

History - federal government moved from indirect to direct (mainly through 16th Amendment which legalized income tax with redistribution)
Hamilton: "not a lot to be gained in distinction of direct vs. indirect"... just history

(2) Value-of-Service Pricing
   Railroad Rates - Interstate Commerce Commission established in 1880s to regulate railroad rates; farmers pushed for regulation to lower shipping rates for farm output; R/R have high fixed cost which need to be allocated so rates exceed marginal cost; ICC decided to discriminate over commodities and keep rates low for commodities with highly elastic demand (i.e., not farm output or coal)

(3) General Theory of Second Best (Lipsey & Lancaster, 1957)
   Said we shouldn't try to counter distortions if they can't be removed

(4) Regulated Utility Pricing - Boitteux
   How do you want to set prices as a multiproduct monopolist; Boitteux worked for France's state-owned utility company looking at different types of customers

Formal Treatment - of optimal commodity tax
   Ramsey, 1927
   Samuelson, 1951 (unpublished)
   Boitteux, 1956 (in French)
   Diamond & Mirrlees, 1971 (explicit general equilibrium framework)... what we'll study
   Stiglitz & Dasgupta, 1971
   Baumol & Bradford, 1970 (specialized to utility pricing)

Tax Reform - Dixit (1975 & 1977); follows theory of 2nd best; what can we do to get local improvement... use duality, but requires lots of conditions so in real world it's hard to guarantee if change will be an improvement

Commodities - \( n + 1 \) of them
   Good Zero - numeraire; think of it as minus labor supply; \( p_0 = 1, \ q_0 = 1, \ t_0 = 0 \)
   (untaxed)... "normalization" (explained in Diamond-Mirrlees)

Commodity Vector - \( (x_0, x) = (x_0, x_1, x_2, \ldots, x_n) \)

Consumer Prices - \( q = (q_1, \ldots, q_n) \)

Producer Prices - \( p = (p_1, \ldots, p_n) \)

Taxes - \( t = q - p \) (difference between what consumers pay and what producers receive
**Production Cost** - \( C(x) \) = numeraire cost of producing \( x \) (think of all output using only labor, \( x_0 \))

**Marginal Cost** - \( C_x = \left( \frac{\partial C}{\partial x_1}, \frac{\partial C}{\partial x_2}, \ldots, \frac{\partial C}{\partial x_n} \right) \)

\( C_{xx} = \left( \frac{\partial^2 C}{\partial x_i \partial x_j} \right) \), \( n \times n \) matrix; assume positive semidefinite (allows fixed cost, but otherwise convex technology)

**Single Consumer** - can think of it as many identical consumers
\( E(1, q, u) = \) minimum expenditure at prices \( (1, q) \) to get utility \( u \)

**Compensated Demands** -
\[
x_0 = E_0(1, q, u) = \frac{\partial E}{\partial q_0} \quad \text{(don't forget } q = \text{ after-tax prices the consumer pays)}
\]

\[
x = E_q(1, q, u) = \begin{bmatrix} \frac{\partial E}{\partial q} \end{bmatrix}
\]

Assume \( E_{qq} \) is negative definite (rules out kinked demand curves)

**Homogeneity of Degree Zero** - \( E_{i0} + \sum_{j=1}^{n} q_j E_{q_j} = 0 \), \( i = 0, 1, \ldots n \)

Note: \( E_{i0} = \frac{\partial^2 E}{\partial q_i \partial q_0} = \frac{\partial x_i^c}{\partial q_0} = \frac{\partial x_0^c}{\partial q_i} \)

Vector notation: \( E_{q0} + q \cdot E_{qq} = 0 \) and \( E_{q0} + q \cdot E_{qq} = 0 \)

**Endowment** - of numeraire is \( Z \)

**Government** - collects commodity taxes and a lump-sum tax to raise amount of \( G \) of numeraire good

**Budget Constraint** - \( t'x + T = G \) (commodity tax revenue + lump-sum revenue = \( G \))

**Producer Profit** - \( P = p'x - C(x) \)
Distributed as lump-sum to consumer

**Consumer Budget** - \( Z - T + P = E(1, q, u) \)

Sub in gov't budget constraint: \( Z - (G - t'x) + P = E(1, q, u) \)

Sub in producer profit: \( Z - G + t'x + (p'x - C(x)) = E(1, q, u) \)

Solve for \( Z \) : \( Z = G - t'x + p'x + C(x) + E(1, q, u) \)

Sub \( t'x = (q - p)'x \) (cancels \( -p'x \) : \( Z = G - q'x + C(x) + E(1, q, u) \))

Sub \( x = E_q(1, q, u) : Z = G + E(1, q, u) - q' E_q(1, q, u) + C(E_q(1, q, u)) \)

Sub \( E_0 = E(1, q, u) - q' E_q(1, q, u) : Z = G + E_0 + C(E_q(1, q, u)) \)

**English** - consumer endowment of labor \( (T) = \) government spending \( (G) \) + amount kept by consumer (leisure; \( E_0 \)) + amount consumed in production \( (C(E_q(1, q, u))) \)

Totally differentiate (first box above): \( T \) and \( G \) are constant

\[
0 = E_q q' dq - dq' E_q dq + C_x q' E_{qq} dq + (E_0_q + C_x q' E_{qq}) du
\]

Combine terms: \( 0 = E_q q' dq - dq' E_q + (C_x q - q' E_{qq}) dq + (E_0_q + C_x q' E_{qq}) du \)
Note $E_q' dq = dq' E_q = 0 = (C_x - q)' E_{qq} dq + (E_{0u} + C_x' E_{qu}) du$

Solve for $du$: $du = \frac{(q - C_x)' E_{qq} dq}{E_{0u} + C_x' E_{qu}} = \frac{(1 \times n)(n \times n)(n \times 1)}{1}$ = scalar

Know $E_{0u} + q' E_{qu} = \frac{\partial E}{\partial u} > 0$ (resource cost at consumer prices increases as utility increases) \(\therefore\) assume $E_{0u} + C_x' E_{qu} > 0$ (i.e., resource cost at producer prices increases as utility increases)

\(\therefore\) $du$ has the same sign as $(q - C_x)' E_{qq} dq$

**What Are We Doing?** - looking at changing consumer prices (rather than tax; actually same as controlling tax with fixed producer prices) and seeing whether consumers are better off ($du > 0$) or worse off ($du < 0$); assume we adjust the lump-sum tax to maintain government revenue so consumer utility is not affected by changes government purchases

**Better Off** - want to find sufficient conditions to guarantee $(q - C_x)' E_{qq} dq > 0$ (so we have $du > 0$... consumers are better off)

**Reduce Tax (Price)** - reducing tax on good $i$ means $dq_i < 0$

\[ n = 2 \text{ Case} \quad (q - C_x)' E_{qq} dq = \left[ \begin{array}{c} q_1 - c_1 \\ q_2 - c_2 \\ q_1 - c_1 \\ q_2 - c_2 \end{array} \right]^T \left[ \begin{array}{cc} E_{11} & E_{12} \\ E_{21} & E_{22} \end{array} \right] \left[ \begin{array}{c} dq_1 \\ dq_2 \\ dq_1 \\ dq_2 \end{array} \right] = \]

\[ = (q_1 - c_1)E_{11} dq_1 + (q_1 - c_1)E_{12} dq_2 + (q_2 - c_2)E_{21} dq_1 + (q_2 - c_2)E_{22} dq_2 \]

**Tax One Good** - suppose $q_2 = c_2$ (i.e., consumer price for good 2 equals marginal cost; that implies price taking on producer side and no taxes on good 2); this means $dq_2 = 0$ and $q_2 - c_2 = 0$ so the expression above simplifies to one term:

$(q_1 - c_1)E_{11} dq_1$

$q_1 - c_1 > 0$ (since good 1 is taxed, the consumer price must be greater than MC)

$E_{11} < 0$ (because we assumed $E_{qq}$ is negative definite)

That means if $dq_1 < 0 \Rightarrow du > 0$

**English** - if only one good is taxed, lowering the tax (i.e., price) on that good toward MC, then consumers are better off; makes sense and is the same result form the partial equilibrium set up (i.e., want price = MC)

**Tax Both Goods** - Now we have $q_1 - c_1 > 0$ and $q_2 - c_2 > 0$; assume we hold price of good 2 constant (i.e., $dq_2 = 0$; don't change the tax); now the expression above simplifies to two terms

$(q_1 - c_1)E_{11} dq_1 + (q_2 - c_2)E_{21} dq_1$

If we lower the price of good 1 ($dq_1 < 0$), we know from previous case that the first term will be negative

The second case is ambiguous:
\[ q_2 - c_2 > 0 \text{ and } dq_i < 0, \text{ but } E_{21} \text{ can go either way} \]

**Another way** to look at it is to factor \( dq_i \):

\[
(q - C_x)'E_{qq}dq = [(q_1 - c_1)E_{11} + (q_2 - c_2)E_{21}]dq_i
\]

We now the first term in brackets is negative and \( dq_i \) is negative; if the second term is sufficiently positive, we could end up with the expression overall being negative (i.e., if \( E_{21} > 0 \), \( dq_i < 0 \Rightarrow du < 0 \))

\[ \Rightarrow \text{ lowering the price of good one may or may not make consumers better off} \]

**Contradiction?** - that sounds weird, but what's happening when we lower the price of good 1 there's a shift in the compensated demand for good 2; if \( E_{21} \) is positive, the shift is to the right (or up) which increases excess burden for good 2 while lowering \( q_1 \) decreases excess burden for good 1

**Yet Another way** to look at it is to recognize \( E_{qq}dq = dx^c \) so

\[
(q - C_x)'E_{qq}dq = (q - C_x)'dx^c, \text{ which means the sign of } du \text{ depends on how compensated demands change}
\]

**Propositions** - these are conditions to guarantee \( du > 0 \) (key is looking for patterns in the "distortion" (i.e., tax or price change: \( q - C_x \)) or in \( E_{qq} \)

1. **Proportional Change** - If consumer prices move toward marginal cost in proportion to existing distortions and the lump-sum tax changes to hold revenue constant then welfare rises

   The proportional change means \( dq = -(q - C_x)dh \), where \( dh \) is a scalar (measures the length of vector in graph; distance traveled by price change) so now the sign of \( du \) is the same as the sign of

   \[
   (q - C_x)'E_{qq}dq = -(q - C_x)'E_{qq}(q - C_x)dh
   \]

   Symmetric quadratic form \( E_{qq} \) is neg. def. so this product is < 0

   We have \((-)(-)(+)\) so \( du > 0 \)

2. **Corlett-Hague Theorem** - holding revenue constant through commodity taxes (i.e., \( (q - C_x)'x = R \); they didn't use lump-sum tax like Dixit did; that makes this more general and realistic than previous proposition), if prices are initially above MC by same factor for all goods (i.e., \( q_i - c_i = \beta c_i \) [proportional to \( c_i \)], welfare increases with small increases in prices of commodities complementary to the numeraire and small decreases in prices

   **Proposition**
   - Wherever we start, we move toward \((c_1, c_2)\)
   - Initially on ray thru origin and \((c_1, c_2)\)
   - Move in either direction, but can't go far ("small changes") because it moves off proportional line and theorem no longer holds
of commodities that are substitutes for numeraire; so sign of \( du \) is the same as the sign of 
\[(q - C_x)' E_{qq} dq = -\beta q' E_{qq} dq \quad \text{(scalar)(1x)(n)(n)(n)(1)}\]

Apply homogeneity of degree zero assumption: \( E_{00} + q'E_{0q} = 0 \) and \( E_{q0} + q'E_{qq} = 0 \)

\[-\beta q' E_{qq} dq = \beta E_{q0}' dq\]

Sign of this term depends on sign of \( E_{q0}' dq \) which is taken care of in the assumption:

- \( E_{i0} > 0 \) (substitutes) \( \Rightarrow dq_i < 0 \)
- \( E_{i0} < 0 \) (complements) \( \Rightarrow dq_i > 0 \)

\[\therefore \beta E_{q0}' dq > 0 \text{ so } du > 0\]

**Improvement** - although this is restrictive in that we have to start with proportional distortions, we don’t have to use a proportional tax (as long as there are substitutes and complements to the numeraire)

(3) **General** - given an arbitrary initial price vector; lowering \( q_j \) toward its marginal cost increases welfare if...

(a) good \( j \) is complementary to all goods with greater proportional distortions
(b) good \( j \) is substitute for all other goods (including numeraire)
(c) we adjust lump-sum taxes to hold revenue constant

Proof: recall sign of \( du \) will be same as sign of \( (q - C_x)' E_{qq} dq \)

Label proportion of distortions with \( \beta_j = \frac{q_j - c_j}{q_j} \)

(similar to Corlett-Hague Thm except each good can have a different distortion)

Sort commodities by distortion so \( \beta_1 \leq \beta_2 \leq \ldots \leq \beta_j \leq \ldots \leq \beta_n \)

We are only changing the price of good \( j \) so \( dq_j < 0 \) and \( dq_k = 0 \) for \( k \neq j \)

That means \( (q - C_x)' E_{qq} dq \) becomes \( \sum_{i=1}^{n} (q_i - c_i) E_{ij} dq_j = \sum_{i=1}^{n} \beta_i q_i E_{ij} dq_j \)

Use homogeneity of degree 0 again: \( E_{j0} + \sum_{i=1}^{n} q_i E_{ij} = 0 \)

We can rewrite this as \( \beta_j \left( E_{j0} + \sum_{i=1}^{n} q_i E_{ij} \right) dq_j = 0 \)

Now subtract it from \( \sum_{i=1}^{n} \beta_i q_i E_{ij} dq_j \)

\[\sum_{i=1}^{n} \beta_i q_i E_{ij} dq_j - \beta_j \left( E_{j0} + \sum_{i=1}^{n} q_i E_{ij} \right) dq_j = \]

\[\sum_{i=1}^{n} \left( \beta_i - \beta_j \right) q_i E_{ij} dq_j \]

To sign this, remember \( dq_j < 0 \); (a) and (b) take care of the rest

- \( E_{ij} > 0 \) (substitutes) \( \Rightarrow \beta_i - \beta_j \leq 0 \) (this include numeraire so \( E_{j0} > 0 \))
- \( E_{ij} < 0 \) (complements) \( \Rightarrow \beta_i - \beta_j \geq 0 \)

We have (-)(-) so \( du > 0 \)
**Good with Largest Distortion** - special case where \( j = n \); lowering the price lessens excess burden if good \( n \) is a substitute for everything... that's a strong condition; if there are any complementary relationships, we can't guarantee lowering price would lower excess burden (it can, we just can't guarantee it)

**Summary** - common 2nd best result: reducing one distortion isn't obviously good (i.e., not clearly an improvement) when other distortions exist

Hamilton: "In the case of multiple distortions, tread carefully with your intuition."

If we know compensated demand derivatives (\( E_{qq} \)) we can get clearer results