Review

Public Economics -
   **Broad** - study of government in modern economy; Hamilton: "a little bit like general surgery;" every decade or so, a new organ gets taken over by a specialist
   **Narrower** - what can/should government do to improve static and dynamic performance of the economy

Macroeconomics - studies tax and expenditure policies; this is NOT what we'll do in public economics

Our Focus - (1) optimal commodity taxation (2nd best); (2) optimal income taxation (classic asymmetric information problem)

Divisions - Richard Musgrave (1950s) split field by branches of government
   **Allocation** - what gets produced and consumed? Direct government action (taxation; public production); government influences on private choices (regulation)
      Note: this doesn't correspond to traditional breakdown of tax and expenditure
   **Distribution** - government influences on who consumes how much (tax policy)
   **Stabilization** - dealt with in macro

Private Enterprise Economies - our focus for this course; government doesn't directly enter production; focus on government setting rules and providing incentives (or disincentives); Europeans are more concerned with public enterprise

Fundamental Theorems -
   **1st FTWE** - a competitive equilibrium is Pareto optimal
   **2nd FTWE** - any Pareto optimal allocation can be sustained as a competitive equilibrium after redistribution of endowments
      Hamilton: "if the conditions of these theorems applied most of the time, we wouldn't be teaching this course"

Failures - monopoly, externalities, public goods
   **Solution** - government intervention may restore efficiency or improve it (2nd Best - can't always get back to Pareto optimal outcome, but can maximize efficiency given some restrictions)

Roadmap - this is what we'll cover in this course
   **Welfare Economics** - welfare functions; interpersonal comparisons; public goods (sum of MRS = MRT)
   **Excess Burden Analysis** - measuring amount of inefficiency; doing dead weight loss correctly (done differently for IO)
   **Commodity Tax Reform** - local improvements from inefficiency allocations
   **Optimal Commodity Taxation** - full blown general equilibrium 2nd best model; answer tells a lot about monopoly regulation and overhead allocation for public enterprise; "how to collect fixed amount of revenue at minimum efficiency cost"
   **Optimal Income Taxation** - classic adverse selection problem; redistribution
   **If Time** - integrate optimal taxation and public goods (changes Samuelson rule); optimal taxation with uncertainty
Welfare Economics

Consumer Theory - studied information revealed by individual choices to derive complete theory of demand; done with ordinal (not cardinal) preferences so it wasn't comparable across individuals

Loosen Assumptions - 1880s to 1960s, economists removed assumptions and got same results

Revealed Preference - worry about consistency in choice; don't need to worry about type or form of utility functions

Welfare Economics - want to talk about good and bad allocations; dealing with "social preferences"

Allocation - complete description of economy; "enough information for everyone concerned"

Limitations to FTWEs -

No Externalities - individual preferences can only depend on own consumption (i.e., Len doesn't care what Josh consumes and vice versa); people are "selfish"; sounds negative, but also means there's no envy;

No Public Goods - no "shared consumption"

Individual Preferences - basic definitions

Weak Preference - \( x R_i y \) ... person \( i \) ranks allocation \( x \) at least as good as \( y \)

Complete - \( x R_i y \) or \( y R_i x \) or both

Transitive - \( x R_i y \) and \( y R_i z \) \( \Rightarrow x R_i z \)

Strict Preference - \( x P_i y \equiv x R_i y \) and Not \( y R_i x \)

Indifference - \( x I_i y \equiv x R_i y \) and \( y R_i x \)

Social Preferences - need to come up with Pareto criterion; we'll assume individual preferences count and are complete and transitive

Pareto Superior - \( x P y \) if \( x R_i y \forall i \) and \( x P_i y \) for at least some \( i \)

Can also say \( x \) Pareto dominates \( y \); "unanimity rule" (if there is one person who does not at least weakly prefer \( x \) to \( y \), then \( x \) isn't Pareto superior to \( y \))

Pareto Optimal - allocation \( x \) is Pareto optimal (efficient) if there does not exist an allocation \( z \) such that \( z P x \)

Problem - PO criterion isn't very useful because there's no way to choose among PO allocations

Revealed Preference - look at data on aggregate consumption of two goods and price ratio when the choice is made; look at graph

A > B A has more of both goods so this is obvious (budget line when B is chosen is irrelevant)

D > A A is available when D is chosen and is not on the budget line

A ≥ E or E ≥ A Neither is available when the other is chosen so we can't compare them

A & F Inconsistent; can't draw tangent indifference curves through F and A without them intersecting

(from ECO 7115 notes)

Weak Axiom of Revealed Preference (WARP) - proposed by Samuelson who argued this is equivalent to preferences satisfying standard properties; can't be done (need transitivity): If \( x \) is chosen under prices \( P' \) and \( P'x \geq P'y \) (i.e., \( y \) is in the budget set) and if \( y \) is chosen under prices \( P'' \), then \( P''y < P''x \)
**Strong Axiom of Revealed Preference (SARP)** - proposed by Houthakker; assumed transitivity holds which allows us to find all properties of preferences

**Aggregate Consumption** - there are too many possibilities; WARP doesn't work with redistribution

**Utilitarianism** - $W = u_1 + u_2$; assumes identical preferences and fixed incomes

**New Welfare Economics** - developed in 1930s; no cardinal utility or interpersonal comparisons, but still have Pareto criterion; trying to compare allocations to determine which is best

**Notation** - $x$ and $y$ are allocations that define "social states" (decisions we might make; e.g., build bridge type $x$ or build bridge type $y$)

- $S(x)$ is the set of allocations accessible from allocation $x$ by redistribution
- $S(y)$ is a different set of allocations accessible from allocation $y$

**Pareto Optimal** - $\exists y \in S(x)$ such that $y \in S(x)$ and $x$ is on the frontier of $S(x)$

**Kaldor Superior** - $x \in S(y)$ means $\exists z \in S(x)$ such that $z \in S(y)$ (i.e., there is an allocation available by redistribution from $x$ that Pareto dominates allocation $y$)

**Makes Sense?** $x \in S(y)$ seems reasonable since moving from $y$ to $x$ has the potential to make everyone better off (by redistributing to get to allocation $z$)

**Problems** - (1) compensation technically doesn't have to take place

- (2) can have $x \in S(y)$ and $y \in S(x)$

Hamilton: this should have your "bullshit meter quivering"

**Scitovsky's "Patch Job"** - $b \in S(a)$ means $b \in S(a)$ and not $a \in S(b)$

**Problems** - (1) relation can change based on endowment (start point);

- (2) still have compensation problem

**Theorem** - if $S(x) = S(y)$ then $x \in S(y)$ iff $x$ is Pareto optimal and $y$ is not

**Proof**:

- (a) $S(x) \Rightarrow x$ is Pareto optimal and $y$ is not

- $x \in S(y)$ means $x \in S(y)$ and not $y \in S(x)$

- $x \in S(y)$ means $\exists z \in S(x) = S(y)$ such that $z \in S(y)$

- $y$ is not Pareto optimal

Suppose $x$ is not Pareto optimal

That means $\exists w \in S(x) = S(y)$ such that $w \in S(x)$ which means $y \in S(x)$, but that

contradicts $x \in S(y)$ so $x$ is Pareto optimal

(b) $x$ is Pareto optimal and $y$ is not $\Rightarrow x \in S(y)$

- $x$ is Pareto optimal means $\exists z \in S(x) = S(y)$ such that $z \in S(x)$

That means not $y \in S(x)$

- $y$ is not Pareto optimal means $\exists w \in S(x) = S(y)$ such that $w \in S(y)$

That means $x \in S(y)$

Since $x \in S(y)$ and not $y \in S(x)$, we have $x \in S(y)$

**Problem?** - this theorem seems bogus; $S(x) = S(y)$ and $x \in S(y)$ cannot be possible; both $x$ and $y$ would have to be on the frontier in order for $S(x) = S(y)$, but that would make both $x$ and $y$ Pareto optimal... after talking to Prof Hamilton, he said defining the feasible set ($S(x)$) doesn't mean $x$ is on the frontier of the set
**Theorem** - if \( S(x) = S(y) \) then \( x \not\sim y \) ⇒ \( y \) is not Pareto optimal

**Problem** - Kaldor and Scitovsky were well intentioned attempts to push things further (i.e., make more comparisons not available with just Pareto optimal criterion), but they don't work

**Samuelson Superior** - \( x \not\sim y \) means \( \forall z \in S(y), \ x \not\sim z \) (i.e., \( S(y) \subset S(x) \)); \( S(y) \) is completely contained in \( S(x) \); any allocaiton that's possible from endowment \( y \) is possible from endowment \( x \)

**Problems** - (1) if \( S(x) = S(y) \), you can't have Samuelson superiority

(2) Still have compensation problem... but if the compensation is actually paid, Samuelson superiority guarantees that \( S(x) \) has a better allocation than everything available with endowment \( y \) (i.e., can beat everything on the frontier of \( S(y) \))

**Revealed Preference** - want to use aggregate data (revealed preference) to draw conclusions about social welfare; some forms work, but in general we can't do this

**Community Indifference Curves** - proposed by Scitovsky; curves satisfy three conditions

(1) based on two goods: \( x \) and \( y \); and two individuals: 1 and 2

(2) consume all goods - total amount of each good available is consumed between all the individuals: \( x = x^1 + x^2 \) and \( y = y^1 + y^2 \)

**Constant Utility** - each individual's utility level is held constant (although they could be different from each other): \( u^1(x^1, y^1) = m^1 \) and \( u^2(x^2, y^2) = m^2 \)

**Same MRS** - the marginal rate of substitution between the goods is the same for all individuals: \( MRS^1 = MRS^2 \)

(From ECO 7120 notes) \( MRS^1 = \frac{\partial u^1}{\partial x^1} / \frac{\partial u^1}{\partial y^1} \)

\( MRS^2 = \frac{\partial u^2}{\partial x^2} / \frac{\partial u^2}{\partial y^2} \)

MRS is the amount of good \( x \) a person has to give up when gaining a specified amount of good \( y \) in order to keep his utility constant

**Community Indifference Curve (CIC)** - level curves defined by \( m^1 \) and \( m^2 \)

(and plotted in commodity space)

**Minimum Total Requirements Curve** - Samuelson's name for the same thing, but he defined it a little differently:

\[
\max_{x^1, x^2, y^1, y^2} \quad u^2(x^2, y^2) \\
\text{s.t.} \quad u^1(x^1, y^1) = m^1, \quad x = x^1 + x^2, \quad y = y^1 + y^2
\]

(This will result in \( MRS^1 = MRS^2 \) and gets to Pareto frontier of \( m^1, m^2 \) utility)

**Income Distribution** - changes in income distribution changes the MRS at all points so the CIC is only useful for a given income distribution... this means we can't use CID for revealed preference

**Trade** - problem with CIC; we can't tell if a new allocation achieved by trade is an improvement because of income distribution problem

**Social Welfare Function**

**Samuelson** - said the "new welfare economics" had gone too far

**Family Analogy** - think of maximizing family welfare; think of family where all consumption is individual consumption; allocation of income isn't independent of prices (in effect, income distribution cushions price shocks that affect one member more than others to equalize their marginal utility of consumption)
Bergson-Samuelson Social Welfare Function (SWF) - \( W(u^1(x^1), u^2(x^2), u^3(x^3)) \)

Notes: (1) \( x^i \) is commodity vector consumed by individual \( i \)
(2) utilities depend only on own consumption

Paretian - \( \frac{\partial W}{\partial u^i} > 0 \) (if anybody's utility increases, social welfare increases)

Individualistic - \( \frac{\partial W}{\partial x^i_j} = \frac{\partial W}{\partial u^i} \frac{\partial u^i}{\partial x^i_j} \) (change in SWF based on individual \( i \)'s consumption of good \( j \) is only based on how \( i \)'s utility changes and how that changes SWF... doesn't impact other individuals)

Graph - SWF defines indifference curves plotted in utility space

Problems -
- Violates "new welfare economics" goals:
  - Interpersonal comparisons added back in
  - Added cardinal utility to "scale" individual utilities
- Excludes some goals (when we max SWF s.t. resource constraints)
  - Individualism excludes merit goods/paternalism (e.g., parents encourage kids to do "what's good for them"; or discourage "what's bad")
  - Rules out specific egalitarianism - doesn't consider whether individuals get certain minimum amount (e.g., food, shelter, healthcare)
  - Thrown out minimalist views of government - maximizing SWF requires redistribution

Stiglitz - "New new welfare economics"; economists job is to determine what's consistent with welfare maximization; policy makers determine the welfare function... or given welfare functions, economists determine whether policy is consistent with welfare maximization

Benefits - sounds bad, but we do get something from SWF... unlike Scitovsky's CIC, SWF lets us make trade offs between individuals

SWF Maximization Problem - 
\[
\max_{x^1, x^2} W(u^1(x^1), u^2(x^2))
\]

s.t. \( x^1_1 + x^1_2 = \bar{x}_1 (\lambda_1) \) and \( x^2_1 + x^2_2 = \bar{x}_2 (\lambda_1) \) (resource constraints)

First Order Conditions -
(1) \( \frac{\partial W}{\partial u^1} \frac{\partial u^1}{\partial x^1_j} - \lambda_1 = 0 \)
(3) \( \frac{\partial W}{\partial u^1} \frac{\partial u^1}{\partial x^2_j} - \lambda_2 = 0 \)
(2) \( \frac{\partial W}{\partial u^2} \frac{\partial u^2}{\partial x^1_j} - \lambda_1 = 0 \)
(4) \( \frac{\partial W}{\partial u^2} \frac{\partial u^2}{\partial x^2_j} - \lambda_2 = 0 \)

First term in each of these is MU of individual's consumption times marginal impact (of individual's utility) on welfare

\[
\frac{\partial W}{\partial u^1} \frac{\partial u^1}{\partial x^1_j} = \frac{\partial W}{\partial u^2} \frac{\partial u^2}{\partial x^1_j} = \frac{\partial W}{\partial u^1} \frac{\partial u^1}{\partial x^2_j} = \frac{\partial W}{\partial u^2} \frac{\partial u^2}{\partial x^2_j}
\]

Take (1)/(3) and (2)/(4):
\[
\frac{\partial u^1}{\partial x^1_j} / \frac{\partial u^1}{\partial x^2_j} = \frac{\partial u^2}{\partial x^1_j} / \frac{\partial u^2}{\partial x^2_j} \quad \text{which means } \text{MRS}^1 = \text{MRS}^2
\]

Result - the solution to the unconstrained social welfare maximization problem is Pareto efficient (because \( \text{MRS}^1 = \text{MRS}^2 \)), but we're also picking a particular point on the contract curve; we're choosing among Pareto efficient allocations to maximize SWF
Maximized Value - $W^*(\bar{x}_1, \bar{x}_2)$; since the only parameters in the SWF model are the resource constraints (given specific individual utility functions), the maximized value is defined by these resource constraints.

Level Curves - can draw level curves of $W^*$ in consumption space similar to CIC, but doesn't take $m_1^*$ and $m_2^*$ as given; instead $W^*$ is maximizing SWF for the given resource levels.

Result - there's a unique curve through each point in resource space (unlike CIC).

Properties of $W^*(\bar{x}_1, \bar{x}_2)$ - Samuelson 1956 and Gorman 1959; assuming:
(i) convex individual preferences
(ii) concave social welfare function (in utilities)
(iii) concave utility functions
(iv) lump sum transfers across individuals (i.e., pass income between individuals)
(1) Aggregate demands are functions of market prices and total income only (different than Scitovsky's CIC)
(2) Aggregate demands satisfy revealed preference properties
(3) Social indifference curves (contours of $W^*$) satisfy all properties of individual indifference curves
(first two properties are most important)

Alternative SWF Maximization Problem - long way or "bury utility functions in SWF"

$$\max_{x_1, x_2} W(u^1(x_1), u^2(x_2)) \quad \quad \max_{x_1, x_2} \bar{W}(x_1, x_2)$$

$$\text{s.t.} \quad P \cdot x_1 + P \cdot x_2 = I^1 + I^2 \quad \quad P \cdot (x_1^* + x_2^*) = I^1 + I^2$$

Significance - looks like regular individual utility maximization problem, except there are 2n goods rather than n goods (have to keep track of who consumes what)

Convexity of SWF Indiff Curves - proof of property (3) above

Pick two allocations $A$ and $B$ with same social welfare:

$$W(u^1(x_A^1), u^2(x_A^2)) = W(u^1(x_B^1), u^2(x_B^2))$$

Define allocation $C$ that's half way between $A$ and $B$:

$$x_C^1 = \frac{1}{2} (x_A^1 + x_B^1) \quad \quad \quad x_C^2 = \frac{1}{2} (x_A^2 + x_B^2)$$

Concavity of utility tells us: $u_C^1 \geq \frac{1}{2} (u_A^1 + u_B^1)$ and $u_C^2 \geq \frac{1}{2} (u_A^2 + u_B^2)$

Concavity of SWF:

$$W(u^1(x_C^1), u^2(x_C^2)) \geq W(u^1(x_A^1), u^2(x_A^2)) + W(u^1(x_B^1), u^2(x_B^2))$$
Indirect Utility Functions - instead of \( W(u^1(x^1), u^2(x^2)) \), we can use indirect utility functions (as long as consumers face the same prices): \( W(V^1(P, I^1), V^2(P, I^2)) \)

**Result** - max social welfare at point where social marginal utilities of income are the same between all individuals... that means ideal income distribution is dependent on market prices

**Summary** - can go from aggregate consumption data to welfare statements, but have to assume we're redistributing income as necessary to maximize social welfare (Bator article)

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**Public Goods**

**Non-Rival** - part that matters to us since we're worried about efficiency

**Non-Excludable** - relates to paying for the good so it's not related to efficiency

**Example** -

\[ H = \text{number of individuals} \]

\[ x^i = \text{private good consumed by individual } i \; \text{using single private good, but could easily be a vector of private goods (single good just makes it easier)} \]

\[ y = \text{level of public good (everyone consumes the same amount by definition)} \]

\[ u^i(x^i, y) = \text{individual } i \;'s \text{ utility is based only on his consumption of the private good and the total amount of the public good} \]

**Pareto Problem** - \[ \max_{x^1, \ldots, x^n, y} u(x^1, y) \]

\[ \text{s.t. } u^i(x^i, y) \geq \bar{u}^j, \; j = 2, \ldots, H \; (\lambda_j) \]

\[ F \left( \sum_{i=1}^H x^i, y \right) \leq 0 \quad (\mu) \quad \text{note that } x = \sum_{i=1}^H x^i \; (\text{total private good}) \]

**First Order Conditions** -

\[ \frac{\partial u^1}{\partial y} + \sum_{j=2}^H \lambda_j \frac{\partial u^j}{\partial y} - \mu \frac{\partial F}{\partial y} = 0 \quad \Rightarrow \quad \frac{\partial u^1}{\partial y} + \sum_{j=2}^H \lambda_j \frac{\partial u^j}{\partial y} = \mu \frac{\partial F}{\partial y} \]

\[ \frac{\partial u^1}{\partial x^1} - \mu \frac{\partial F}{\partial x} = 0 \quad \Rightarrow \quad \frac{\partial u^1}{\partial x^1} = \lambda_1 \frac{\partial u^1}{\partial x^1} = \mu \frac{\partial F}{\partial x} \]

\[ \lambda_j \frac{\partial u^i}{\partial x^j} - \mu \frac{\partial F}{\partial x} = 0 \]

**Samuelson Rule** -

\[ \sum_{j=1}^H \frac{\partial u^j}{\partial x^j} = \frac{\partial F}{\partial x} \Rightarrow \sum_{j=1}^H \text{MRS}^j = \text{MRT} \]

**Graphically** - assume only 2 individuals (argument works for more)

Modify Samuelson Rule to go from top graph to bottom:

\[ \text{MRS}^1 = \text{MRT} - \text{MRS}^2 \]

\( y^* \) and \( \text{MRS}^1 \) depend on \( \bar{u}^2 \)

**Result** - level of public good (for efficiency) depends on income distribution... we'll have to modify the Samuelson rule when we account for paying for the public good