Private Provision of Public Goods

BBV - most important paper on this subject; also applies to giving to any charity, political party, the arts, etc.; we'll focus on pure public good

Pure Altruism Model - consumers only care about total amount of contributions, not their individual contribution: \( u_i(x_i, G) \) vs. \( u_i(x_i, G, g_i) \) from Andreoni's Impure Altruism (or Warm Glow) model which we'll study later

- Assumptions on \( u_i(x_i, G) \): (i) strictly increasing, (ii) quasiconcave (i.e., convex better-than sets), (iii) differentiable

- Nash equilibrium in gifts (i.e., contributions to public good) from consumer \( i \) is \( g_i, \) such that

\[(x^*_i, g^*_i) \text{ solves:} \quad \forall \, i = 1, 2, \ldots, n \]

\[ \max_{x_i, g_i} u_i(x_i, g_i + G^*_i) \quad \text{where} \quad G_{-i} \equiv \sum_{j=1}^{n} g_j = \text{public good not counting } i \text{'s contribution} \]

s.t. \[ x_i + g_i = \omega_i \]
\[ g_i \geq 0 \]
\[ x_i = \text{amount of private good purchased by consumer } i \]
\[ \omega_i = \text{consumer } i \text{'s endowed wealth} \]

Note: taking everyone else's equilibrium gift as given

Effective Wealth - other people's contributions effectively increase consumer \( i \)'s wealth (although with a truncated budget set); re-write the problem:

\[ \max_{x_i, G} u_i(x_i, G) \]

s.t. \[ x_i + G = \omega_i + G^*_i \]

\[ G \geq G^*_i \]

2 kinds of consumers:
(1) those who contribute: \( g_i > 0 \)
(2) those who don't: \( g_i = 0 \)

Neutrality Result - (Theorem 1, p.29) Assume that consumers have convex preferences and that contributions are originally in a Nash equilibrium. Consider a redistribution of income among contributing consumers such that no consumer loses more income than his original contribution. After the redistribution there is a new Nash equilibrium in which every consumer changes the amount of his gift by precisely the change in his income. In this new equilibrium, each consumer consumes the same amount of the public good and the private good that he did before the redistribution.

Proof: let \( s \) denote initial equilibrium and \( \Delta\omega_i \) is the change in consumer \( i \)'s wealth

(\( \text{theorem requires } g_j + \Delta\omega_j \geq 0 \) ... can't take more than person is currently giving)

Assume all consumers \( j \neq i \) change their contribution by \( \Delta\omega_j \)

Then \( \Delta G_{-i} = \sum_{j=1}^{n} \Delta\omega_j = -\Delta\omega_i \) (from the assumption and because \( \sum_{j=1}^{n} \Delta\omega_n = 0 \) )
Graphically, consumer $i$’s budget constraint is truncated (if $\Delta \omega_i < 0$) or lengthened (if $\Delta \omega_i > 0$), but not enough to change the result; since consumer $i$ doesn’t change $(x_i^*, g_i^*)$ when others don’t change their decisions, this is still a Nash equilibrium.

Non Contributors - not addressed in theorem, but they don’t change their decision either

Another View - using demand curves:
Standard utility maximization problem: $\max_{x,y} u_i(x, y)$ s.t. $x + p_x y = \omega_i$ (using $p_x = 1$)

Solution yields demand function: $y = f(\omega_i, p_x)$

Apply to equilibrium problem for contributor:
Similar to standard utility problem with:
- $p_G = p_x = 1$ (everything in $\$)
- Effective income (social wealth): $\omega_i + G_{-i}^*$
- Ignore $G \geq G_{-i}^*$

Solution yields $G = f_i(\omega_i, G_{-i})$ (suppress the price argument since it’s constant)

For contributors ($G > G_{-i}^*$) the amount of public good demanded at $p_G = 1$ is the amount provided
For non-contributors ($G = G_{-i}^*$) the amount of public good demanded at $p_G = 1$ is less than the amount provided ($f_i(\omega_i, G_{-i}) < G_{-i} = G$)

Neutrality Result - since $\Delta G_{-i} = -\Delta \omega_i$, effective wealth doesn’t change; that means demand curve doesn’t change (solution to utility maximization problem is the same)

Income Effects - everything in this model operates through income effects

Demand - combining results for contributors and non-contributors:
$G = \max \{f_i(\omega_i, G_{-i}), G_{-i}\}$

Optimal Response - just subtract $G_{-i}$ from both sides of the demand equation:
$g_i = G - G_{-i} = \max \{f_i(\omega_i, G_{-i}) - G_{-i}, 0\}$
Existence Assumptions -
(1) $u_i(x_i, G)$ is quasiconcave
(2) $f_i'(\omega) \in (0,1)$...

Existence - (Theorem 2, p.33) A Nash equilibrium exists

Proof: standard application of Brower's Fixed Point Theorem:
domain $(W = \{x \in R^n : 0 \leq x_i \leq \omega_i \forall i = 1,...,n\})$ is a compact and convex set; $g_i$ is a continuous function from $W$ into itself

Note: if the function doesn't map into itself or if it's not continuous there may not be a fixed point. For more painful details on fixed point theorems, see ECO 7405 notes "Single Play Nash Equilibrium" and "Weaken Assumptions of Nash Theorem"

Contributors Set - $C$ is set of consumers who contribute in equilibrium; the number of consumers is $c$

Fact 1 - A configuration of gifts is a Nash equilibrium if and only if the following conditions are satisfied:
$G = f_i(\omega_i + G_{-i})$ for $i$ in $C$
$G \geq f_j(\omega_j + G_{-j})$ for $j$ not in $C$ ... we covered why on the previous page

Fact 2 - There exists a real valued function $F(G,C)$, differentiable and increasing in $G$, such that in a Nash equilibrium: $F(G,C) = \sum_{i \in C} \omega_i$ (the sum of the wealth of all contributors)

(poor notation: using a set $C$ as an argument for a function)

Proof: $f_i(\omega)$ is a strictly increasing function (by assumption), so it has an inverse $\phi_i = f_i^{-1}$

Apply the inverse to the first equation from fact 1: $\phi_i(G) = \omega_i + G_{-i} \forall i \in C$
(This is like an inverse demand function; it gives the effective wealth necessary to get consumer $i$ to demand $G$ amount of the public good)

Add these equations: $\sum_{i \in C} \phi_i(G) = \sum_{i \in C} \omega_i + \sum_{i \in C} G_{-i}$

The last term can be simplified: $\sum_{i \in C} G_{-i} = \sum_{i \in C} (G - g_i) = \sum_{i \in C} G - \sum_{i \in C} g_i = cG - G$

Put that back in: $\sum_{i \in C} \phi_i(G) = \sum_{i \in C} \omega_i + cG - G \Rightarrow \sum_{i \in C} \phi_i(G) + (1-c)G = \sum_{i \in C} \omega_i = F(G,C)$

This part is "less than obvious; that's why I have a job" (Romano)

$\frac{\partial F}{\partial G} = \sum_{i \in C} \phi_i'(G) + (1-c)$
Recall derivative of an inverse function: \( (f^{-1}(y_0))' = \frac{1}{f'(x_0)} \) so \( \phi_i'(G) = \frac{1}{f_i'(\omega_i + G_{-i})} \)

Since \( 0 < f_i'(\omega) < 1, \phi_i'(G) > 1 \Rightarrow \sum \phi_i'(G) > c \)

Put that back in and we see that \( \frac{\partial F}{\partial G} = \sum \phi_i'(G) + (1 - c) > c + (1 - c) = 1 \) or \( \frac{\partial F}{\partial G} > 1 \) so \( F(G, C) \) is strictly increasing in \( G \)

**Uniqueness** - (Theorem 3, p.34) There is a unique Nash equilibrium with a unique quantity of public good and a unique set of contributing consumers.

**Proof**: uses \( F(G, C) \); paper skips a step

**Fact 3** - Let \( g = (g_1, \ldots, g_n) \) and \( g' = (g_1', \ldots, g_n') \) be Nash equilibria given the wealth distributions \( \omega = (\omega_1, \ldots, \omega_n) \) and \( \omega' = (\omega_1', \ldots, \omega_n') \) (Note: these are different distributions of the same aggregate level of wealth; no new wealth is created), let \( C \) and \( C' \) be the corresponding sets of contributing consumers, and let the function \( F(G, C) \) be as defined in fact 2, then: \( F(G', C) - F(G, C) \geq \sum_{i \in C} (\omega_i' - \omega_i) \)

**Proof**: from fact 1, \( f_i(\omega_j + G_{-j}) \leq G' \quad \forall \quad i \in C \) (there's no guarantee that \( i \in C \) is also \( i \in C' \))

Apply \( \phi_i = f_i^{-1}: \omega_j + G_{-j} \leq \phi_i(G) \quad \forall \quad i \in C \)

Sum these up: \( \sum_{i \in C} \omega_i' + \sum_{i \in C} G_{-i}' \leq \sum_{i \in C} \phi_i(G') \)

As before, we can simplify \( \sum_{i \in C} G_{-i}' = \sum_{i \in C} (G'-g_i') = \sum_{i \in C} G' - \sum_{i \in C} g_i' \leq cG' - G' \)

(this last step is because the set of contributors may be different; if members from the original set are not in \( C' \), then \( g_i' = 0 \) so in effect, it’s a subset of the contributions of the new contributors)

Put that back in: \( \sum_{i \in C} \omega_i' \leq \sum_{i \in C} \phi_i(G') + (1 - c)G' \)

From fact 2: \( F(G', C) = \sum_{i \in C} \phi_i(G') + (1 - c)G' \quad \therefore \quad F(G', C) \geq \sum_{i \in C} \omega_i' \)

From fact 2: \( F(G, C) = \sum_{i \in C} \omega_i \)

Subtract this from the first one: \( F(G', C) - F(G, C) \geq \sum_{i \in C} (\omega_i' - \omega_i) \)

**Comparative Statics** - (Theorem 4, p.35) In a Nash equilibrium:

(i) Any change in the wealth distribution that leaves unchanged the aggregate wealth of current contributors will either increase or leave unchanged the equilibrium supply of public good.

**Math** - any redistribution that keeps \( \sum_{i \in C} \omega_i' = \sum_{i \in C} \omega_i \) results in \( G' \geq G \)

**Proof**: From fact 3, \( F(G', C) - F(G, C) \geq \sum_{i \in C} (\omega_i' - \omega_i) = 0 \) (unchanged agg. wealth)

From fact 2, \( F(G, C) \) is increasing in \( G \) so \( F(G', C) - F(G, C) \geq 0 \) \( \Rightarrow \ G' \geq G \)
Note 1: this isn't the same as neutrality result because we didn't restrict wealth redistribution to the contributions \( g_i + \Delta \omega_i \geq 0 \)

Note 2: not saying anything about private consumption, but that changes too

Example - \( g_1 = \$2 \), \( g_2 = \$1 \), \( g_3 = \$0 \) so \( G = \$3 \)

Step 1 - take \$1 from person 2 and give it to person 1; neutrality result says public good is unchanged (now \( g_1 = G \) and \( g_2 = 0 \))

Step 2 - take \$0.50 from person 2 and give it to person 1; person 1's wealth has risen so we'll by more public good \( \therefore G' > G \)

(ii) Any change in the wealth distribution that increases the aggregate wealth of current contributors will necessarily increase the equilibrium supply of the public good.

Math - any redistribution that has \( \sum_{i \in C} \omega_i ' > \sum_{i \in C} \omega_i \) results in \( G' > G \)

Proof: From fact 3, \( F(G', C) - F(G, C) \geq \sum_{i \in C} (\omega_i ' - \omega_i ) > 0 \)

From fact 2, \( F(G, C) \) is increasing in \( G \) so \( F(G', C) - F(G, C) > 0 \Rightarrow G' > G \)

(iii) If a redistribution of income among current contributors increases the equilibrium supply of the public good, then the set of contributing consumers after the redistribution must be a proper subset of the original set of contributors.

Math - redistribution among \( i \in C \) and \( G' > G \), then \( C' \subseteq C \) (i.e., fewer contributors in new equilibrium)

Proof: From fact 1, if \( G' > G \), people who didn't contribute before, won't contribute now so \( C' \subseteq C \)

If \( C' = C \), then the change from the income distribution leaves the total income of contributors in \( C' \) unchanged and part (i) of this theorem says \( G' \geq G \), but we know \( G' > G \) so we can't have \( C' = C \) \( \therefore C' \subseteq C \)

(iv) Any simple transfer of income from a consumer to a currently contributing consumer will either increase or leave constant the equilibrium supply of the public good.

Proof: follows directly from (i) and (ii)

Government Provision - assume government uses lump sum taxes on consumers (i.e., not conditioned on anything) to provide some of the public good and consumers are allowed to voluntarily contribute to purchase more of the public good

Neutrality - more general form of Theorem 6, part i, p.42; theorem starts with absence of government supply, but this result doesn't make that assumption

If government places a lump sum tax on contributors that is less than their contributions, there is a 100% crowding out (i.e., contributors reduce donations by the amount of the tax and there is no change in the total amount spent on the public good)

Proof:

Define \( t_i = \) lump sum tax on contributor \( i \) (\( g_i - \Delta t_i \geq 0 \)... tax less than contribution)

Define \( g_0 = \sum_{i \in C} t_i = \) government provision of public good

For any contributor \( G = g_i + G_{-i} + g_0 \)

Budget constraint of consumer \( i \) will be \( x_i + g_i = \omega_i - t_i \) or \( x_i + G = \omega_i - t_i + G_{-i} \)

Follow reasoning from Facts 1 & 2 and we see in Nash equilibrium we have \( G = f_i(\omega_i - t_i + G_{-i} + g_0) \)

Suppose government changes tax on consumer \( i \) such that \( g_i - \Delta t_i \geq 0 \)
Suppose all \( j \neq i \) have \( \Delta g_j = -\Delta t_j \) (i.e., other contributors reduce donations by the amount of their respective taxes)

\[
\Delta g_0 = \sum_{i \in C} \Delta t_i \quad \text{and} \quad \Delta G_{-i} = \sum_{j \neq i} \Delta t_j \quad \Rightarrow \quad \Delta G_{-i} + \Delta g_0 = \Delta t_i
\]

Put these into consumer \( i \)'s demand for public good:

\[
G = f_i(\omega_i - t_i + G_{-i} + g_0 - \Delta t_i + \Delta G_{-i} + \Delta g_0) = f_i(\omega_i - t_i + G_{-i} + g_0)
\]

There is no change in the consumer's demand (holds for all other consumers since we picked a generic \( i \))

**Theorem 6** (p.42) - Suppose that starting from an initial position where consumers supply a public good voluntarily, the government supplies some amount of the public good which it pays for from taxes. Then:

(i) If the taxes collected from each individual do not exceed his voluntary contribution to the public good in the absence of government supply, then the government's contribution results in an equal reduction in the amount of private contributions.

Math - \( t_i \geq 0 \) and \( \Delta t_i \leq g_i \) ... slightly different than neutrality result we just proved

(ii) If the government collects some of the taxes that pay for its contribution from non-contributors, then, although private contributions may decrease, the equilibrium total supply of the public good must increase.

Proof: taxing non-contributors is equivalent to a transfer that increases the effective wealth of the set of contributors... by part (ii) of Theorem 4, that means equilibrium supply of the public good increases

(iii) If the government collects some of the taxes that pay for its contribution by taxing any contributor by more than the amount of this contribution, the equilibrium total supply of the public good must increase.

Proof: first take the contributor the entire amount he's contributing; part (i) says amount of public good doesn't change and contributor reduces his contribution to zero; now he's a non-contributor so if government keeps taking him, wealth is transferred as in part (ii) so amount of public good increases

Bernheim - "On the Voluntary and Involuntary Provision of Public Goods" (1989); generalizes to other forms of taxation; neutrality holds in general case... difficult to prove empirically

Epple & Romano - (don't have title) IER 2003; looks more at dual provision of public good

**Identical Tastes** - make model more specific to get more specific results; look at case where individuals are identical except for endowment (i.e., \( u_i(x_i, G) \) is same so we can drop the \( i \) from \( u_i \); only difference between consumers is endowment \( \omega_i \)... we'll use this model for the remainder of the discussion of the paper

Assumptions - as before \( u(x_i, G) \) is (i) strictly increasing, (ii) quasiconcave, (iii) differentiable; also have \( f_i'(\omega) \in (0,1) \)

Subscripts - also drop subscripts from \( f_i(\omega) \) and \( \phi_i = f_i^{-1} \)

**Equalizing Redistribution** - redistribution of wealth is said to be equalizing if it is equivalent to a series of bilateral transfers in which the absolute value of the wealth difference between the two parties to the transfer is reduced (e.g., $60K and $50K; transfer no more than $10K); results in less disperse distribution of wealth

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**Fact 4** - If all consumers have identical preferences and $G^*$ is an equilibrium supply of the public good, then there is a critical wealth level $w^* = \phi(G^*) - G^*$ such that all consumers with wealth $w_i \leq w^*$ contribute nothing and every consumer with income $w_i > w^*$ contributes the amounts $g_i^* = w_i - w^*$ to the supply of the public good.

**Math** - $G^* \equiv$ equilibrium supply of public good

$w^* = \phi(G^*) - G^*$ ... cut off level of income for contributors

$w_i > w^* \Rightarrow$ contribute $g_i^* = w_i - w^*$

$w_i \leq w^* \Rightarrow$ don't contribute ($g_i^* = 0$)

**Proof:** From Fact 1, in equilibrium a contributor has $G^* = f_i (w_i + G_i^*)$ and a non-contributor has $G^* \geq f_j (w_j + G_j^*)$

$\therefore f(w_i + G_i^*) \leq G^*$ with equality if $g_i^* > 0$

Take inverse of both sides: $w_i + G_i^* \leq \phi(G^*)$ (with equality if $g_i^* > 0$ throughout)

Recall $G_i^* = G^* - g_i^*$: $w_i + G^* + g_i^* \leq \phi(G^*) \Rightarrow w_i - g_i^* \leq \phi(G^*) - G^*$

Let $w^* = \phi(G^*) - G^*$

$\therefore$ if person $i$ contributes, $w_i - g_i^* = w^* \Rightarrow g_i^* = w_i - w^*$

if person $i$ doesn't contribute, $w_i - g_i^* \leq w^*$ and $g_i^* = 0 \Rightarrow w_i \leq w^*$

**Equilibrium Properties for Identical Tastes** - (Theorem 5, p.38) If preferences are identical, then in a Nash equilibrium:

(i) All contributors will have greater wealth than all non-contributors.

**Math** - if $i$ contributes and $j$ doesn't, $w_i > w_j$

**Proof:** immediate from Fact 4

Romano: "If you think about it, these things follow immediately, which means you don't have to think about it"

(ii) All contributors will consume the same amount of the private good as well as of the public good.

**Math** - all contributors consume $w^*$ of the private good (contribute $g_i^* = w_i - w^*$ to public good); all consume $G^*$ of the public good

**Proof:** immediate from Fact 4

(iii) An equalizing wealth redistribution will never increase the voluntary equilibrium supply of the public good.

**Proof:** follows from (iv) and (v)

(iv) **Neutrality** - Equalizing wealth redistributions among current non-contributors or among current contributors will leave the equilibrium supply unchanged.

**Proof:** "obvious" from Fact 4; for non-contributors, redistribution keeps incomes $\leq w^*$ so they still don't contribute; for contributors, redistribution keeps incomes $> w^*$ so they still contribute... appeal to part 1 of Theorem 4

(v) Equalizing income redistributions that involve any transfers from contributors to non-contributors will decrease the equilibrium supply of the public good.

**Proof:** outline: contributor's wealth ↓ so he contributes less; non-contributors wealth ↑,

but if it's below $w^*$ he still doesn't contribute so $G^* \downarrow$ (not a vigorous proof because $w^*$ is also changing)
**Identical & Homothetic Tastes** - now add homothetic tastes (i.e., income consumption curve is a straight line; individual consumes goods in same proportion; \( \text{MRS}_{x,y} = f(x/y) \))

**Demand** - \( f = \alpha(\text{wealth}) \) ... proportional to effective wealth

Recall, \( f'(x) \in (0,1) \) so we must have \( \alpha \in (0,1) \)

**Sort** - label individuals such that lower number is higher wealth: \( \omega_1 \geq \omega_2 \geq \ldots \geq \omega_n \)

**When Only Richest Type Contributes**

Assume only person 1 contributes: \( G = \alpha \omega_1 \) (i.e., \( G_{-1} = 0 \))

Make sure person 2 doesn't: \( G \geq \alpha(\omega_2 + G_{-2}) = \alpha(\omega_2 + G) \) ... only 1 does so \( G_{-2} = G \)

Sub \( G = \alpha \omega_1 : \ \alpha \omega_1 \geq \alpha(\omega_2 + \alpha \omega_1) \Rightarrow \omega_1 \geq \omega_2 + \alpha \omega_1 \Rightarrow (1-\alpha) \omega_1 \geq \omega_2 \\
\therefore \) depends on both \( \alpha \) and the difference in wealth

Consider \( \alpha = 0.5 \) ... that means \( \omega_1 \geq 2 \omega_2 \) (the richest guy has to have more than twice the wealth of the second guy in order for only the richest guy to contribute)

**Wider Distribution of Wealth = More Voluntary Provision** - just applying part (v) of Theorem 5

Fix total wealth in the economy at \( W \)

(1) Widest wealth distribution is \( \omega_1 = W > \omega_2 = \omega_3 = \ldots = \omega_n = 0 \)  \[ \text{Subscript for case 1} \]

In this case only the richest type contributes which we just solved: \( G_1 = \alpha \omega_1 = \alpha W \)

(2) Split wealth equally between top 2: \( \omega_1 = \omega_2 = \frac{W}{2} \geq \omega_3 = \ldots = \omega_n = 0 \)

Equilibrium is symmetric: \( g_1 = g_2 = G_2 / 2 \)

\[ G_2 = \alpha(\omega_1 + G_{-1}) = \alpha(W/2 + G_2 / 2) \Rightarrow G_2 (1/2 - \alpha) = \alpha W/2 \Rightarrow \ G_2 = \frac{\alpha}{2-\alpha} W \]

So we have \( G_2 < G_1 \)

(3) Split wealth equally between top \( k \) : \( \omega_1 = \omega_2 = \ldots = \omega_k = \frac{W}{k} \geq \omega_{k+1} = \ldots = \omega_n = 0 \)

Equilibrium is symmetric: \( g_1 = g_2 = \ldots = g_k = G_3 / k \)

\[ G_3 = \alpha(\omega_1 + G_{-1}) = \alpha \left( \frac{W}{k} + \frac{k-1}{k} G_3 \right) \Rightarrow G_3 = \frac{\alpha}{(1-\alpha)k - \alpha} W \]  \[ \text{(typo in paper)} \]

Note: \( G_3 < G_2 \) and is decreasing in \( k \)

**Growing Economy** - want to show in large economy, very few types will contribute; this is the key flaw in the BBV model (doesn't match the real world where lots of types contribute)

**How to Grow** - replicate (clone) consumers so we increase the number of members of each type; denote \( k \) \( \in \{1,2,\ldots\} \) as number of each type of individual

**Assumptions** - model we just used: identical & homothetic tastes; keep individuals ordered as before (but assume strict inequalities): \( \omega_1 > \omega_2 > \ldots > \omega_n \)

**Only Richest Contributes** - find condition where only type 1 contributes (like we just did at the top of the page):

**Contributor** - \( G = \alpha \left( \omega_1 + \frac{k-1}{k} G \right) \Rightarrow G = \frac{\alpha}{1-\alpha - \alpha / k} \omega_1 \)

**Non-Contributor** - \( G \geq \alpha(\omega_2 + G) \)
Sub $G$ from above: \[
\frac{\alpha}{1 - \alpha} \omega_1 \geq \alpha \left( \omega_2 + \frac{\alpha}{1 - \alpha} \omega_1 / k \right).
\]

Do a little algebra: \[
\frac{(1 - \alpha)\omega_1}{1 - \alpha - \alpha / k} \geq \omega_2 \Rightarrow (1 - \alpha)\omega_1 \geq \omega_2 - \alpha \omega_2 - \omega_2 / k \quad \Rightarrow
\]
\[
(1 - \alpha)\omega_1 - \omega_2 (1 - \alpha) \geq -\alpha \omega_2 / k \quad \Rightarrow (1 - \alpha)(\omega_1 - \omega_2) \geq -\alpha \omega_2 / k \quad \Rightarrow
\]
\[
k \geq \frac{\alpha \omega_2}{(1 - \alpha)(\omega_1 - \omega_2)}.
\]

Large $\alpha$ means public good is highly valued (i.e., less likely to have single contributor)

Consider $\alpha = 0.5$: if $\omega_1 = 1.10 \omega_2$, then $k \geq 10$ means only richest contributes

If $\omega_1 = 1.01 \omega_2$, then $k \geq 100$ means only richest contributes

**Result** - these are not very big economies. Don't need a big economy before only the richest type is the only contributor

**General Case** - this result holds in general for the Pure Altruism Model (i.e., $u_i(x_i, G)$)

Andreoni uses a continuum of types

Romano, Fries, & Golding (IER, 1986?) use discrete types