Dynamic Price Competition
(Repeated Games)

**Varian** - model of sales with informed and uninformed consumers; Hamilton says we covered it previously, but I couldn't find it anywhere (we did cover it in Game Theory; see "Applications of Nash Equilibrium" p.1)... we'll apply this idea of informed and uninformed buyers (e.g., some consumers buy newspaper with sales inserts in it; others don't know which supermarkets have lowest price)

**Duopoly** - two firms (Publix and Winn Dixie)

**Bertrand** - Edgeworth used capacity constraint to solve \( P = MC \) result, but he used a static (one period) model

**Chamberlin** - thought firms would avoid \( P = MC \) through collusion... but Sherman Anti-Trust Act prevents it and is especially strict on price fixing

**Collusion** - firms colluding at monopoly price is unstable; in order to keep \( P > MC \), firms must figure out a way to detect cheaters

**Stigler** - "A Theory of Oligopoly" (JPE 1964); looked at cartel stability and "secret price cuts" (not good for retail, but common practice for industrial sales)

**Own Demand** - firms may detect cheating from their own demand fluctuations (if there's a sudden drop, someone's probably cheating), BUT if demand is fluctuating naturally, it's more difficult to detect (: more difficult to preserve the cartel)

**Subgame Perfect** - after Stigler's time

**Finite Horizon** - always repeat one-shot game

**Infinite Horizon** - have to worry about punishment strategies (want to get back lost sales and punish firm that cheated)... treats must be credible

**Friedman** - "Oligopoly and the Theory of Games" (1977)

**Folk Theorem** - for low enough discount rates, any payoff pair on the Pareto frontier is a SPNE outcome of infinitely repeated game

Quantity Competition with inverse demand \( P = a - Q \)

Identical firms with \( MC = ATC = c \)

Monopoly profit: \( \pi^m = \frac{(a - c)^2}{4}; \ q^m = \frac{a - c}{4} \)

Two firms colluding: \( \pi^d = 1/2 \pi^m = \frac{(a - c)^2}{8} \)

Cournot Equilibrium: \( \pi^C = \frac{(a - c)^2}{9} \) (from "Entry and Exit, Strategic Moves" p.2)

**Deviate?** - if opponent is cooperating (i.e., producing \( q^m = \frac{a - c}{4} \), firm wants:

\[
\max_q \pi = \left( a - \frac{a - c}{4} - q - c \right) q
\]

\[
\frac{\partial \pi}{\partial q} = \left( a - \frac{a - c}{4} - q - c \right) - q = a - \frac{a - c}{4} - c - 2q = 0 \implies q_c = \frac{3}{8} (a - c)
\]

\[
\hat{\pi} = \frac{9}{64} (a - c)^2, \quad \frac{9}{64} > \frac{1}{8} \implies \text{better than colluding... but we have to account for repeated game}
\]
**Discount Factor** -  \( \delta = \frac{1}{1+r} \) (where \( r = \) discount rate \( \in [0,1] \) so \( \delta \in [1/2,1] \) so we have to look at \( \pi_1 + \delta \pi_3 + \delta^2 \pi_3 + \ldots + \delta^{n-1} \pi_n \)

Collude forever: \( \pi^d = \frac{(a-c)^2}{8(1-\delta)} (1 + \delta + \delta^2 + \ldots = \frac{1}{1+\delta}, \text{ for } \delta < 1) \)

Cheat once and return to Cournot: \( \pi = \frac{9}{64} (a-c)^2 + \frac{\delta}{1+\delta} \frac{(a-c)^2}{9} \)

∴ firm will choose to collude if \( \delta > \frac{9}{17} \) ... that's pretty big discount factor

**Bertrand** - Tirole did this same thing for Bertrand (price setting) model and got \( \delta > 1/2 \) ... firms less likely to collude than in Cournot (quantity setting); gains from cheating are larger, but so is the loss from cheating

**Smaller \( \delta \)** -

**Short Periods** - the shorter the time period between rounds, the smaller \( \delta \) is ∴ firms aren't likely to collude

**End Game** - if there's positive probability game ends, effect of \( \delta \) goes down