Monopolistic Competition Models

**Chamberlinian Models** - monopolistic competition; assumes varieties (products) are "located" symmetrically and each faces a downward sloping demand curve (so any brand can raise price and not lose all of its sales)
- **Monopolistic** - have product differentiation
- **Competition** - free entry (but have fixed cost to entry)
- **Problem** - Chamberlinian models don't allow us to endogenize differentiation

**Big D** - demand curve faced by a variety when all varieties are charging the same price

**Little d** - demand curve faced by a variety when it raises its price and all others are charging the same price it was originally charging; Note: there are many little d's that intersect every point of the big D

**Entry** - big D shifts left (each firm has fewer sales); consumers value new entrant's product (variety) enough to switch to it

**Max Profit** - for a given little d curve, firm will max profit... MR = MC results in \( p^* > MC \) (short-run inefficiency) and \( p^* > \min \text{ATC} \) (long-run inefficiency)

**Excess Variety** - Chamberlin's interpretation of this result was that each firm was producing at too small a scale so the market is inefficient

**Dixit & Stiglitz** - said excess variety argument ignores value of variety

**Symmetry** - not like address models (Hotelling & location) most of which do not result in symmetric equilibria (except circle model which is structured to result in symmetric equilibria); Chamberlinian models assume a symmetric result (i.e., varieties will be spaced optimally; firms compete equally strongly [or weakly] with all other varieties), but doesn't say anything about the number of varieties which is what we're interested in

**Linear Product Space** - can have multiple equilibria with free entry; consider 2 firms, should firm 3 enter? Depends on location of 1 and 2

**Philosophical Difference** - when is firm location fixed?
- Bikes on beaches (Hotelling) - easily changed
- Produced space (product differentiation) - not easily changed
- Retail competition (location) - once store is built, location is fixed

**Chamberlinian Model** - not good for asking where firms locate; purpose is to determine whether there are too many or too few firms assuming they are located symmetrically

**Circle vs. Chamberlinian** - in a circle location model, firm 7 only worries about price from firms 6 and 8, but in Chamberlinian model, every variety interacts with the other varieties (i.e., no "close neighbors" like a circle model)

**Realistic?** - does Mercedes care about Kia prices?

**Details of Chamberlinian Model** - 2 types of goods
- **Outside Good** - representative of "all other goods"; usually used as a numeraire; these goods are assumed to have \( P = MC \)
- **Varieties** - differentiated products that compete directly; assume maximum of \( n \) of them

**Representative Consumer** - buys some of every variety (fractional quantities)
- **Consumer Utility** - \( u = U(q_o, V(q_1, \ldots, q_n)) \); \( q_o \) is outside good; \( V(q_1, \ldots, q_n) \) is subutility for the \( n \) varieties

**Symmetry** - assumes \( V(q_1, \ldots, q_n) \) is symmetric with constant returns to scale; want it to be easy to vary the number of goods; Dixit & Stiglitz solution:
\[ u = U \left( q_0, \left( \sum_{i=1}^{n} q_i^\rho \right)^{\frac{1}{\rho}} \right) \] ... results in constant elasticity of substitution

\[ 0 < \rho \leq 1 \] builds in taste for variety

**Consumer Optimization** - \( \max_{q} u \) s.t. \( p_0 q_0 + \sum_{i=1}^{n} p_i q_i = I \) (budget constraint)

**Simplify** - can remove \( q_0 \) by making it the numeraire (\( p_0 = 1 \)) and substituting the budget constraint into the objective:

\[ \max_{q} U \left( I - \sum_{i=1}^{n} p_i q_i, \left( \sum_{i=1}^{n} q_i^\rho \right)^{\frac{1}{\rho}} \right) \] (unconstrained optimization)

**FOC** - \( \frac{\partial U}{\partial q_i} = -U_1 p_i + U_2 \left( \sum_{i=1}^{n} q_i^\rho \right)^{\frac{1}{\rho} - 1} \rho q_i^{\rho - 1} = 0 \) ...

\( U_1 = \frac{\partial U}{\partial q_0}, U_2 = \frac{\partial U}{\partial V(q_1, \ldots, q_n)} \)

\[ U_1 p_i = U_2 \left( \sum_{i=1}^{n} q_i^\rho \right)^{\frac{1}{\rho} - 1} q_i^{\rho - 1} \]

**Competition Assumption** - assume any one firm is negligible in market so firm \( i \) doesn’t influence \( U_1 \) or \( U_2 \) (don’t need to worry about type of utility function);

also doesn’t influence \( \sum_{i=1}^{n} q_i^\rho \)

\( \therefore \) FOC simplifies to \( p_i = K q_i^{\rho - 1} \) (for some constant \( K \)) \( \Rightarrow q_i = kp_i^{\frac{-1}{\rho}} \)

**Result** - constant elasticity of demand

**Firm Optimization** - assume all firms have same MC; \( \pi_i = (p_i - c) \left[ kp_i^{\frac{-1}{\rho}} \right] - F \)

**FOC** - \( \frac{\partial \pi_i}{\partial p_i} = kp_i^{\frac{-1}{\rho}} + (p_i - c) \left( \frac{-1}{1 - \rho} kp_i^{\frac{-1}{\rho}} \right) = 0 \)

\[ kp_i^{\frac{-1}{\rho}} + p_i \left( \frac{-1}{1 - \rho} kp_i^{\frac{-1}{\rho}} \right) - c \left( \frac{-1}{1 - \rho} kp_i^{\frac{-1}{\rho}} \right) = 0 \]

\[ kp_i^{\frac{-1}{\rho}} + \left( \frac{-1}{1 - \rho} kp_i^{\frac{-1}{\rho}} \right) + c \left( \frac{1}{1 - \rho} kp_i^{\frac{-1}{\rho}} \right) = 0 \]

\[ \left( \frac{1}{1 - \rho} \right) kp_i^{\frac{-1}{\rho}} + c \left( \frac{1}{1 - \rho} kp_i^{\frac{-1}{\rho}} \right) = 0 \]

\[ \left( \frac{1}{1 - \rho} \right) kp_i^{\frac{-1}{\rho}} = -c \left( \frac{1}{1 - \rho} kp_i^{\frac{-1}{\rho}} \right) \]
\[
\left(1 - \frac{1}{1-\rho}\right) = \frac{1-\rho}{1-\rho} - \frac{1}{1-\rho} = -\rho = -c \frac{1}{1-\rho} p_i^{-1}
\]

\[
p_i = -c \frac{1}{1-\rho} \left(1 - \frac{1}{1-\rho}\right) = \frac{c}{\rho} \geq c
\]

Result - \( p_i^* = \frac{c}{\rho} \geq c \); price proportional markup of MC (marginal pricing rule)

Perfect Substitutes - \( \rho = 1 \), then \( \left( \sum_{i=1}^{n} q_i^\rho \right)^{1/\rho} \) becomes \( \sum_{i=1}^{n} q_i \), and \( p_i^* = \frac{c}{1} = c \); the varieties are perfect substitutes so it doesn't matter how much of each the consumer buys, just the total amount; results in perfectly competitive output (\( P = MC \))

Lower \( \rho \) - as \( \rho \downarrow \) to zero, goods are not as good a substitutes for each other (consumer will want equal amounts of each); \( p_i^* \) (infinte at the limit)

Price Independent - \( p_i^* \) is not dependent on the number of varieties, just on marginal cost and taste for variety (\( \rho \))

Free Entry - implies \( \pi_i = (p_i - c)q - F = 0 \) \( \Rightarrow \) \( (p_i - c)q_i = F \)

Sub \( p_i = \frac{c}{\rho} \left( \frac{c}{\rho} - c \right)q_i = F \) \( \Rightarrow \) \( q_i = \left( \frac{F}{\frac{c}{\rho} - c} \right) \)

Technology

Utility

Quantity Independent - long run quantity produced is independent of the number of firms

# Firms - original question: "is there excess entry?" answer: compared to what?

Full Optimum - 1st Best; social planner sets \( n \) and \( q_i^* \) in order to get \( P = MC \) so \( \pi_i = -F \); firms can't recover fixed costs so have to tax consumers to subsidize firms (i.e., must subtract \( nF \) from consumer's budget constraint)

Finding \( n \) - consumer chooses the same quantity from each firm to maximize

\[
\max_{n,q} U \left( I - nF - \sum_{i=1}^{n} p_i q_i, \frac{\sum_{i=1}^{n} q_i^\rho}{\rho} \right) = U \left( I - nF - nCq, qn^{1/\rho} \right)
\]

Constrained Outcome - 2nd Best; \( \pi_i = 0 \) (so no subsidy has to be paid); let firms choose price and quantity in order to maximize profit like we did above; assume all firms produce same amount (from symmetry assumption); then solve FOC for \( n \):

\[
U_1 p = U_2 \left( \sum_{i=1}^{n} q_i^\rho \right)^{1-\rho} q^{\rho-1} \ldots \text{sub } p = \frac{c}{\rho} \text{ and } q = \frac{F}{\frac{c}{\rho} - c}
\]

\[
U_1 \frac{c}{\rho} = U_2 \frac{1-\rho}{\rho} \ldots \text{where the derivatives } U_1 \text{ or } U_2 \text{ are evaluated at }
\]

\[
\left( I - \sum_{i=1}^{n} pq, \frac{\sum_{i=1}^{n} q_i^\rho}{\rho} \right) = \left( I - \frac{nCq}{\rho}, qn^{1/\rho} \right) \quad (\text{Tirole p.299})
\]
Other Option - instead of the above constraint, we may want to control the number of firms, but not prices (i.e., not limiting to $\pi_i = 0$)

Result - number of firms is the same either way; $n_{1st} = n_{2nd}$

Ottaviano, Tabuchi & Thisse Model - Dixit & Stiglitz model above not very general (sensitive to utility function); OT&T developed model that is easy to solve because FOC are linear

Competition - firms small so we ignore each firm's effect on the market

Representative Consumer - buys some of every variety (same as Dixit & Stiglitz model)

Finite Firms - $k + 1$ firms (varieties)

Quasilinear Utility - $U(x) = \alpha \sum_{j=1}^{k+1} x_j - \frac{\beta - \gamma}{2} \sum_{j=1}^{k+1} x_j^2 - \frac{\gamma}{2} \sum_{i=1}^{k+1} \sum_{j=1}^{k+1} x_i x_j + y$ → Expenditure on all other goods

Utility from consumption of "the varieties"

$\alpha > 0$, $\beta > 0$ and $\gamma < \beta$ are constants

Budget Constraint - $\sum_{i=1}^{k+1} p_i x_i + y = I$ (solve for $y$ and plug into utility function)

FOC - $\frac{\partial U}{\partial x_i} = \alpha - \beta x_i - \gamma \sum_{j \neq i} x_j - p_i = 0$ if $x_i > 0$ ($i = 1, \ldots, k + 1$

Note: $\frac{\partial}{\partial x_i} \left( \sum_{j=1}^{k+1} \sum_{h=1}^{k+1} x_j x_h \right) = \frac{\partial}{\partial x_i} \left( x_1 x_i + x_1 x_2 + \ldots + x_1 x_{k+1} + x_2 x_1 + x_2 x_2 + \ldots \right) = 2 \sum_{j=1}^{k+1} x_j$

Interpretation - $\alpha = MU$ of first unit of consumption

$-\beta x_i \Rightarrow$ diminishing MU

$-\gamma \sum_{j \neq i} x_j \Rightarrow$ diminishing MU if $\gamma > 0$ (i.e., goods are all substitutes for each other)

If $\gamma = 0$ goods are not related

This term (along with $\alpha$) determines the intercept for the inverse demand for good $i$; slope will be $\beta$... note as goods are closer substitutes ($\gamma \uparrow$), demand decreases (intercept moves down)

All $x$'s appear in each FOC $\therefore$ have to solve simultaneously

Solution - $x_i = A - B p_i + c \sum_{j \neq i} p_j$ (linear in own price and rivals' prices)

Where $A = \frac{\alpha}{\beta + k \gamma}$, $B = \frac{\beta - \gamma + k \gamma}{(\beta - \gamma)(\beta + k \gamma)}$, $C = \frac{\gamma}{(\beta - \gamma)(\beta + k \gamma)}$

Continuum of Firms - this is a "more rigorous" analysis, but you have to use your imagination to use a continuum of firms; another alternative is to write it out Dixit-Stiglitz style, but then we have to ignore some terms

$q(i) = a - bp(i) + c \int_{0}^{N} [p(j) - p(i)] dj$ ($N =$ "number of firms")
\[
\int_0^N [p(j) - p(i)]dj = Np(i) + \int_0^N p(j)dj \quad \text{(changing } p(i) \text{ doesn't change } p(j))
\]

Define \( P = \int_0^N p(j)dj \) as "price index"

\[ q(i) = a - (b - cN)p(i) + cP \]

**Parameters** - utility function that derives this demand curves gives us: (Hamilton didn't cover how we got these)

\[
a = \frac{\alpha}{\beta + (N-1)\gamma}, \quad b = \frac{1}{\beta + (N-1)\gamma}, \quad c = \frac{\gamma}{(\beta - \gamma)(\beta + (N-1)\gamma)}
\]

**Monopoly Behavior** - \( q(i) = a - (b - cN)p(i) + cP \) ... firm takes \( P \) (index of all other firms in market) as given so it behaves like a monopoly wrt its own variety

**Regular Demand** - if all firms are identical and charge the same price, demand facing an individual firm is \( q_i = a - (b - cN)p_i + cP \), but \( P = Np_i \) so demand becomes

\[ q_i = a - bp_i \]

**Applications** - "quite amenable to studying equilibria with two types of firms"; one example is home vs. foreign firms and studying the effect of market size

**Random Utility** - empirically useful because it's "tractable in the right sort of way" (direct quote from Hamilton; don't ask me what that means!); don't need to use representative consumer; doesn't have starkness of horizontal models (i.e., discontinuities where \( p \uparrow \Rightarrow q = 0 \) and rival takes all sales)

**Conditional Utility** - of good \( j \) for consumer \( k \): \( u^k_j = y - p_j + \varepsilon^k_j, \quad j = 1, \ldots, n \)

**Match Value** - \( \varepsilon^k_j \) determines what it is about product that gives consumer his utility; we won't specify what's in \( \varepsilon^k_j \) (i.e., don't say why consumer prefers one product over another); consumer evaluations about the quality of the good are drawn from a distribution

**Buy 1 Good** - consumer only buys 1 good; the one that gives him maximum conditional utility: \( u^k = \max_j u^k_j \); some variations of the model allow consumers the choice of not buying any goods so \( u^k = \max \left[ 0, \max_j u^k_j \right] \)

**Symmetric** - adding symmetry (like a Chamberlinian model) means distributions for match values are the same for all consumers; also assume there is no correlation between varieties and firms have same cost (fixed cost \( K > 0 \) and MC = 0)

**Density for Variety** - probability density function for good \( j \) is \( f(\varepsilon_j) \); assume \( \varepsilon_j \in [a,b] \):

\[
\int_a^b f(\varepsilon_j) = 1 \quad \text{(definition of a density function)}
\]

\[
F(x) = \int_a^x f(x)dx \quad \text{(cumulative distribution function; this will be used to determine how many consumers an individual firm will have)}
\]
Joint Density \( \prod_{i=1}^{n} f(\varepsilon_i) \)

**Why Buys What** - firm \( i \) will sell to any consumer for whom
\[
u^k_i = y - p_i + \varepsilon^k_i \geq u^j_k = y - p_j + \varepsilon^j_k \quad (j \neq i, \; j = 1, \ldots, n) \Rightarrow \varepsilon^j_k \leq p_j - p_i + \varepsilon^k_i \; \forall \; j
\]
Probability this holds for a single firm \( j \) and given value of \( \varepsilon^k_i \):
\[
\Pr[\varepsilon^j_k \leq p_j - p_i + \varepsilon^k_i] = F(p_j - p_i + \varepsilon^k_i)
\]
Probability it holds for all firms and given value of \( \varepsilon^k_i \):
\[
\Pr[\varepsilon^j_k \leq p_j - p_i + \varepsilon^k_i \; \forall \; j \neq i] = F(p_j - p_i + \varepsilon^k_i)
\]
Demand = probability it holds for all firms for all values of \( \varepsilon^k_i \):
\[
D_i = \int_a^b f(x) \prod_{j=1}^{n} F(p_j - p_i + x) \; dx
\]

**Specify Density** - everything to this point just used micro foundations, but to get any results we need to specify the form of the density which will determine the properties of the demand function; Caplin and Nalebuff (Econometrica, 1991) discuss different distributions

- **Normal** - results in probit demand model; nice properties, but Normal distribution is hard to work with
- **Double Exponential** - results in logit demand model

**Density** - \( F(x) = \exp \left[ - \exp \left( \frac{x - \eta}{\mu} \right) \right] \)

**Parameters** - \( \eta \) ("eta") is location parameter; \( \mu \) ("mu") is scale parameter
\[
\mu \uparrow \Rightarrow \text{Var} \uparrow \text{ (tastes are more diverse; less response to price change)} \\
\eta \uparrow \Rightarrow \text{Mean} \uparrow \text{ (better matches; firm sells more on average)}
\]

**Mean** - \( E[x] = \gamma \mu + \mu \) (Euler's constant: \( \gamma \approx 0.577 \))

**Variance** - \( \text{Var}[x] = \mu^2 \pi^2 / 6 \) (\( \pi \approx 3.14 \... \))

**Demand** - \( D_i = \frac{\exp \left( \frac{a_i - p_i}{\mu} \right)}{\sum_{j=1}^{n} \exp \left( \frac{a_j - p_j}{\mu} \right)} \quad \text{or} \quad D_i = \frac{\exp \left( \frac{a_i - p_i}{\mu} \right)}{\sum_{j=1}^{n} \exp \left( \frac{a_j - p_j}{\mu} \right) + \exp \left( \frac{a_0}{\mu} \right)} \)

- \( a_i \) = deterministic match value; observable quality that shifts distribution (e.g., add better radio to car); for symmetric model \( a_j = 0 \; \forall \; j = 1, \ldots, n \)

**Problem** - McFadden transportation demand (BART); red-bus vs. blue-bus problem; view good 1 as blue bus; good 2 is car; good 3 is red bus; adding the red bus (i.e., other alternatives) does not change the ratio of demands between blue bus and car (not realistic)

2 firms (varieties): \( D_i = \frac{\exp \left( \frac{-p_1}{\mu} \right)}{\exp \left( \frac{-p_1}{\mu} \right) + \exp \left( \frac{-p_2}{\mu} \right)} \)

Second version is if consumer is given choice to hot buy good; \( a_0 \uparrow \Rightarrow \text{less likely to buy} \)
or multiply by \( \frac{\exp\left(\frac{p_1}{\mu}\right)}{\mu} \) to get \( D_1 = \frac{1}{1 + \exp\left(\frac{p_1 - p_2}{\mu}\right)} \) (don't need this though)

\[
D_2 = 1 - D_1 = 1 - \frac{\exp\left(-\frac{p_2}{\mu}\right)}{\exp\left(-\frac{p_1}{\mu}\right) + \exp\left(-\frac{p_2}{\mu}\right)}
\]

\[
D_1 = \frac{\exp\left(-\frac{p_1}{\mu}\right)}{\exp\left(-\frac{p_2}{\mu}\right)}
\]

3 firms (varieties): \( D_1 = \frac{\exp\left(-\frac{p_1}{\mu}\right)}{\exp\left(-\frac{p_1}{\mu}\right) + \exp\left(-\frac{p_2}{\mu}\right) + \exp\left(-\frac{p_2}{\mu}\right)} \)

\[
D_1 = \frac{\exp\left(-\frac{p_2}{\mu}\right)}{\exp\left(-\frac{p_1}{\mu}\right) + \exp\left(-\frac{p_2}{\mu}\right) + \exp\left(-\frac{p_2}{\mu}\right)}
\]

\[
D_1 = \frac{\exp\left(-\frac{p_1}{\mu}\right)}{\exp\left(-\frac{p_2}{\mu}\right)} ... \text{same as with two varieties}
\]

**Result** - logit framework may not be well-suited to analyzing entry of new varieties unless all varieties are symmetric in the sense that a new variety attracts customers away from existing varieties in proportion to pre-entry sales

**Solution** - nested logit; pick best bus option (red vs. blue), then compare that bus to the car option; another example: pick best brand of DVD player and best brand of VHS player, then compare DVD to VHS

**Applying Logit** - profit: \( \pi_i = (p_i - c)D_i - F \)

\[
\frac{\partial \pi_i}{\partial p_i} = D_i + (p_i - c)\frac{\partial D_i}{\partial p_i} = 0
\]

Simplify \( D_i \) first: \( D_i = \frac{\exp\left(-\frac{p_i}{\mu}\right)}{\sum_{j=1}^{n} \exp\left(-\frac{p_j}{\mu}\right)} = \frac{1}{1 + (n-1)\exp\left(\frac{p_i - p_j}{\mu}\right)} \)
\[ \frac{\partial D_i}{\partial p_i} = -D_i^2 \frac{n-1}{\mu} \exp\left(\frac{p_i - p_j}{\mu}\right) \]

Plug that back into FOC:

\[ \frac{\partial \pi_i}{\partial p_i} = D_i - (p_i - c)D_i^2 \frac{n-1}{\mu} \exp\left(\frac{p_i - p_j}{\mu}\right) = D_i \left[ 1 - (p_i - c)D_i \frac{n-1}{\mu} \exp\left(\frac{p_i - p_j}{\mu}\right) \right] = D_i \left[ 1 - (p_i - c) \frac{n-1}{1 + (n-1)} \right] = 0 \]

Wave your hands and this results in symmetric outcome: \( p^* = c + \frac{n\mu}{n-1} \)

Difference between price and marginal cost is proportional to \( \mu \) (taste for variety) for given number of firms

\[ \mu \uparrow \Rightarrow p^* \uparrow \]

\[ \lim_{n \to \infty} p^* = c + \mu \ldots \text{same as Dixit-Stiglitz result (taste for variety allows each firm to charge a premium over marginal cost)} \]

**Interpretation** - value of adding a firm is that consumers draw their max utility from a bigger sample; firms aren't just slicing market into smaller pieces, but giving consumers what they want; result is loyal customers so \( P > MC \)

**Free Entry** - \( \pi_i = (p_i^* - c)D_i - F = 0 \)

From symmetry assumption, we can assume \( D_i = 1/n \) (firms evenly split market)

Plug in equilibrium price: \( \pi_i = \left(\frac{\mu}{n-1}\right) \frac{1}{n} - F = 0 \Rightarrow n^* = \frac{\mu}{F} + 1 \)

**Optimal Entry** - Anderson, de Palma, and Thisse show...

1st Best - use lump sum taxes to subsidize

2nd Best - firms must break even (zero profits; \( p^* = c + \mu \))

**Result** - \( n_{1st} = n_{2nd} = \frac{\mu}{F} \ldots \text{have 1 extra firm (not bad unless } n^* \text{ is small)} \)

**Price Discrimination** - Spence (Review of Economic Studies, 1976) argued excess entry depends on whether price discrimination is feasible

**Perfect Price Discrimination** - firm captures entire surplus, so the surplus the firm brings by entry equals firm's fixed cost \( \therefore \) no excess entry

**Not Perfect** - if not perfect discrimination, we have to worry about what signal a firm gets; results in firms maximizing wrong surplus function
Business Stealing - Mankiew and Whinston (RAND, 1986) argued about inefficiency for firm entry, but made assumptions about endogenous variables to get their results

Assumptions:
(1) Firm Entry - total output increases with # of firms \( N \) but has a finite bound
(2) “Business Stealing” Effect - each firm’s output in a symmetric equilibrium falls as \( N \) ↑
(3) Positive Profits - (before fixed costs) \( P(Nq_N) \geq C'(q_N) \ \forall \ N \)

Symmetric, Homogeneous Case - output per firm \( (q_N) \) decreases with \( N \) \( (dq_N/dN < 0) \)
\[ \pi_N = P(Nq_N)q_N - C(q_N) - K = 0 \] (from free entry)... this assumes \( N \) is continuous
Discrete case is: \( \pi_N \geq 0 \) and \( \pi_{N+1} < 0 \)

Optimal # Firms - \[ \max W(N) \equiv \int_0^{Nq_N} P(s)ds - NC(q_N) - NK \]

Interpretations -
\[ W(N) = \text{gross surplus} - \text{costs} \]
\[ W(N) = \text{consumer surplus} + \text{firm profit} \]

Follows from adding and subtracting \( P(Nq_N)q_N N \) (price times quantity per firm times number of firms = total revenue)
\[ W(N) = \int_0^{Nq_N} P(s)ds - P(Nq_N)q_N N + P(Nq_N)q_N N - NC(q_N) - NK \]

consumer surplus
totla profit (all firms)

FOC - use Leibnitz Rule... serious magic derivative wand waving here
\[ \frac{\partial W}{\partial N} = \left[ \int_0^{Nq_N} \frac{\partial P(s)}{\partial N} ds + P(Nq_N) \left( q_N + N\frac{dq_N}{dN} \right) \right] - C(q_N) - NC'(q_N)\frac{dq_N}{dN} - K = \]
\[ P(Nq_N)q_N + P(Nq_N)N\frac{dq_N}{dN} - C(q_N) - NC'(q_N) - K = \]
\[ [P(Nq_N)q_N - C(q_N) - K] + NP(Nq_N)\frac{dq_N}{dN} - NC'(q_N)\frac{dq_N}{dN} = \]
\[ \pi_N + N[P(Nq_N) - C'(q_N)]\frac{dq_N}{dN} \]

From free entry \( \pi_N = 0 \), know \( P(Nq_N) \geq C'(q_N) \) (or firm’s wouldn’t sell), and Mankiew & Whinston assume \( dq_N/dN < 0 \) \( \therefore dW/dN < 0 \)

Result - if firm enters and takes sales away from other firms (i.e., \( dq_N/dN < 0 \) while firms are making positive profit before deducting fixed costs (i.e., \( P(Nq_N) > C'(q_N) \)), then there are too many firms; if \( P = MC \) or \( dq_N/dN = 0 \) then \( \partial W/\partial N = \pi_N \) (firm’s profit equals change in social welfare so number of firms is efficient)
**Heterogeneous Case** - rather than one demand curve we have many (1 per firm) so write
gross surplus as \( G \left[ \sum_i f(q_i) \right] \) ... in symmetric case: \( G[Nf(q_N)] \) (\( G \) is a function that builds
in taste for variety)
Assume \( f(0) = 0 \) and \( f' > 0 \) & \( f'' < 0 \) for \( q_N > 0 \)
Price \( P = G' f'\)

\[
\text{FOC} - \quad \frac{dW}{dN} = G\left[ Nf'(q_N)\frac{dq_N}{dN} + f(q_N) \right] - C(q_N) - NC'(q_N)\frac{dq_N}{dN} - K = \\
NG' f'(q_N)\frac{dq_N}{dN} + G' f(q_N) - C(q_N) - NC'(q_N)\frac{dq_N}{dN} - K + G' f'q_n - G' f'q_n = \\
[ G' f'q_n - C(q_N) - K ] + NG' f'\frac{dq_N}{dN} - NC'(q_N)\frac{dq_N}{dN} + G' f - G' f'q_n = \\
\pi_N + N[ G' f' - C'(q_N) ]\frac{dq_N}{dN} + G'[ f - f'q_n ]
\]

From free entry \( \pi_N = 0 \)

\( G' f' - C'(q_N) = P - C' \geq 0 \) & \( \frac{dq_N}{dN} < 0 \) (assumption) \( \therefore \) second term is negative

Know \( f - f'q_n > 0 \) (assumptions about \( f \)), but can't sign the third term without
knowing the structure of \( G \)

**Result** - sign of \( \frac{dW}{dN} \) (i.e., whether entry is efficient or not) depends on strength of taste
for variety (structure of \( G \))... a very obscure way to go about deriving a common sense answer

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**Leibnitz Rule** - differentiating an integral of one variable with respect to another

\[
F(t) = \int_{g(t)}^{h(t)} f(x,t) dx \\
F'(t) = \int_{g(t)}^{h(t)} \frac{\partial f(x,t)}{\partial t} dx + h'(t) f(h(t),t) - g'(t) f(g(t),t)
\]