Common Knowledge Failures

Common Knowledge - most game (even imperfect information games) assume common knowledge: all players know the structure of the game; and each player knows the other players know the structure; and each player knows the other players know that he knows that they know; etc.

Lack of Common Knowledge - Rubinstein modeled this using (lack of) communication and showed that it's equivalent to having imperfect information (one player knows something about the game the other guy doesn't know)

E-Mail Game - player 1 sends a message to player 2; player 2 then replies to confirm he got the message; player 1 replies to confirm the reply, etc.; this process continues until one of the players doesn't receive a reply; at this point there are 2 possibilities: (1) his last message did not reach his opponent, or (2) his opponent's reply did not reach him

Sent vs. Received - can keep track of messages sent or received; get same result

Ends with 1 - if player 1 doesn't get a reply and he has sent x messages, then he believes player 2 either sent x (and player 1 didn't get it) or x - 1 (because he didn't get player 1's last message); but then to know how each of these types of player 2 will respond player 1 has to figure out each player 2 type's beliefs (see tree)

Ends with 2 - if player 2 doesn't get a reply and he has sent x messages, then he believes player 1 either sent x (because player 2's last message didn't arrive) or x + 1 (and player 2 didn't receive player 1's last message)

Infinite Regress - players have to have beliefs about beliefs about beliefs...

Player 1 knows:

Player 1's belief about Player:

Player 2's belief about Player:

Player 1's beliefs about Player:

Etc.

Example - suppose there are two games that can result as a chance of nature; player 1 knows which game is played, but player 2 doesn't (i.e., coordination problem); player 1 sends a message to player 2 to inform him if they face game A or game B (e.g., send message if A, but no message if B; that way the content of the message is irrelevant); player 2 replies to confirm; player 1 replies to confirm the confirmation, etc.
Discontinuity - Rubinstein used induction proof to say player 1 will play game A (with probability 1) regardless of the number of messages; the number of messages doesn't matter until it gets to $\infty$ (i.e., "common knowledge"); at the limit, equilibrium changes so this boils down to a discontinuity.

Limit - Rubinstein showed that at the limit, communication is effectively removed and this is a game of incomplete information.

Controversy - not all game theorists like this model of common knowledge.

"To go on would require many more hours."

Real World Communication Failure - theory paper in AER (Slutsky doesn't remember the content of the paper, just the controversy); someone wrote in saying that he agreed with the model assumptions, but there was a math error; the original author wrote back saying there was no math error; AER took the easy way out and published both comments so there was no resolution; the math was either right or wrong so what happened:  
(a) Did the first critic get a chance to reply? or
(b) Did the first critic concede (i.e., agree with the author’s reply)?