**Costs, Technology, Productivity**

**Christensen & Greene.** "Economies of Scale in U.S. Electric Power Generation."

**Ai's Take** - this paper has some errors (it was written in 1976 so there have been lots of advancements in econometrics), but it is very well written

**Economies of Scale** - double output results in costs that are less than double... based on cost

**Scale Economies** - double inputs, more than double output; also called returns to scale...

   based on technology; SE implies EOS, but not vice versa

   * C&G use the terms interchangeably in paper, but they mean economies of scale

**Exogenous** - firm has no say/decision on variable; in this paper, power companies have no say in their output (regulation requires them to meet demand)

**Endogenous** - firm decides the value of the variable

**Literature Review** - Nerlove (1963) did early work on scale economies in electricity generation using data from 1955

**Contribution** - C&G use a more general model (translog cost function) that doesn't impose homotheticity or unitary elasticities of substitution; use Nerlove's data from 1955 and more recent data from 1970

**Why Use Cost** - C&G have "fancy talk" reasoning for using cost function instead of production function (lots of stuff about duality, p.658), but Ai says, definition of economies of scale is centered on cost so cost function makes more sense

   There exists a production function that results in the translog cost function, but we don’t need to know what it is

**Translog Cost Function** - big contribution of C&G paper; places no restrictions on substitution among factors of production and allows scale economies to vary with the level of output (i.e., classical U shape ATC)... also contains all forms estimated by Nerlove (1963)

**Cost Function** -  

\[
C = f(P, Y) \quad \text{based on input prices and output}
\]

**Log Linear** -  

\[
\ln C = \alpha_0 + \alpha_Y \ln Y + \sum_{i=1}^n \alpha_i \ln P_i \quad \text{... this is Cobb-Douglas (restrictive assumptions)}
\]

**Translog** - adds quadratic and interaction terms:

\[
\ln C = \alpha_0 + \alpha_Y \ln Y + \frac{1}{2} \gamma_{YY} (\ln Y)^2 + \sum_{i=1}^n \alpha_i \ln P_i + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} \ln P_i \ln P_j + \sum_{i=1}^n \gamma_{yi} \ln Y \ln P_i
\]

**Long-Run** - this doesn't include last period's capital stock so it is a long-run cost function

**Homogeneity of Degree 1 in P** - for this to be real cost function, it must satisfy this property:  

\[
f(\lambda P, Y) = \lambda f(P, Y) \quad \text{... this implies the following relationships among the parameters:}
\]

\[
\sum_{i=1}^n \alpha_i = 1, \quad \sum_{i=1}^n \gamma_{yi} = 0, \quad \text{and} \quad \sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} = \sum_{i=1}^n \gamma_{yi} = \sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} = 0 \quad \text{(Note: } \gamma_{ij} = \gamma_{ji})
\]

(We'll show how to incorporate this in the model shortly)
Share Function - % of \( i \)th factor of production is \( S_i = \frac{PX_i}{C} \)

Sub in Shephard’s Lemma: \( \frac{\partial C}{\partial P_i} = X_i \cdot S_i = \frac{P_i}{C} \frac{\partial C}{\partial P_i} = \frac{\partial C}{\partial P_i} / P_i = \frac{\partial \ln C}{\partial \ln P_i} \)

Taking the derivative gives a linear function (part of appeal of translog cost function):

\[
S_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln P_j + \gamma_{ii} \ln Y
\]

Restriction - by definition, \( \sum_{i=1}^{n} S_i = 1 \) so one of the share equations is redundant

Problems -

Error Terms - C&G add error terms \( U \) and \( \varepsilon_i \) to \( \ln C \) and \( S_i \), respectively and assume disturbances have joint normal distribution and zero correlations across firms:

\[
(U, \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n) \sim N(0, \Omega) ... \text{pdf: } \frac{1}{\sqrt{|\Omega|(2\pi)^n}} \exp\left(-\frac{1}{2} \eta^T \Omega \eta \right), \text{ where } \eta = \begin{bmatrix} U \\ \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{bmatrix}
\]

but \( U \) and \( \varepsilon_i \) can’t be independent because \( S_i \) is a partial derivative of \( \ln C \) ... the error term in \( \ln C \) will drop out when taking the derivative unless \( U = \tilde{U} + \sum_{i=1}^{n} \varepsilon_i \ln P_i \);

but this error term may cause heteroskedasticity (something C&G never addressed)

MLE - C&G solved model using maximum likelihood estimation (MLE); in this case they misspecified the distribution of the error term so it’s "quasi-MLE", but it’s still consistent (just might not get the right standard error)

Share Function - \( S_i \in [0,1] \)... using regular regression or even MLE doesn’t guarantee this will hold... need logit or probit

Scale Economies - \( \text{SCE} = 1 - \frac{\partial \ln C}{\partial \ln Y} \)

\( \text{SCE} > 0 \Rightarrow \) economies of scale (\( Y \uparrow \) causes \( C \uparrow \) by smaller proportion: \( \partial \ln C / \partial \ln Y < 1 \))

\( \text{SCE} < 0 \Rightarrow \) diseconomies of scale (\( Y \uparrow \) causes \( C \uparrow \) by larger proportion: \( \partial \ln C / \partial \ln Y > 1 \))

\( \text{SCE} = 0 \Rightarrow ? \)

Take the derivative:

\[
\text{SCE} = 1 - \left( \alpha_Y + \gamma_{YY} \ln Y + \sum_{i=1}^{n} \gamma_{yi} \ln P_i \right)
\]

Goal - use \( \ln C \) and \( S_i \) to estimate parameters to calculate SCE

Theory vs. Econometrics - \( \ln C \) and \( S_i \) are equivalent in theory (no new information), but for econometrics they’re not necessarily equivalent; can use data from both models to estimate
the parameters (prevents possible multicollinearity issues estimating ln C by itself because it has a large number of regressors that don't vary greatly across firms, p.662)... ideally they're estimated at the same time using Seemingly Unrelated Regression (SURE) Model

**Problem** - we have to drop one of the share equations because it's redundant; SURE results will depend on which equation gets dropped

**Solution** - C&G use MLE so it doesn't matter which equation gets dropped

**Applying Restrictions** - restrictions to guarantee homogeneity of degree 1 in prices from bottom of page 1:

\[ \sum_{i=1}^{n} \alpha_i = 1 \Rightarrow \alpha_i = 1 - \sum_{i=2}^{n} \alpha_j \quad (1) \]
\[ \sum_{i=1}^{n} \gamma_{yi} = 0 \Rightarrow \gamma_{y1} = -\sum_{i=2}^{n} \gamma_{yi} \quad (2) \]
\[ \sum_{i=1}^{n} \gamma_{yj} = 0 \Rightarrow \gamma_{1j} = -\sum_{i=2}^{n} \gamma_{yj} \quad (3) \]
\[ \sum_{j=1}^{n} \gamma_{yj} = 0 \Rightarrow \gamma_{i1} = -\sum_{j=2}^{n} \gamma_{yj} = \gamma_{1j} \quad \text{(Recall: } \gamma_{yj} = \gamma_{yj}) \]
\[ \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{yj} = 0 \ldots \text{this is redundant} \]

Plugging these into the cost function:
\[
\ln C = \alpha_0 + \alpha_Y \ln Y + \frac{1}{2} \gamma_{YY} (\ln Y)^2 + \sum_{i=1}^{n} \alpha_i \ln P_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln P_i \ln P_j + \sum_{i=1}^{n} \gamma_{yi} \ln Y \ln P_i
\]

**Impose (1):**
\[
\left( 1 - \sum_{i=2}^{n} \alpha_i \right) \ln P_i + \sum_{i=2}^{n} \alpha_i \ln P_i
\]

**Impose (2):**
\[
\frac{1}{2} \sum_{i=1}^{n} \gamma_{yi} (\ln P_i)^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln P_i \ln P_j
\]

**Impose (3):**
\[
\frac{1}{2} \left[ \sum_{i=2}^{n} \gamma_{yi} (\ln P_i)^2 - \sum_{i=2}^{n} \gamma_{yi} (\ln P_i)^2 \right]
\]

**Impose (3):**
\[
\sum_{i=2}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln P_i \ln P_j - \sum_{i=2}^{n} \gamma_{ij} \ln P_i \ln P_j
\]

**Impose \gamma_{ij} = \gamma_{ij}:**
\[
-\gamma_{ij} - \sum_{i=\max(2, j)}^{\max(2, j)-1} \gamma_{ij} - \sum_{i=2}^{\max(2, j)} \gamma_{ij} \quad \text{(Recall: } \gamma_{ij} = \gamma_{ij})
\]

**Result** - it's complicated; we have to combine terms and simplify this, but Ai gave up on it (and I'm not about to do it!); this was a very painful part of class; we also have to impose the restrictions on \( S_i \)
Model Results - estimate coefficients, then use them to estimate:

\[
\hat{SCE} = 1 - \left( \hat{\alpha}_Y + \hat{\gamma}_{YY} \ln Y + \sum_{i=1}^{n} \hat{\gamma}_{yi} \ln P_i \right)
\]

Can do this for each firm individually or group them by size; C&G used 5 sizes

Other Restrictions - these were assumed by Nerlove (1963), but can actually be tested with the translog specification... C&G found most of these terms are significantly different from zero so the cost function does not have any of these properties

Homothetic Production - cost function is separable function in output and factor prices

(i.e., not interaction terms between \(\ln Y\) and \(\ln P_i\) so \(\gamma_{yi} = 0\))

Homogeneous Production - homothetic production where elasticity of cost wrt output is constant (i.e., drop the quadratic term so \(\gamma_{YY} = 0\), plus \(\gamma_{yi} = 0\) form homothetic)

Unitary Elasticity of Production - \(\gamma_{ij} = 0\)

Final Restriction - cost function should have \(C > 0\) and be concave in prices


Issue - compare efficiency of two types of firms; in this case, looking at ownership (public vs. private) and regulation

Theory - usually focuses on only one factor; most theorists argue private is more efficient because it maximizes profit (CEO compensation is tied to profit and other things); public firms don't use profit

Regulation - imposes distortion so firms will be less efficient, but firms a regulated in the first place because of an inefficiency

Empirical - "those who do theory are very stubborn" (Ai); if empirical results don't agree with theory, the empiricist did something wrong, it's not a problem with the theory

Fantasy World - implication of simple theory is irrelevant because real world is more complicated

Types of Empirical Research - both are useful, but some argue one is better than the other

Reduced Form Approach - take theory's prediction and try to show the implication of the theory holds in practice (e.g., look at cost functions); this works well with classical microeconomics where there are very general assumptions, but with newer theories (like game theory), results change dramatically when assumptions change so they're harder to verify empirically

Structural Approach - test theory directly... this is what A&H do

Intro Section

Theory - consistently concludes (1) privately-owned firms are more efficient than publicly-owned and (2) unregulated firms are more efficient than regulated

Econometric Studies - test joint effects of ownership and regulation (whereas theoretical literature generally only looks at partial effects rather than joint effects)

Technique - A&H show behavior of utility-maximizing regulated firms is equivalent to cost minimization subject to shadow (rather than market) prices of inputs
Theoretical Model

Managerial Utility - function of both profit and quantities of inputs: $U(\pi, X)$

Profit - manager’s compensation depends on firm’s profit; assume positively related: $\frac{\partial U}{\partial \pi} > 0$; for public firms, profit measurement is usually not emphasized as much so the manager’s utility is not impacted as much by profit ($[\frac{\partial U}{\partial \pi}]_{\text{private}} > [\frac{\partial U}{\partial \pi}]_{\text{public}}$)

Inputs - this is a generic way to say manager’s utility depends on things other than the firm’s profit: $\frac{\partial U}{\partial X_i}$ can be <, =, or > 0

Direct Pecuniary Benefits - e.g., manager’s income may increase with number of employees supervised

Non-Pecuniary Benefits - e.g., business entertainment

Non-Pecuniary Costs - e.g., manager’s own effort

Profit - assume firm faces downward-sloping demand curve so $\pi = R(X) - \sum_{i=1}^{n} P_i X_i$

Regulation - use traditional rate of return regulation from Averch & Johnson (1962):

$$R(X) - \sum_{i=1}^{n} P_i X_i \leq (\overline{P}_K - P_K) X_K = G^K (X^{G^K}) ,$$

where $P_K$ = firm’s cost of capital and $\overline{P}_K$ = allowed rate of return

Modern Regulation - U.S. moving to price cap or incentive/profit share regulation; still have some countries using rate of return regulation

Manager’s Decision -

$$\max_{X} U(\pi, X)$$

s.t. $\pi = R(X) - \sum_{i=1}^{n} P_i X_i$, profit

$$R(X) - \sum_{i=1}^{n} P_i X_i \leq G^K (X^{G^K})$, regulation

Lagrangian - $\ell = U(\pi, X) - \lambda \left[ \pi - R(X) + \sum_{i=1}^{n} P_i X_i \right] - \sum_{k} \theta_k \left[ R(X) - \sum_{i=1}^{n} P_i X_i - G^K (X^{G^K}) \right]$

FOC for Input - reduces to marginal revenue product of input $i$ equals shadow price of input $i$

$$\frac{\partial R}{\partial X_i} = P_i - \frac{\partial U / \partial X_i + \sum_{k} \theta_k \frac{\partial G^K / \partial X_i}}{\partial U / \partial \pi - \sum_{k} \theta_k} = P_i^*$$

Private Owner Without Regulation - this should be $\frac{\partial R}{\partial X_i} = \frac{\partial U / \partial X_i}{\partial U / \partial \pi}$

Implication for Input - we want to get the theory to something we can test empirically and the only things we observe are outputs and market prices; theory says if one price of input is lower, then the firm will use more of that input; if there are multiple prices (e.g., capital and labor) that are lower, theory doesn’t say anything about the impact so we’ll look at ratios of outputs
A&H’s way: $\frac{\partial R}{\partial X_i} = \frac{\partial R}{\partial Q} \frac{\partial Q}{\partial X_i}$

Ai’s way: $R = Q(X) \cdot r$, so $\frac{\partial R}{\partial X_i} = \frac{\partial Q}{\partial X_i}$

Ratio $\frac{\partial R}{\partial X_i} = \frac{\partial Q}{\partial X_i} = \frac{\partial Q}{\partial X_j}$

\[ \text{Ratio} \quad \frac{\partial R}{\partial X_i} \quad \frac{\partial Q}{\partial X_i} = \frac{\partial Q}{\partial X_j} \quad \text{MRTS}_{ij} = \frac{P_i^*}{P_j^*} \]

marginal rate of technical substitution equals ratio of shadow prices

A&H say this means utility-maximizing behavior under regulation can be modeled as cost minimization subject to appropriately defined shadow prices

Previous Theoretical Studies

**Alchian (1965)** - non-transferability of ownership in publicly-owned firms decreases concentration of rewards and costs on persons responsible for them and inhibits specialization of ownership → adversely affects efficiency

**Alchian & Kessel (1962)** - effects of regulation would dominate those of ownership type

**Consensus** - shadow price of labor is lower for public firms; Niskanen (1971), Stigler (1971), Tullock (1972)

**No Consensus** - effect of public ownership on shadow price of capital; De Alessi (1969) & Niskanen (1971) said it would be lower for public firms; Cran & Zardkoohi (1980) said it would be higher

**Averch & Johnson (1962)** - analyzed effects of rate of return constraint on privately-owned firms; countered by Atkinson & Halvorsen (1980); cited by some studies as grounds for expecting publicly-owned utilities to be more efficient than privately-owned utilities... A&H say this needs to be studied empirically

Previous Empirical Studies - yield conflicting evidence

Summary

**Electric Utilities**

- Ki Lorenzo & Robinson (1982) Linear cost equations No significant difference
- Färe, Grosskopf and Logan (1985) Non-parametric No significant difference
- Meyer (1975) Quadratic cost function Public firms more efficient
- Moore (1970) Linear input & cost equations Private firms 5% better
- Neuberg (1977) Cobb-Douglas cost function Public firms 6-10% better
- Pescatrace & Trapani (1980) Translog cost function Public firms 33% better

**Water Utilities**

- Bruggink (1982) Cobb-Douglas cost function Public firms 21% better
- Crain & Zardkoohi (1978) Cobb-Douglas cost function Private firms 22% better
- Feigenbaum & Teeples (1983) Translog cost function No significant difference

**Meyer & Neuberg** - results clouded by failure to control for differences in cost of capital to publicly-owned and privately-owned utilities

**Pescatrace & Trapani** - did control for differences in input prices, but their model is overly restrictive (assumes publicly-owned firms are efficient and only source of inefficiency for privately-owned firms is rate of return constraint)

**Färe, Grosskopf and Logan** - non-parametric approach doesn't provide information on possible differences in the relative use of inputs
Econometric Model - build on theory, but making something that's observable

**Shadow Prices** - not observed; assume their proportional to market prices: \( P_i^* = k_i P_i \)

**Shadow Cost Function** - use translog function (Eqn 8):

\[
\ln C^S = \alpha_0 + \alpha_Q \ln Q + \gamma_Q \ln (\ln Q)^2 + \sum_{i=1}^{n} \alpha_i \ln P_i^* + \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln P_i^* \ln P_j^* + \sum_{i=1}^{n} \gamma_Qi \ln Q \ln P_i^*
\]

**Shephard's Lemma** - firms' shadow cost function is \( C^S(kP, Q) \), where \( kP \) = vector of shadow prices; Lau & Yotopolous (1971) showed actual input demand functions can be derived using Shephard's Lemma: \( \partial C^S / \partial k_i P_i = X_i \)

**Actual Cost** - \( C^A = \sum_{i=1}^{n} P_i X_i = \sum_{i=1}^{n} P_i (\partial C^S / \partial k_i P_i) \); use this and (Eqn 8) to get actual cost function (Eqn 9):

\[
\ln C^A = \alpha_0 + \alpha_Q \ln Q + \gamma_Q \ln (\ln Q)^2 + \sum_{i=1}^{n} \alpha_i \ln Q + \sum_{i=1}^{n} \gamma_{ij} \ln (k_i P_i) + \ln \left\{ \sum_{i=1}^{n} k_i^{-1} \left[ \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln (k_j P_j) + \gamma_{Qi} \ln Q \right] \right\}
\]

**Share Equations** - (Eqn 10):

\[
M_i^A = \frac{\alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln (k_j P_j) + \gamma_{Qi} \ln Q}{\sum_{i=1}^{r} \left[ \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln (k_j P_j) + \gamma_{Qi} \ln Q \right] k_i^{-1}}
\]

**Technology** - reflected in coefficients in cost function; Ai says all of them potentially change, but A&H assumed only the coefficients on linear terms change (probably because they didn’t have enough data to use more parameters):

\( \alpha_Q^i = a_Q + b_Q D^r \) and \( \alpha_i^r = a_i + b_i D^r \) (i = 1, ..., n)

where \( r = \begin{cases} \text{P for privately-owned utilities} \\ \text{G for publicly-owned utilities} \end{cases} \)

\( D^P = 0 \) and \( D^G = 1 \)

\( a_i, b_i, a_Q, \) & \( b_Q \) are parameters to be estimated

**Other Modifications** - linear homogeneity in prices (same restrictions as C&G paper), cost shares sum to one (so one of the share equations is dropped); additive disturbance term

**Inputs** - like C&G paper, only use three inputs: capital (K), labor (L) and fuel (F); only worried about price ratio, so normalized \( k_L = 1 \)

**Price Efficiency** - requires that all MRTS are equal to the corresponding ratios of market prices (i.e., \( k_i = k_j \)); use firm-specific estimates of \( k_i \)’s:

\( k_i' = d_i + g_i D^r \) (i = 1, ..., n)

where \( r \) & \( D^r \) are same as before; \( d_i \) & \( g_i \) are parameters to be estimated

Efficiency: \( H_0: d_k = d_F = 1.0 \) for private firm

\( d_k + g_k = d_F + g_K = 1.0 \) for public firm
Cost Efficiency - requires price efficiency and technical efficiency (i.e., obtain maximum output possible from any given set of inputs); allow for differences in technical efficiency between public and private firms:

\[ \alpha' = d_0 + g_0 D' \]

where \( r \) & \( D' \) are same as before; \( d_0 \) & \( g_0 \) are parameters to be estimated

Data - 1970 data for 123 privately-owned and 30 publicly-owned steam-electric utilities

Method -
(1) first tested for equal cost (in)efficiency in publicly-owned and privately-owned electric utilities (i.e., \( g_K = g_F = 0 \)); imposed equal technical efficiency (\( g_0 = 0 \))

\textbf{Result} - null hypothesis of equal cost efficiency is easily accepted at the 5% level

(2) given acceptance of equal price efficiency for types of firms (public vs. private), now determine if price efficiency of inputs is the same (i.e., \( d_K = d_F = 1.0 \))

\textbf{Result} - null hypothesis of price efficiency is rejected at the 5 percent level

(3) given acceptance of equal technical and price efficiency, test for price efficiency between each pair of inputs; 3 tests:
- Capital and labor: \( d_K = 1.0 \)
- Fuel and labor: \( d_F = 1.0 \)
- Capital and Fuel: \( d_K = d_F \)

\textbf{Result} - pairwise price efficiency is rejected for capital-labor and fuel-labor, but is accepted for capital-fuel

\textbf{Didn't Test} - firms using same technology (i.e., \( b_Q = b_i = 0 \))

Other Stuff -
(a) monotonicity is checked by determining if the calculated values of the shadow cost shares are positive
(b) concavity is checked by determining if the principal minors of the Hessian matrix have the correct (alternating) signs
(c) comparison with fitted actual total costs indicates that price inefficiency increases total (and average) costs by an average of 2.4 percent for both private & public firms
(d) price inefficiency substantially increases the quantity demanded of capital (average increase is 23.4% for both); price inefficiency also increase the average quantity demanded of labor (10.2% for private; 16.9% for public) and decreases the average quantity demanded of fuel (2.4% for private; 2.8% for public)