1. Assume an individual consumes only two commodities, apples and oranges. The individual picked a bundle with 6 apples and 0 oranges (6,0) over one with 5 apples and 3 oranges (5,3). The individual then chose a bundle with 3 apples and 6 oranges (3,6) over the one with 6 apples and 0 oranges (6,0). Is this behavior consistent with the five basic assumptions of consumer theory? In particular, does it satisfy "transitivity", "more is better", and "convexity"? (Hint: It might help to draw a graph and see if you can draw an indifference map with normal properties which is consistent with the specified choices.)

Transitivity - $\forall x, y, z, x \succ y$ and $y \succ z \Rightarrow x \succ z$
In this case $A \succ B$ and $C \succ A$; transitivity would imply that $C \succ B$. Although there is no information in the problem to confirm this we can see from the upper graph that the "least" convex (i.e., linear) indifference curve through $C$ that has $C \succ B$, would also have $C \succ A$ which violates the consumer's stated preferences $\therefore$ these preferences are not transitive.

Monotonicity (more is better) - $x > y \Rightarrow x \succ y$
Looking at $A$ and $B$: $A_1 > B_1$, but $A_2 < B_2$
Looking at $C$ and $A$: $C_1 < A_1$
$\therefore$ we can't say anything about monotonicity.

Convexity - $R^\geq(y)$ is a convex set
The "least" convex (i.e., linear) indifference curve through $A$ that has $A \succ B$ is pictured on the lower graph. The point $C$ does not lie on the right side of this indifference curve (or we could argue that since $C \in R^\geq(A)$, it is apparent that $R^\geq(A)$ is not a convex set.)
$\therefore$ the individual's behavior is not consistent with well behaved preferences.

2. Draw the indifference curves implied by each of the following behavior patterns. Indicate the direction of increasing utility. (Note: In some of these examples the indifference curves may not have the "normal" shape.)
(a) Two goods are useful only in fixed proportions (right and left shoes).
(b) Two goods are considered by the consumer as perfect substitutes (red and green stamps to a color blind person).
(c) Steak and potatoes assuming you are a vegetarian.
(d) Vodka and Sherry, assuming you care only about the alcohol content (vodka is 40% alcohol, sherry is 20%).
(e) One good is a bad (smelly garbage) and gourmet hamburgers.
(f) Garbage removal and hamburgers are the two commodities. 
(g) "I like onion on my hamburger, but you can overdo a good thing."
(h) The two commodities are the average temperature to which you heat your home in the middle of a Gainesville winter and all other commodities.
Hint: The best way to do these is to pick a point on the graph and find all points superior and inferior to it, then postulate a point of indifference. Here's one to give you the idea. "I can't tell the difference between butter and margarine."

Start with a point such as A. Clearly, A < P B, C < P A, D < P A and so on. In fact, from the statement that the consumer can't tell the difference, we know that he will only trade a unit of margarine for a unit of butter, no more, no less. Hence, a point such as E is indifferent to A. Finally, the set of indifference curves is a set of parallel straight lines, with the MRS = 1 at any point on any line.

(a) (b) (c) (d) (e) (f) (g) (h)
3. Consider the utility functions defined by:

\[ U(x) = x_1 x_2 \]
\[ V(x) = \log(x_1) + \log(x_2) \]
\[ W(x) = x_1^2 x_2^2 \]
\[ F(x) = x_1^a x_2^b + k[\log(x_1) + \log(x_2)] \]

Do these utility functions represent the same or different preferences?

Let \( c_i \) be a constant \( (i = 1, 2, 3, 4) \)

\[ U(x) = x_1 x_2 = c_1 \Rightarrow x_2 = \frac{c_1}{x_1} \text{ (indifference curve)} \]

\[ V(x) = \log(x_1) + \log(x_2) = c_2 \Rightarrow x_2 = \frac{10^{c_2}}{x_1} \text{ same indifference curve if } c_1 = 10^{c_2} \]

Could also argue that \( V(x) = \log(U(x)) \) (an increasing transformation)

\[ \therefore \text{ } V(x) \text{ does represent the same preferences as } U(x) \]

\[ W(x) = x_1^2 x_2^2 = c_3 \Rightarrow x_2 = \frac{\sqrt{c_3}}{x_1} \text{ same indifference curve if } c_1 = \sqrt{c_3} \]

Could also argue that \( W(x) = (U(x))^2 \) (an increasing transformation)

\[ \therefore \text{ } W(x) \text{ does represent the same preferences as } U(x) \]

\[ F(x) = x_1^a x_2^b + k[\log(x_1) + \log(x_2)] = c_4 \Rightarrow x_2 = \text{ something ugly} \]

Another way to look at this is \( F(x) = (U(x))^a + kV(x) \), as long as \( a \) and \( k > 0 \) this is a valid transformation.

\[ \therefore \text{ } F(x) \text{ does represent the same preferences as } U(x) \]

4. Do the following utility functions represent strictly convex preference orderings?

(a) \( U(x) = x_1^a x_2 \)
(b) \( U(x) = x_1^2 + x_2^2 \)
(c) \( U(x) = x_1 + x_2 + 2x_1 x_2 \)
(d) \( U(x) = -ax_1^2 - bx_2^2 \)

To show strictly convex preferences we need to show that \( U(x) \) is strictly quasiconcave. Since there are only two variables, that means we need to show:

\[ [BH] = \begin{bmatrix} 0 & U_1 & U_2 \\ U_1 & U_{11} & U_{12} \\ U_2 & U_{21} & U_{22} \end{bmatrix} = 2U_{12}U_{21} - U_{11}^2 - U_{12}^2 - U_{22}^2 > 0 \]

\[ \begin{align*}
(a) \quad U_1 &= ax_1^{a-1}x_2 \\
U_2 &= x_1^a \\
U_{11} &= a(a - 1)x_1^{a-2}x_2 \\
U_{22} &= 0
\end{align*} \]
\[ U_{12} = ax_1^{a-1} \]

\[ |BH| = 2(2ax_1^{a-1}x_2)x_1^{a-1} - (x_1^{a-1}) - (x_1^{a-1}) - (ax_1^{a-1}x_2)^2(0) = 2(a^2x_1^{3a-2}x_2) - ((a^2-a)x_1^{3a-2}x_2) = a^2x_1^{3a-2}x_2 + ax_1^{3a-2}x_2 > 0 \text{ if } a \notin (-1,0) \]

\[ . . . \text{ } U(x) = x_1^ax_2 \text{ does represent strictly convex preferences} \]

(b) \[ U_1 = 2x_1 \quad U_{11} = 2 \]
\[ U_2 = 2x_2 \quad U_{22} = 2 \]
\[ U_{12} = 0 \]

\[ |BH| = 2(2x_1)(2x_2)(0) - (2x_2)^2(2) - (2x_1)^2(2) = -8x_2 - 8x_1 \leq 0 \]

\[ . . . \text{ } U(x) = x_1^2 + x_2^2 \text{ does not represent strictly convex preferences} \]

(c) \[ U_1 = 1 + 2x_2 \quad U_{11} = 0 \]
\[ U_2 = 1 + 2x_1 \quad U_{22} = 0 \]
\[ U_{12} = 2 \]

\[ |BH| = 2(1 + 2x_2)(1 + 2x_1)(2) - (1 + 2x_2)^2(0) - (1 + 2x_1)^2(0) = 4(1 + 2x_2 + 2x_1 + 4x_1x_2) > 0 \]

\[ . . . \text{ } U(x) = x_1 + x_2 + 2x_1x_2 \text{ does represent strictly convex preferences} \]

(d) \[ U_1 = 2ax_1^{-3} \quad U_{11} = -6ax_1^{-4} \]
\[ U_2 = 2bx_2^{-3} \quad U_{22} = -6bx_2^{-4} \]
\[ U_{12} = 0 \]

\[ |BH| = 2(2ax_1^{-3})(2bx_2^{-3})(0) - (2bx_2^{-3})^2(-6ax_1^{-4}) - (2ax_1^{-3})^2(-6bx_2^{-4}) = 24abx_1^{-4}x_2^2 + 24a^2bx_1^{-6}x_2^{-4} > 0 \text{ if } a \text{ and } b > 0 \]

\[ . . . \text{ } U(x) = -ax_1^{-2} - bx_2^{-2} \text{ does represent strictly convex preferences} \]

\[ \quad \]

Documentation.

I checked my answers with Josh Kneifel.

Burcin said that adding a utility function to another utility function representing the same preferences is a valid transformation (for 3d). She also said to interpret 2g as onions being the good that you can overdo (not burgers).