1. Suppose the time series pattern in yearly income is explained by the following equation:

\[ Y_t = 1000 + e_t \] where \( e_t = u_t + 0.5e_{t-1} \) and \( u_t \) is "white noise".

(a) This is an ARIMA\((p,d,q)\). What are the values for \( p, d, \) and \( q \)?

\[
\begin{align*}
p &= 1 & \text{There is 1 lagged } e \text{ term} \\
d &= 0 & \text{There are no difference terms (i.e., lagged } Y) \\
q &= 0 & \text{There are no lagged } u \text{ terms}
\end{align*}
\]

(b) If \( e_{2003} = 100 \) what is \( E_{2003}(Y_{2004}), E_{2003}(Y_{2005}), E_{2003}(Y_{2031}) \)?

\[
\begin{align*}
E_{2003}(Y_{2004}) &= E_{2003}[1000 + u_{2004} + 0.5e_{2003}] = 1000 + 0.5e_{2003} = 1000 + 0.5(100) = 1050 \\
E_{2003}(Y_{2005}) &= E_{2003}[1000 + u_{2005} + 0.5e_{2004}] = E_{2003}[1000 + u_{2005} + 0.5u_{2004} + 0.5e_{2003})] = 1000 + 0.5^2e_{2003} = 1000 + 0.5^2(100) = 1025 \\
E_{2003}(Y_{2031}) &= 1000 + 0.5^{28}e_{2003} \\
\end{align*}
\]

Suppose, instead, that \( Y_t = Y_{t-1} + e_t \) where \( e_t = u_t + 0.5e_{t-1} \)

(c) What are the values for \( p, d, \) and \( q \)?

\[
\begin{align*}
p &= 1 & \text{There is 1 lagged } e \text{ term} \\
d &= 1 & \text{There is 1 difference term (i.e., lagged } Y) \\
q &= 0 & \text{There are no lagged } u \text{ terms}
\end{align*}
\]

(d) If \( Y_{2003} = 1000 \) and \( e_{2003} = 100 \), what is \( E_{2003}(Y_{2004}) \)?

\[
E_{2003}(Y_{2004}) = E[Y_{2003} + u_{2004} + 0.5e_{2003}] = Y_{2003} + 0.5e_{2003} = 1000 + 0.5(100) = 1050
\]

(e) If the average propensity to consume out of permanent income is 0.9 and consumers are "rational", what amount of consumption should occur in 2003?

\[
C_{2003} = 0.9(\text{Average expected income})
\]

\[
\text{Average expected income} = \frac{1}{(N - 2003 + 1)} \sum_{t=2003}^{N} E_{2003}(Y_t)
\]

\[
\text{Permanent Income Theory} \Rightarrow N \to \infty
\]

\[
E_{2003}(Y_{2005}) = E_{2003}[Y_{2004} + u_{2005} + 0.5e_{2004}] = E_{2003}[Y_{2004} + u_{2005} + 0.5(u_{2004} + 0.5e_{2003})] = E_{2003}[Y_{2004}] + 0.5^2e_{2003} =
\]

\[
C_{2003} = 990
\]
$$Y_{2003} + 0.5e_{2003} + 0.5^2e_{2003} =$$
$$Y_{2003} + (0.5 + 0.5^2)e_{2003}$$

$$E_{2003}(Y_{2006}) = E_{2003}[Y_{2003} + u_{2006} + 0.5e_{2005}] =$$
$$E_{2003}[Y_{2003} + u_{2006} + 0.5(u_{2005} + 0.5e_{2004})] =$$
$$E_{2003}[Y_{2005}] + 0.5^2E_{2003}[e_{2004}] =$$
$$(Y_{2003} + (0.5 + 0.5^2)e_{2003}) + (0.5^2E_{2003}[u_{2004} + 0.5e_{2003}]) =$$
$$Y_{2003} + (0.5 + 0.5^2 = 0.5^3)e_{2003}$$

$$E_{2003}(Y_{\infty}) = Y_{2003} + \left(\sum_{i=1}^{\infty} 0.5^i\right)e_{2003} = Y_{2003} + e_{2003} = 1000 + 100 = 1100$$

As \( N \to \infty \)
$$C_{2003} = 0.9 \cdot E_{2003}(Y_{\infty}) = 0.9(1100) = 990$$

(f) Under the same assumptions, by how much should consumption change in 2004 if \( u_{2004} = 10 \)?

$$Y_{2004} = Y_{2003} + u_{2004} + 0.5e_{2003} =$$
$$1000 + 10 + 0.5(100) = 1060$$

By similar argument as 1e, we get \( E_{2004}(Y_{\infty}) = Y_{2004} + e_{2004} = 1060 + 60 = 1120 \)

As \( N \to \infty \)
$$C_{2004} = 0.9 \cdot E_{2004}(Y_{\infty}) = 0.9(1120) = 1008$$

2. Suppose that the inflation rate, \( \pi \), is defined as the change in the log of the price level, \( P \). That is \( \pi_t = P_t - P_{t-1} \). Suppose further that inflation is described by the ARIMA process:

$$\pi_t = 0.03 + e_t$$

where \( e_t = u_t + 0.6e_{t-1} \) and \( u \) is a "white noise" error term.

(a) If the price level is described as an ARIMA(\( p, d, q \)), what are the values for \( p, d, \) and \( q \)?

\[
\begin{align*}
  p &= 1 & \text{There is 1 lagged } e \text{ term} \\
  d &= 1 & \text{There is 1 difference term (i.e., lagged } P) \\
  q &= 0 & \text{There are no lagged } u \text{ terms}
\end{align*}
\]

(b) If \( P_{2003} = 1.05 \) and \( P_{2002} = 1.00 \), what is the conditional forecast \( E_{2003}(P_{2004}) \)?

\( E_{2003}(P_{2004}) \)?

$$P_t = P_{t-1} + 0.03 + u_t + 0.6e_{t-1}$$

$$P_{2003} = P_{2002} + 0.03 + e_{2003}$$

\( e_{2003} = P_{2003} - P_{2002} - 0.03 = 1.05 - 1.00 - 0.03 = 0.02 \)

$$E_{2003}(P_{2004}) = E_{2003}[P_{2003} + 0.03 + u_{2004} + 0.6e_{2003}] =$$
$$P_{2003} + 0.03 + 0.6e_{2003} = 1.05 + 0.03 + 0.6(0.02) = 1.092$$

$$E_{2003}(P_{2005}) = E_{2003}[P_{2004} + 0.03 + u_{2005} + 0.6e_{2004}] =$$
$$E_{2003}[P_{2004} + 0.03 + u_{2005} + 0.6(u_{2004} + 0.6e_{2003})] =$$

2 of 3
\[ E_{2003}[P_{2004}] + 0.03 + 0.6^2e_{2003} = \]
\[ (P_{2003} + 0.03 + 0.6e_{2003}) + 0.03 + 0.6^2e_{2003} = \]
\[ P_{2003} + 2(0.03) + (0.6 + 0.6^2)e_{2003} = \]
\[ 1.05 + 2(0.03) + (0.6 + 0.6^2)0.02 = 1.1292 \]

**Documentation.**

Prof Bomberger told me consumption equals MPC times the value where income levels off in problems 1e and 1f.

I reviewed my work with Josh Kneifel.