Inflation and Unemployment

No Growth Model - IS-LM model we looked at before
\[
\frac{dP}{dM} = \frac{M}{P} > 0 \quad \text{(Price changes in proportion to } M) \\
\frac{dY}{dM} = \frac{di}{dM} = 0 \quad \text{(No effect on } Y \text{ or } i \text{ in long run)}
\]

Growth Model - assumes output is growing over time; we'll look at specific case where production function is Cobb-Douglas and \( \%ΔIS = \%ΔLM \Rightarrow \text{constant interest rate} \)

Purpose - we want to be able to look at changing the growth rate of the money supply \( (\mu = \frac{dM}{dT}/M) \), not just a one time change in the money supply \( (M) \)

Equations -

Static  
Specific/Growing
\[
Y = F(K, \bar{N}) \\
W/P = F_N(K, N) \\
N = \bar{N} \\
C = C(Y - T) \\
T \text{ exogenous} \\
G = gY \\
I = I(i - \pi^e) \\
M/P = L(i, Y)
\]
\[
Y = AK^\alpha \bar{N}^{1-\alpha} \\
W/P = (1 - \alpha)AK^\alpha N^{-\alpha} \\
N = \bar{N} \\
C = c(Y - T) \\
T = tY \\
G = gY \\
I = h(i - \pi^e)\bar{Y} \\
M/P = l(i)Y
\]

Math Tricks -
\[
\%Δ(M/P) = \%ΔM/M - \%ΔP/P \\
\%ΔM/M = (dM/dT)/M = \mu \text{ (here } T \text{ stands for time, not taxes)} \\
\%ΔP/P = (dP/dT)/P = \pi \\
%Δll/l = %ΔY/Y \\
%Δll/l = 0 \text{ (because we're assuming } i \text{ doesn't change)} \\
%ΔY/Y = (dY/dY)/Y = Y
\]
\[
\therefore \pi = \mu - \pi^e \quad \text{(rate of growth equals money growth rate minus inflation rate)}
\]

Result - rearrange terms: \( \pi = \mu - y \) to get formal relationship between \( \pi \) and \( \mu \) shown below (crosses \( \mu \) axis at positive \( \mu \) and has slope of 1)

Changing Variables - look at changes in growth model

Static  
Specific/Growing
\[
M \text{ fixed} \\
\bar{N} \text{ fixed} \\
K \text{ fixed} \\
A \text{ fixed} \\
\bar{N} \text{ fixed}
\]
\[
(dM/dT)/M = \mu \\
(d\bar{N}/dT)/\bar{N} = n \\
(dK/dT)/K = k \\
(dA/dT)/A = a \\
(d\bar{N}/dT)/\bar{N} = n
\]

Before consumption was generic function \( C \) of net income \( (Y - T) \); now it's a specific function \( c \) times \( (Y - T) \); \( c \) = marginal propensity to consume

Taxes \( (T) \) change proportionately with actual output \( Y \)

Gov't purchases \( (G) \) change proportionately with potential output \( \bar{Y} \)

Investment \( (I) \) changes proportionately with potential output \( \bar{Y} \); \( h' < 0 \)

Money demand \( (L) \) changes proportionately with output \( Y; l'' < 0 \)

Not the same \( k \) from Solow model (output per worker); now \( k \) = growth rate of capital
Output Growth - get a result similar to Solow model... output growth depends on technology, capital, and labor growth rates

\[ y = \frac{dY/dT}{Y} = \frac{d(AK^\alpha N^{1-\alpha})/dT}{AK^\alpha N^{1-\alpha}} = \frac{K^\alpha N^{1-\alpha} (dA/dT) + \alpha AK^{\alpha-1} N^{-\alpha} (dK/dT) + (1-\alpha) AK^\alpha N^{-\alpha} (dN/dT)}{AK^\alpha N^{1-\alpha}} \]

\[ = \frac{dA/dT}{A} + \frac{dK/dT}{K} + (1-\alpha) \frac{dN/dT}{N} = a + \alpha k + (1-\alpha) n \]

**Note:** \( \mu \) is not in this equation

Labor Growth - confirms another result of the Solow model

\[ \%\Delta (W/P) = \%\Delta W/W - \%\Delta P/P \]

\( \%\Delta W/W = (dW/dT)/W = w \) (% change in nominal wages)

\( \%\Delta P/P = (dP/dT)/P = \pi \) (inflation rate)

\[ \%\Delta (W/P) = \frac{d(W/P)/dT}{W/P} = \frac{d((1-\alpha) AK^\alpha N^{-\alpha})/dT}{(1-\alpha) AK^\alpha N^{-\alpha}} \]

\[ = \frac{K^\alpha N^{-\alpha} (dA/dT) + \alpha AK^{\alpha-1} N^{-\alpha} (dK/dT) - \alpha AK^\alpha N^{-\alpha-1} (dN/dT)}{AK^\alpha N^{-\alpha}} \]

\[ = \frac{(dA/dT)}{A} + \frac{(dK/dT)}{K} - \frac{(dN/dT)}{N} = a + \alpha k - \alpha n \]

**Result** - \( \%\Delta (W/P) \) (% change in real wages) = \( w - \pi = a + \alpha k - \alpha n = y - n \) (substituting equation \( y \) we found above); also, using the math trick above with %\Delta of ratios, %\Delta(\bar{Y}/\bar{N}) [i.e., output per worker] = \( y - n \); that confirms the Solow result: the growth rate of real wages = the growth rate of worker productivity

Money Growth - suppose constant money growth \( \mu \); this means LM curve steadily shifts right

Inflation - if \( \mu > \%\Delta IS \), there will be inflation

Example - \( \mu = 6\% \& \%\Delta IS = 3\% \Rightarrow \pi = 3\% \)

Stationary Graph - assume we're panning so IS-LM curves don't shift; just have arrows showing direction of change; if we print money faster (e.g., \( \mu \uparrow \) from 6\% to 10\%), result is growth in inflation (\( \pi \uparrow \) from 3\% to 7\%... amount is same: 4\%)

\( \Delta \pi \) causes \( \pi \) to change too (\( \uparrow \)), which effectively lowers real interest rate (\( i_0 - \pi \))... shifts IS right

IS\( \uparrow \) countered by \( P \uparrow \) which shifts LM left so \( i \uparrow \)

Prices increase faster than inflation this year, then settle down so \( \Delta i = \Delta \pi \Rightarrow \Delta \pi = \Delta \mu \)

Note: this illustrates the equation we had earlier: \( y = \mu - \pi \)
Increase $M$ vs. Increase $\mu$

**Monetary Neutrality** - no change in real variables ($Y, C, I, i, W/P, M/P$) in long run from $\Delta M$

**Monetary Superneutrality** - no change in real variables ($Y, C, I, i, W/P, M/P$) in long run from $\Delta \mu$; doesn't hold because $d(M/P)/d\mu \neq 0$

**Graphs** - solid lines show instantaneous changes; dashed lines show more realistic changes over time

\[
\begin{align*}
\Delta M & \quad \Delta \mu \\
Y & \quad dY/dM = 0 \quad dY/d\mu = 0 \\
C & \quad dC/dM = 0 \quad dC/d\mu = 0 \\
I & \quad dI/dM = 0 \quad dI/d\mu = 0 \\
i & \quad di/dM = 0 \quad di/d\mu = 1 \\
M/P & \quad d(M/P)/dM = 0 \quad d(M/P)/d\mu = l'Y < 0 \\
P & \quad dP/dM = P/M \quad dP/d\mu = -P l'Y l > 0 \\
\pi & \quad d\pi/dM = 0 \quad d\pi/d\mu = 1
\end{align*}
\]

$\Delta M$ - change money supply; affects $P$, but not $Y$ or $i$ (real variables); monetary neutrality (realistic)

$\Delta \mu$ - change rate of growth; affects $\pi$, but not $Y$ or $i$

$\pi$ vs. $\mu$ - notice intercept isn't at (0,0); that's because there's real growth so you can have increasing money supply ($\mu$) without inflation ($\pi$); we found the formal relationship earlier: $\pi = \mu - y$

$y$ vs. $\mu$ - note that there doesn't appear to be a relationship

between money growth rate and real output growth

**Phillip's Curve**

**Phillip's Curve** - plotted rate of change of money wage rates vs. unemployment (data from Brittan, 1861 to 1913) and concluded that there's a trade off between inflation and
unemployment; higher inflation means lower unemployment

**Why Care** - Okin's Law says for each % drop in unemployment rate, output (Y) increases by 2%; Phillips curve looks like we can trade off higher inflation in order to get lower unemployment

**Influential** - suggests increasing money growth rate in order to causes inflation and lower unemployment

**Realistic?** - data fit very well for U.S. in 60s, but not afterwards

**Theory** - if wages are increasing (W↑), it appears we started above \( \bar{N} \); that suggests we can have \( w > 0 \) or \( U > 0 \), but not both... not realistic because of structural unemployment

### Structural Unemployment

Adding structure to labor market can result in unemployment even when total demand for labor equals total supply \( (N^D = N^S) \)

**Numerical Example** - assume 4 different labor markets, each with supply of 100 workers (total supply of labor = 400 workers); with single labor market, as soon as demand for labor equals 400, unemployment goes to zero; with structured markets, it's possible to have demand exceed supply and still have unemployment (workers not in right market)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Overall</th>
<th>Single Market</th>
<th>Structured Market</th>
</tr>
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<tbody>
<tr>
<td>( N^D )</td>
<td>70</td>
<td>75</td>
<td>80</td>
<td>85</td>
<td>90</td>
<td>100</td>
<td>340</td>
</tr>
<tr>
<td>( w )</td>
<td>-6%</td>
<td>-4%</td>
<td>-2%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
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</tr>
</tbody>
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**Wage Stickiness** - \( w = \beta \left( \frac{N^D - \bar{N}}{\bar{N}} \right) \)

\( \beta \) is measure of how responsive wages are; higher value means more responsive; table above uses \( \beta = 0.4 \)

**Downward Stickiness** - graph shows stickiness for wages (they don't go down much); has different \( \beta \) when \( N^D - \bar{N} \) is negative vs. positive

**Non-linearity of Phillips Curve** - caused by different labor markets and sticky wages
Saving Phillips Curve - Friedman article ("The Role of Monetary Policy," 1968) explained why Phillips curve does fit U.S data

\[ w = \beta \left( \frac{N^D - \bar{N}}{\bar{N}} \right) + \pi^e \]

... usually have inflation (\( \pi \)) = wage growth rate (\( w \)) so changes in unemployment rate (\( N^D - \bar{N} \)) result from disparity between \( \pi \) and \( \pi^e \)

Sticky Wages - usually wages determined by contracts which try to anticipate \( w \) by estimating \( N^D - \bar{N} \) and \( \pi \) (using \( \pi^e \))

Short-Run Phillips Curve - plots unexpected \( \pi \) (i.e., \( \pi - \pi^e \)) vs. \( u \)

\( \pi^e < \pi \Rightarrow w \) lower in real terms so \( N^D > \bar{N} \) (tight labor market) \( u \downarrow \)

\( \pi^e > \pi \Rightarrow w \) higher in real terms so \( N^D < \bar{N} \) (excess supply) \( u \uparrow \)

Long-Run Phillips Curve - vertical line at natural rate of unemployment

Natural Rate of Unemployment - natural doesn't necessarily mean unchanging, just not influenced by M or \( \mu \); changes based on mismatches in structure of labor markets

Goes Down - retrain/move workers; wages adjust

Goes Up - price shocks (e.g., oil crisis in 70s); technology (changes increase mismatches in labor markets; e.g., programmers vs. typists)

Loops - if \( \pi \) keeps going up and down, expect clockwise loops, just like we see in U.S. data

Conclusion - can't use monetary policy to influence unemployment in long-run

Original Phillips Curve - very stable because Britain was on the gold standard so \( \pi^e \) was very stable (single curve)

Illusion - since there was only a single curve, the original paper by Phillips seemed to suggest we could permanently stay at high \( \pi \) and low \( u \) (which Friedman and U.S. data argued we can't)

Changes in Natural Rate - study by Lilien trying to measure impact of shocks on unemployment (1982)

\[ u_t = 52.6 \sigma_t - 15.3(\pi_t - \pi^e_t) - 16.6(\pi_{t-1} - \pi^e_{t-1}) + 0.728u_{t-1} \]

\[ R^2 = 0.739 \]

\( \sigma_t \) = standard deviation of employment in 11 broadest classifications of industry; evidence of shock when \( \sigma_t \) is big

Negative Coefficients - make sense because actual inflation > expected inflation means unemployment goes down

Higher Unemployment in Europe - various suggestions to explain it

- regulation in labor market
- high unemployment benefits
- reluctant to move (more mismatch in labor markets)
Role of Monetary Policy - in addition to fixing the Phillips Curve, Friedman’s article ("The Role of Monetary Policy") talked about the proper role; looked at 3 cases:

"Peg" u - try to keep unemployment rate (u) at u* by printing money faster (or slower)

Events - μ↑ ⇒ LM↑ ⇒ LM↓ (from P↑, but not all the way back to original level if you believe in Phillips Curve [trading π for u]) and IS↑ (from π↑ [i.e., I↑]) ⇒ LM↓ (from additional P↑ or W↓)

Results - Δπ = Δπ* = Δi = Δμ ⇒ real interest rates (i) don't change; works in short run (temporary fix), but eventually return to u0 (natural rate or "full" employment); to stay at u*, need to keep increasing M resulting in accelerating inflation (not the same as Phillips' original conclusion of a one-time trade off between π and u)

"Peg" i - try to keep interest rate (i) at i* by printing money faster (or slower)

Events - same as trying to peg u; note that i drops initially, but ends up with i1 > i0

Results - worse than trying to peg u; not only does it lead to accelerating inflation, but it ends up driving interest rates the wrong way

Real World - Fed doesn't really target interest rate in and of itself; interest rate targets are a short run way of getting economy to full employment; that's why Fed is always changing the target interest rate

"Peg" π (or μ) - try to keep inflation under control to π*

Events - assume π too high; want to get it down so μ↓ ⇒ LM↓ ⇒ IS↓ (from π↓ [i.e., I↓]) ⇒ LM↑ (from P↓; higher real money balances); this is exact opposite of IS-LM used for peg u above (that would be the case of wanting to increase inflation)

Results - achieving specified inflation rate is feasible; Fed doesn't target inflation because it’s necessarily most important; it’s just the only thing that can be controlled; Note: trying to bring inflation down causes a temporary increase in unemployment

Note: If Fed want to increase of decrease money supply (M), usually equivalent to increase of decrease rate of growth of money supply (μ) so "decrease" doesn't necessarily mean less money in the economy
Why not peg $\pi$ at zero? (or even negative?)

**Non-Vertical Phillips Curve** - Tobin, "Inflation and Unemployment"; argued that long-run Phillips curve is not vertical because wages are sticky downward; \( \therefore \text{as } i \downarrow \text{below a certain level}, u \uparrow \)

**Advantage of $\pi$** - inflation adds flexibility to the labor market; allows wages to adjust faster (people don't like a cut in nominal wages, but not adjusting for inflation is equivalent to a decrease in real wages)

**Liquidity Trap** - if there's a downturn from a change in aggregate demand (i.e., IS\( \downarrow \)), could get to \( i = 0 \) so there's no room to use monetary policy

**Where to Set Inflation** - we know we can't purposely set inflation higher to improve unemployment as Phillips proposed; we also saw that we don't want inflation at zero; what's the right level? Is there a benefit from high or low inflation? That's what we'll cover in the next section