Investment

Investment - looked at it before as a function of real interest rate: \( I = I(i - \pi) \), were \( I' < 0 \)
(i.e., \( (i - \pi) \)↑ \( \Rightarrow \) I↓)

3 Ways to Finance

Debt - costs interest on principle
Equity - costs dividends paid to stockholders
Internal - use retained earnings; most popular form for U.S. firms

Why Invest? - trying to get some return on the money; have to measure against what that money could do otherwise; assuming stockholders would save the money (i.e., get a return on it), we compare present value of future profits associated with the investment to the amount needed for the investment

Addition to Profits - depends on...
- Depreciation - profits decline over time
- Diminishing Returns to Capital - profits decline as you add more of the same investment
- Economic Conditions - expected future profits change based on expectations of economic condition (could go up or down)

\[ \text{PV of Profits} = \sum_{t=1}^{n} \frac{P_t}{(1 + r)^t} \quad (\text{PV} = \text{present value}) \]

Internal Rate of Return (IRR) - equivalent to treating investment like a saving's account; IRR is the interest rate that account would have (on average); found by solving PV of profits equation for \( r \); benefit of IRR is you don't have to recompute PV when interest rate changes to make investment decision

Investment Decision -
- PV - if PV of profit > cost of investment, do it
- IRR - if IRR > cost of capital (interest rate), do it

Example

<table>
<thead>
<tr>
<th>Project</th>
<th>Cost</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>PV of Profit (7%)</th>
<th>IRR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truck 1</td>
<td>-45000</td>
<td>18000</td>
<td>18000</td>
<td>15000</td>
<td>12000</td>
<td>53944</td>
<td>16.2%</td>
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<tr>
<td>Truck 2</td>
<td>-45000</td>
<td>15000</td>
<td>15000</td>
<td>15000</td>
<td>12000</td>
<td>48519</td>
<td>10.5%</td>
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<tr>
<td>Truck 3</td>
<td>-45000</td>
<td>3000</td>
<td>6000</td>
<td>16500</td>
<td>30000</td>
<td>44400</td>
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<tr>
<td>Truck 4</td>
<td>-45000</td>
<td>1500</td>
<td>3000</td>
<td>16500</td>
<td>24000</td>
<td>35801</td>
<td>0.0%</td>
</tr>
<tr>
<td>Desk 1</td>
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<td>2700</td>
<td>2700</td>
<td>2700</td>
<td>9145</td>
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<tr>
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<td>2250</td>
<td>2250</td>
<td>2250</td>
<td>7621</td>
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<tr>
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<td>2100</td>
<td>1500</td>
<td>750</td>
<td>0</td>
<td>3885</td>
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<td>20000</td>
<td>24000</td>
<td>28000</td>
<td>73374</td>
<td>3.6%</td>
</tr>
<tr>
<td>Light Bulb</td>
<td>-1.5</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1.81</td>
<td>21.5%</td>
</tr>
</tbody>
</table>
Internal Financing

Accelerator Model of Investment - not the same as doing multipliers which look at comparative statics (equilibrium conditions); this model instead looks at how investment changes over time so we see the in between steps, not just the end result; we’ll look at two versions: one period and multiple periods (partial adjustment)

Internal Financing - firm is taking money that it would otherwise give to shareholders in order to increase future income for shareholders; this means investment is based on intertemporal budget constraint; investment lowers \( Y_t \) in the hopes of increasing \( Y_t + 1, Y_t + 2, \ldots \) enough to increase consumption

\[
C_1 + \frac{C_2}{1 + r} + \frac{C_3}{(1 + r)^2} + \cdots = Y_1 + \frac{Y_2}{1 + r} + \frac{Y_3}{(1 + r)^2} + \cdots
\]

**Alternative** - assume shareholders have a risk free asset with yield \( i \) where they could put the money in; this is what we'll compare the investment's IRR to

Production Function, \( Y = F(K,N) \) - since value of investment (i.e., IRR) depends on future income \( (Y) \), we can use the production function to link \( I \) to \( Y \) through \( K \) (capital)

Limiting Factor - when we used \( F(K,N) \) before we assumed \( K \) was fixed, now it's variable; to constrain the problem we'll assume the limiting factor is expected demand, \( Y^e \)

Capital \( (K) \) - capital stock this year = capital stock from last year plus addition to capital (i.e., investment) minus depreciation of last year's capital stock; \( \delta = \) depreciation rate

\[ K_t = K_{t-1} + I_t - \delta K_{t-1} \]

Account for Inflation - if current price is \( P_t \) and there is a constant inflation rate \( \pi^e \), then

\[ E(P_{tm}) = P_t(1 + \pi^e)^t \]

Increase Capital Stock - to permanently increase capital stock by \( dK \), we have to...

Pay for It Now - \(-P_t dK \) (this is already a PV because it happens now)

Pay to Maintain - \(-P_t(1 + \pi^e)(\delta dK)/(1 + i)^t \) (to get PV of \( P_t \) for \( t = 1,2,\ldots \))

Get More Income - change in capital leads to greater output \( Y \) based on \( F_k = \partial F(K,N)/\partial K \);

\[ P_t(1 + \pi^e)(F_k dK)/(1 + i)^t \] (want to add up PV for \( t = 1,2,\ldots \))

Put it together: \( \Delta PV = -P_t dK + \frac{P_t(1 + \pi^e)(F_k dK - \delta dK)}{(1 + i)^t} + \frac{P_t(1 + \pi^e)^2(F_k dK - \delta dK)}{(1 + i)^{2t}} + \cdots \)

**Real Interest Rate** - \( r = (i + \pi^e)/(1 + \pi^e) \); we usually estimate this by \( i + \pi^e \)

Geometric series simplifies (don't worry about it): \( \Delta PV = -P_t dK + P_t(F_k - \delta dK)/r \)

Optimal Capital Stock \( (K^*) \) - set \( \Delta PV = 0 \) and solve for \( K \); two notes: (1) \( \Delta PV \) levels off from diminishing returns; (2) \( K \) will be embedded in \( F_k \); to solve for \( K \), we need a specific production function:

\[-P_t dK + P_t(F_k - \delta dK)/r = 0 \Rightarrow F_k = \delta + r \text{ (or } F_k + \delta = r) \]

**Gross IRR** - \( F_k \); invest up to point that gross IRR equals gross cost of capital: direct cost of investment (\( \delta \)) plus opportunity cost of investment (\( r \))

Net IRR - \( F_k + \delta \); invest up to point that net IRR equals net cost of capital (\( r \))

Relate to Investment - use formula for capital above to solve for investment:

\[ K_t = K_{t-1} + I_t - \delta K_{t-1} \Rightarrow I_t = K_t - K_{t-1} + \delta K_{t-1} = dK_t + \delta K_{t-1} \]

additional investment to get to \( K_t \) from \( K_{t-1} \) additional investment to maintain \( K_{t-1} \)
To get further results, we need a specific production function to find $F_K$

Cobb-Douglas Production Function - $Y = AK^\alpha N^{1-\alpha} \Rightarrow F_K = \alpha AK^\alpha N^{1-\alpha} = \alpha Y/K$

**Optimal Capital Stock** - $F_K = \alpha Y/K^* = \delta + r \Rightarrow K^* = \alpha Y/((\delta + r))$

Note: from here we can see directly that $Y^{\uparrow} \Rightarrow I^{\uparrow}$ and $r^{\uparrow} \Rightarrow I^{\downarrow}$

**One Period Adjustment** - assumes you invest enough to get from current capital stock ($K_{t-1}$, which we assume was optimal before) to new optimal value in a single period

$I_t = K_t^* - K_{t-1} + \Delta K_{t-1} = K_t^* - (1 - \delta)K_{t-1} = \alpha Y_{t-1}/((\delta + r_t)) - (1 - \delta)\alpha Y_{t-1}/((\delta + r_{t-1}))$

Through a little algebra magic (adding and subtracting $\alpha Y_{t-1}/((\delta + r_{t-1}))$) we can "simplify" to:

$I_t = \alpha \left( \frac{Y_t - Y_{t-1}}{\delta + r_t} \right) - \alpha \left( \frac{(r_t - r_{t-1})Y_{t-1}}{\delta + r_t} \right) + \alpha \left( \frac{\Delta Y_{t-1}}{\delta + r_{t-1}} \right) = \alpha \left( \frac{Y_t - Y_{t-1}}{\delta + r_t} \right) - \alpha \left( \frac{(r_t - r_{t-1})Y_{t-1}}{\delta + r_t} \right) + \alpha \left( \frac{\Delta Y_{t-1}}{\delta + r_{t-1}} \right)$

**Replacement Investment** - $I$ when $r_t = r_{t-1}$ and $Y_t = Y_{t-1}$; additional investment to maintain capital stock so $K_t = K_{t-1}$

**Example** - $\alpha = 0.25$, $\delta = 0.05$, $r = 0.05 \Rightarrow I_t = 2.5\Delta Y_t - 25\Delta r Y_{t-1} + 0.125Y_{t-1}$

$\Delta Y = -$1B $\Rightarrow I_t$ by 2.5GDP $\Delta r = -1% \Rightarrow I_t$ by 2.5GDP

**Example** - oven makes 1 loaf/hour; limited to 50 hr/week and lasts 1000 hr; $Y^* = 1000/wk$;

$K^* = 1000/50 = 20$ ovens; in long-run, expect to replace 1 oven per week ($I = 1$); if $Y^* \uparrow$ to 1100 (10% increase), now $K^* = 22$ ovens so investment includes 1 oven for depreciation plus 2 new ovens ($I = 3$)... that's 200% increase; long-run will be 1.1 ovens/week

**Realistic?** - average investment about 12.5% with large variations from average... that's realistic, although amount of variation with on period adjustment may be too much

**Partial Adjustment** - assumes you only invest a fraction ($\lambda$) of the gap between current capital stock and new capital stock; eqns included for completeness (don't need to know them)

$I_t = \lambda (K_t^* - K_{t-1}) + \delta K_{t-1},$ where $K_t = \lambda K_{t-1} + \lambda(1 - \lambda)K_{t-2} + \lambda(1 - \lambda)^2K_{t-3} + ...$

$I_t = \delta K_{t-1} + \lambda(K_{t-1} - K_{t-2}) + \lambda(1 - \lambda)(K_{t-2} - K_{t-3}) + \lambda(1 - \lambda)^2(K_{t-3} - K_{t-4}) + ...$

**Link to Recession?** - $Y^* \downarrow$ (recession) and $r \downarrow$ (expansionary monetary policy) have opposite effect so $I$ may $\uparrow$ or $\downarrow$; $I$ won't $\uparrow$ with certainty until $Y^* \uparrow$ (with $r$ unchanged)

**Cause?** - so is recession caused by $I \downarrow$ or does $I \downarrow$ because of recession ($Y^* \downarrow$); it's a self-fulfilling prophesy: if firms anticipate $Y^* \downarrow$, then $I \downarrow$ which will causes $Y \downarrow$ even if it wasn't going to happen!

**Other Methods of Financing**

**Example** - $10,000$ investment now pays $1,500 per year for 10 years.

**Internal Financing** - decision depends on real interest rate at which stockholders can reinvest their money; $r_{TB}$ (for Treasury Bills); if $r_{TB} = 10\%$

$\Delta PV = -10000 + \frac{1500}{1.1} + \frac{1500}{(1.1)^2} + \ldots + \frac{1500}{(1.1)^{10}} = -$783 \:. don't do project

**Debt Financing** - decision depends on interest rate firm gets to borrow at ($r_{DEBT}$); it will be higher than $r_{TB}$ to compensate lenders for the risk that the firm doesn't pay back the loan; for this example, let's assume there's no extra risk so $r_{TB} = r_{DEBT} = 10\%$; in this case firm doesn't pay anything in year 1; it pays interest on the loan each year (10% of 10,000 = $1,000$)
1,000); then in year 10, the firm may pay back the amount borrowed ($10,000); the change in the present value of income for shareholders in the same

\[ \Delta PV = 0 + \frac{1500 - 1000}{1.1} + \frac{1500 - 1000}{1.1^2} + \cdots + \frac{(1500 - 1000) - 10000}{(1.1)^{10}} = -783 \]

**Equity Financing** - new stockholders take share of firm's profits (not just from investment being financed and not just for duration of project); for now assume \( r_{\text{EQUITY}} = r_{\text{TB}} = 10\% \); in year 1 the firm pays nothing, but firm pays 1000 every period; investment decision is the same as other forms of financing

\[ \Delta PV = 0 + \frac{1500 - 1000}{1.1} + \frac{1500 - 1000}{1.1^2} + \cdots + \frac{(1500 - 1000) - 1000}{(1.1)^{11}} + \frac{1000}{(1.1)^{12}} \cdots = -783 \]

**Summary** - investment decision is identical regardless of method of financing if the rates are the same; in reality, however, \( r_{\text{TB}} < r_{\text{DEBT}} < r_{\text{EQUITY}} \), but investment decision is still the same because the differences in interest rates are accounting for differences in risk

**Risk & Return**

**Basic Gamble** - say you have $50,000; if you wager $25,000 on an even with 0.50 probability which results your losing or gaining $25,000 (i.e., total available to you is $25,000 or $75,000)

**Expected Utility** - probability of each outcome times the utility of each payoff in that outcome
(e.g., \( E(U) = 0.5U(25K) + 0.5U(75K) \))

**Risk Averse** - given the choice between a fixed amount of money and a gamble with an expected payoff of the same amount, a risk averse person would take the fixed amount; this is because of "diminishing returns" (i.e., concave utility function)

**Example** - \( U(C) = C^{1/2} \)

Fixed amount (no gamble) - \( U(50K) = 50K^{1/2} = 233 \)
Gamble - \( E(U \text{ of gamble}) = 1/2U(25K) + 1/2U(75K) = 1/2(158) + 1/2(274) = 216 \)
\( \therefore \) person with this utility function would take the fixed amount (i.e., risk averse)

**Risk Premium** - can look at it as either the amount by which the fixed amount needs to be lowered, or the amount by which the expected payoff of the gamble needs to be raised in order to get a risk averse person to take the gamble; actual value will be depend on the utility function (i.e., level of risk aversion); can write as $ or %

**Example** - want \( pU(25K) + (1-p)U(75K) = U(50K) \); solve for \( p \):

\[ p(U(25K) - U(75K)) = U(50K) - U(75K) \Rightarrow p = (U(50K) - U(75K))/(U(25K) - U(75K)) = 0.434 \]

\( \therefore \) Value - \( E($) \text{ of gamble} = 0.434(25K) + 0.566(75K) = $53,300 \)

\% Value - (53,300 - 50000)/50000 = 6.6%

**More Risk Averse** - more concave the utility function, the more risk averse, hence the higher risk premium (e.g., if \( U(C) = C^{1/5} \), risk premium is 10.5%)

**Risk Neutral** - \( U(C) = C \) (linear); risk premium is 0%

**Graphically** - need higher payoff to make up for increased risk:
Modigliani Miller Theorem

**MM Theorem** - if firms have same expected stream of income and same variance (risk), then market value of equity plus debt is constant (e.g., firm 1 has no debt and firm 2 has debt $D_2$:

$$V_1 = V_2 + D_2$$

**Example** -

**Firm A**: 0.5 chance of $8K/year and 0.5 chance of $12K/year; expected earnings $10K/year (± 20%); expected earnings over lifetime ($r_{TB} = 10\%$) is $10K/0.1 = $100K; risk averse investor would be willing pay less than this; assume risk premium is 2% so value of firm is $V_A = 10K/12\% = $83,333$

**Firm B**: required rate of return 12\% (same as Firm A); expected earnings of $10K ± 20\%$ (same as Firm A), but has $50K debt (at 10\%);∴ income is actually 0.5 chance of $8K - $5K = $3K and 0.5 chance of $12K - $5K = $7K; expected income is $5K ± 40\%$ (i.e., debt adds risk), use MM Theorem to calculate value of firm: $V_B + 50K = V_A$ ⇒ $V_B = 83,333 - 50K = $33,333; required return to account for risk: $5K/r = 33,333$ ⇒ $r = 15\%$

**Firm C**: required return is 12\% and expected earnings are $4K/year; given $V_C = 4K/0.12 = $33,333, what is risk class? based on info above a 12\% return is required for ± 20\%, ∴ payoffs for this firm are 0.5 chance of 3200 and 0.5 chance of 4800 because 800 is 20\% of $4K$

**Firm D**: Add $50K capital to firm C to raise earnings to $10K/year without changing risk (i.e., still ± 20\%)

Debt Financing - becomes same scenario as firm B so $V_{D1} = $33,333; since there is no change in value of firm (vs. firm C), stockholders are indifferent to the investment

Equity Financing - $V_{D2} = $83,333 = $33,333 + $50K (equity); old shares still worth $33,333 so original stockholders are not better off

Paradox? - 10\% for debt financing and 12\% for equity financing came to same investment decision; this is because difference in risk between different financing options; for debt financing, bank doesn't assume any of the risk (always gets its 10\%); for equity financing, new stockholders have to bear some of the risk (don't collect if firm doesn't make enough profit)