Consumption

\(C(Y - T)\) - for IS-LM models in previous section, we looked at consumption as function of after-tax income only

\(C(Y - T, i - \pi)\) - consumption as function of after-tax income and real interest rate

- if interest rates increase, consumption decreases; from IS-LM: now \(G \uparrow \Rightarrow I \downarrow \& C \downarrow\); there is "crowding out" of investment and consumption

**Shape of Consumption Function** - Keynes argued for two things:

- \(0 < \frac{C}{Y} < 1\) - spend more as \(Y\) increases, but not as much as \(\Delta Y\)
- \(\frac{\text{APC}}{\Delta Y} = \frac{C}{(Y - T)}\); same as saying average propensity to save decreases (i.e., rich people save more and consume less as proportion of income than poor people)

**Significance**

- increasing output/worker \((Y)\) means increasing standard of living means we’re getting richer as a society; should have \(\text{APC} \downarrow\) (i.e., more savings)

**Problem**

- \(\text{APC} \downarrow (\text{APS} \uparrow) \Rightarrow i \downarrow\); eventually get to lower limit on interest rate (zero); now economy can’t get to potential by lowering wages

**Keynes’ Solution**

- \(G \uparrow\) to fix economy

**Utility**

- firms are easy to explain; they seek profit; households are more difficult; we pretend we can quantify happiness or satisfaction (utility)

**Marginal Utility of Consumption**

- \(U' = \frac{\partial U}{\partial C} > 0\); if consumption increases, so does utility ("more is better")

\(U(C_1, C_2, \ldots, C_T)\) - people don’t derive utility from savings; savings are used to finance future consumption; \(\therefore\) look at utility as function of time series of consumption

**Future Consumption**

- assumes people can plan for future consumption (i.e., rational & calculating); Bomberger: “There are 100,000 versions of irrational; that’s too much work so we’ll assume rational” (rough paraphrase)

**Real Consumption** - accounts for price level

**Saving**

- \(S_1 = Y_1 - C_1\); labor income minus consumption; ignore other income for now because it’s endogenous (depends on saving); all three \((S, Y, C)\) are flows

**Assets**

- stock of accumulated savings; \(A_T = A_0(1 + r) + (Y_1 - C_1)(1 + r)\)

\((r = \text{real interest rate}; \text{assumes saving is done at beginning of year})\)

**Time Horizon**

- last year for planning

**Bequest**

- assets remaining at time \(T\); what you leave as your inheritance; must have \(A_T \geq 0\) or problem won’t be constrained (always happier if you die in greater debt); but if \(A_T \neq 0\) (i.e., \(A_T > 0\)), this becomes an intergenerational problem (hard to solve)

**Intertemporal Budget Constraint**

- have to choose consumption plan \((C_1, C_2, \ldots, C_T \& A_T)\) that you can afford; present value of spending and bequest must equal financial wealth plus present value of future labor; if you assume \(r\) is constant and saving is done at beginning of year: \(A_2 = (Y_2 - C_2)(1 + r) + A_1(1 + r)\)
Substitute $A_1$ from above so $A_2 = (Y_2 - C_2)(1 + r) + [(Y_1 - C_1)(1 + r) + A_0(1 + r)](1 + r) = (Y_2 - C_2)(1 + r) + (Y_1 - C_1)(1 + r)^2 + A_0(1 + r)^2$

Keep this up through the end of the time horizon and get:

$$A_T = (Y_T - C_T)(1 + r) + (Y_{T-1} - C_{T-1})(1 + r)^2 + ... + (Y_2 - C_2)(1 + r)^{T-1} + (Y_1 - C_1)(1 + r)^T + A_0(1 + r)^T$$

Put endogenous (choice) variables $(C_1, C_2, ... , C_T & A_T)$ on left side:

$$C_1 + \frac{C_2}{(1 + r)} + ... + \frac{C_2}{(1 + r)^{T-2}} + \frac{C_T}{(1 + r)^{T-1}} + \frac{A_T}{(1 + r)^{T-1}} = A_0 + Y_1 + \frac{Y_2}{(1 + r)} + ... + \frac{Y_T}{(1 + r)^T}$$

Book Version - assumes interest added at end of year (not start); leaves out $A_T$, but since uses $\leq$ (rather than $=$), it assumes $A_T \geq 0$

$$\sum_{i=0}^{T} \frac{1}{(1 + r)^i} C_i \leq A_0 + \sum_{i=0}^{T} \frac{1}{(1 + r)^i} Y_i$$

No interest rate ($r = 0$): $\sum_{i=0}^{T} C_i \leq A_0 + \sum_{i=0}^{T} Y_i$

Intertemporal Utility Function - $U(C_1, C_2, ..., C_T, A_T)$

More is Better - $\partial U/\partial C_i = u_i > 0$; usually also assume $\partial U/\partial A_T = u_A > 0$ (but could be $< 0$)

Diminishing Marginal Utility - $\partial^2 U/\partial C_i^2 = u''(i) < 0$

Cross Terms - $\partial^2 U/\partial C_i \partial C_j$ could be anything; usually assume $= 0$; if $\neq 0$, then consumption in one year leads to greater (or less) utility in some other year.

Book Version - assumes $u_{ij} = 0 (i \neq j)$; assumes utility function $(u)$ is same form year to year

$$U = \sum_{i=0}^{T} u(C_i) = u(C_1) + u(C_2) + ... + u(C_T) \ ; \ u''(i) > 0, \ u''''(i) < 0$$

Analogy to Consumer Theory - Max $U(Q_1, Q_2, ...)$ subject to $P_1 Q_1 + P_2 Q_2 + ... = Y$; basically doing same thing except now we're doing it over time.

Optimal Solution - consumption plan $C_1^*, C_2^*, ..., C_T^*$ that maximizes utility subject to budget

Example - (p345)

$$U = \sum_{i=0}^{T} \frac{1}{(1 + \rho)^i} C_i^{1-\theta} = \frac{1}{1-\theta} \left[ \frac{C_1^{1-\theta}}{1 + \rho} + \frac{C_2^{1-\theta}}{(1 + \rho)^2} + ... + \frac{C_{T-1}^{1-\theta}}{(1 + \rho)^T} + \frac{C_T^{1-\theta}}{(1 + \rho)^T} \right]$$

Risk Aversion $(\theta)$ - how severe are diminishing returns; determines willingness to shift consumption between different periods; smaller $\theta$ means marginal utility falls slower as consumption rises (more willing to allow consumption to vary over time); $\theta$ is coefficient of relative risk aversion (the inverse of the elasticity of substitution between consumption at different dates).

Discount Rate $(\rho)$ - similar to interest rate in time value of money; $\rho$ measures "time value of consumption"; consumption now is better then consumption later; larger $\rho$ means more impatient (consumption now even better than later)

Check More is Better - $U_1 = \frac{C_1^{1-\theta}}{1 + \rho} > 0$

Check Diminishing $MU - U_{11} = -\frac{\theta C_1^{1-\theta-1}}{1 + \rho} < 0$
Simple Case - \( T = 2, A_0 = 0; A_T = A_2 = 0; \partial U/\partial A_T = U_A = 0 \)

Budget Constraint: \( C_1 + \frac{C_2}{(1+r)} = Y_1 + \frac{Y_2}{(1+r)} \)

Solve constraint for \( C_2 \): \( C_2 = Y_1(1+r) + Y_2 - C_1(1+r) \)

Only 1 Choice: if you pick \( C_1 \), \( C_2 \) is automatically determined

Indifference Curve - all combinations of \( C_1 \) and \( C_2 \) that result in same amount of utility

Optimal Solution - indifference curve just tangent to budget line; from 1st order condition:

\[ U_1 - (1 + r)U_2 = 0 \implies \text{you can't make yourself better off by saving more (or less)} \]

Interpretation - extra dollar spent in \( C_1 \) gains \( U_1 \), but you lose \( (1 + r) \) dollars in \( C_2 \) which is a loss of \( (1 + r)U_2 \)

What Makes People Save - take total differential of 1st order condition:

1st Order Condition: \( U_1(C_1) - (1 + r)U_2(Y_1(1 + r) + Y_2 - C_1(1 + r)) = 0 \)

Differential: \( U_1dC_1 - U_2dr - (1 + r)U_2(Y_1dr + (1 + r)dY_1 + dY_2 - C_1dr - (1 + r)dC_1) = 0 \)

Combine terms & solve for \( dC_1 \):

\[ dC_1 = \frac{U_{22}(1+r)^2}{U_{11} + U_{22}(1+r)^2} dY_1 + \frac{U_{22}(1+r)}{U_{11} + U_{22}(1+r)^2} dY_2 + \frac{(Y_1 - C_1)(1+r)U_2}{U_{11} + U_{22}(1+r)^2} dr \]

Interest Effect - earning income now \( (Y_1) \) is better than next year (i.e., \( dC_1/dY_1 > dC_1/dY_2 \)); evident in \( (1 + r)^2 \) term vs. \( (1 + r) \); if \( r = 0 \), this effect goes away \( (dC_1/dY_1 = dC_1/dY_2) \)

Increase \( r \) - budget line is steeper because consumption next year \( (C_2) \) is cheaper relative to this year; budget line rotates on point where interest is irrelevant (i.e., \( C_1 = Y_1 \) and \( C_2 = Y_2 \); consume exactly what you make; zero savings); effect on \( C_1 \) depends on savings in year 1 \( (Y_1 - C_1) \)

Negative Savings (Borrowing) - \( Y_1 - C_1 < 0; dC_1/dr < 0 \); makes sense because borrowing will be more expensive

No Savings - \( Y_1 - C_1 = 0 \implies Y_2 = C_2 \) (consuming at rotation point on graph); \( dC_1/dr < 0 \); make sense because you'll be better off saving

Positive Savings - \( Y_1 - C_1 > 0; dC_1/dr \) can be > 0 or < 0

Income Effect - more income available in year 2; can transfer some back to year 1; could have \( C_2↑ \) and \( C_1↑ \) with positive savings; negative savings will have \( C_1↓ \) (similar to consumer theory if \( p_{\text{Beef}} \downarrow \), buying same amount of beef, you now have more income to spend on other goods)

Substitution Effect - since consumption in year 2 is relatively cheaper than year 1, buy more of it (similar to consumer theory if \( p_{\text{Beef}} \uparrow \), substitute chicken with beef)

Interest \( (r) \) vs. Discount Rate \( (\rho) \) -

\( r = 0 \implies u_1 = u_2 = u_3 = ...; \) with no discounting \( (\rho = 0) \), spend same amount every year

\( r > 0 \) and \( \rho > 0 \) imply opposite effects; may cancel; people tend to “smooth out” consumption over time (borrow in lean years and save in good years);

two models to explain this: life-cycle and permanent income
Life-Cycle Model

**Basics** - forward looking person can smooth out consumption; borrow early; repay debt and save during peak years; graph shows how assets ($A$) change over time ($A'$)

**Numerical Example** -

<table>
<thead>
<tr>
<th></th>
<th>Young</th>
<th>Middle</th>
<th>Old</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poor</td>
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<td>25K</td>
<td>5K</td>
<td>15K</td>
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<td>Middle</td>
<td>25K</td>
<td>35K</td>
<td>15K</td>
<td>25K</td>
</tr>
<tr>
<td>Rich</td>
<td>35K</td>
<td>45K</td>
<td>25K</td>
<td>35K</td>
</tr>
</tbody>
</table>

**Assumptions** - no uncertainty; each period is same amount of time; no interest rate

**Consumption for** $A_T = 0$ - i.e., $MPC = 0$; for poor person: $(15K + 25K + 5K)/3 = $15K; for middle person: $(25K + 35K + 15K)/3 = 25K; rich person: $(35K + 45K + 25K)/3 = 35K

**Budget Study** - record income and consumption

<table>
<thead>
<tr>
<th>Income</th>
<th>Consumption</th>
<th>Saving</th>
</tr>
</thead>
<tbody>
<tr>
<td>5K</td>
<td>15K</td>
<td>-10K</td>
</tr>
<tr>
<td>15K</td>
<td>20K</td>
<td>-5K</td>
</tr>
<tr>
<td>25K</td>
<td>25K</td>
<td>0</td>
</tr>
<tr>
<td>35K</td>
<td>30K</td>
<td>5K</td>
</tr>
<tr>
<td>45K</td>
<td>35K</td>
<td>10K</td>
</tr>
</tbody>
</table>

**Problem** - budget study makes it look like rich people (higher income) save more; plot above indicates $MPC = 0.5$, but model has no difference (all have $MPC = 1$)

**Modigliani & Brumberg Model** (1954) -

\[
C_t = \frac{Y_t + (T - 1)Y_t^e + A_t}{T}
\]

$Y_t^e$ = expected income on average from $t + 1$ to future; can't measure it so by assumption:

\[
Y_t^e = \beta Y_t
\]

**Regression** - used $C_t = a_1Y_t + a_2A_t$, where $a_1 = \frac{1 + \beta(T - 1)}{T}$ and $a_2 = \frac{1}{T}$

**Result** - $C_t = 0.7Y_t + 0.06A_t$ $\Rightarrow$ $C_t = 0.7 \frac{Y_t}{GDP} + 0.06 \frac{A_t}{GDP} = 0.71$

Long-Run $MPC = 0.71 >$ Short-Run $MPC = 0.52$

**Interpretation of Short-Run** - if GDP↓, each $1↓ \Rightarrow Y_t \downarrow$ $0.75 \therefore C↓$ by $0.7(0.75) = 0.52$; agrees with real world experience (consumption usually doesn't go down much during recessions)
Permanent Income Theory

Friedman Model (1957) - people plan with very long time horizon; indefinite because of heirs

\[ \text{Wealth} = A_0 + \frac{Y_1}{1 + r} + \frac{Y_2}{(1 + r)^2} + \cdots \] (no end)

- Financial Capital
- Human Capital
  (PV of labor income)

**Permanent Income** \((Y_p)\) - sustainable level of consumption without diminishing wealth; \(Y_p = r \cdot \text{wealth} \)

**Transitory Income** \((Y_{TR})\) - difference between permanent income and actual income; \(Y = Y_p + Y_{TR} \)

**Permanent & Transitory Consumption** - \(C = C_p + C_{TR} \)

**Model** - \(C_p = kY_p\), \(k\) is factor of proportionality \((\text{MPC in long-run})\)

**Assumptions** -
1. \(C_{TR} \sim C_p\) (deviations from average consumption not correlated)
2. \(Y_{TR} \sim Y_p\) (same)
3. \(k \sim Y_p\) (rich & poor have same MPC)
4. \(C_{TR} \sim Y_{TR}\) (consumption varies from different circumstances than income)

**Numerical Example** - using \(k = 0.9\); no circumstances so \(C_p = C\)

<table>
<thead>
<tr>
<th></th>
<th>Yp</th>
<th>Bad</th>
<th>Avg</th>
<th>Good</th>
<th>C_p</th>
<th>C</th>
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</thead>
<tbody>
<tr>
<td>Poor</td>
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<td>0</td>
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<td>20K</td>
<td>9K</td>
<td>9K</td>
</tr>
<tr>
<td>Middle</td>
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<td>10K</td>
<td>20K</td>
<td>30K</td>
<td>18K</td>
<td>18K</td>
</tr>
<tr>
<td>Rich</td>
<td>30K</td>
<td>20K</td>
<td>30K</td>
<td>40K</td>
<td>27K</td>
<td>27K</td>
</tr>
</tbody>
</table>

**Budget Study** -

<table>
<thead>
<tr>
<th>Income ((Y))</th>
<th>Consumption</th>
<th>MPC</th>
<th>Saving</th>
<th>(Y_p)</th>
<th>(Y_{TR})</th>
<th>MPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9K</td>
<td>-9K</td>
<td>10K</td>
<td>-10K</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10K</td>
<td>13.5K</td>
<td>0.45</td>
<td>-3.5K</td>
<td>15K</td>
<td>-5K</td>
<td>0.9</td>
</tr>
<tr>
<td>20K</td>
<td>18K</td>
<td>0.45</td>
<td>2K</td>
<td>20K</td>
<td>0</td>
<td>0.9</td>
</tr>
<tr>
<td>30K</td>
<td>22.5K</td>
<td>0.45</td>
<td>7.5K</td>
<td>25K</td>
<td>5K</td>
<td>0.9</td>
</tr>
<tr>
<td>40K</td>
<td>27K</td>
<td>0.45</td>
<td>13K</td>
<td>30K</td>
<td>10K</td>
<td>0.9</td>
</tr>
</tbody>
</table>

\[ \frac{(9K + 18K + 27K)/3}{(22.5K - 18K)/10K} = \frac{\Delta C/\Delta Y_p}{\Delta C/\Delta Y_{p}} = \frac{\text{MPC}}{\text{MPC}} \]

**Problem** - incorrect conclusion from single year’s data (“rich save more than poor”); it’s not just rich & poor, but people with good and bad years

**Regression** - \(C = kY_p\)

**Problem** - can’t measure permanent income

**Solution** - use expectation weighted average of past income: \(Y_{pt} = w_0Y_t + w_1Y_{t-1} + w_2Y_{t-2} + \cdots\)

**Assumptions** -
1. \(w_1 = w_2 = w_3 = \cdots = \lambda < 0\) (declining weights; recent history is more important)
2. \(\sum_{i=1} w_i = 1\)

**Results** - back before computers, Friedman ran regression for \(\lambda = 0.1, 0.2, \ldots, 0.9\); best result was \(\lambda = 0.7, k = 0.9, w_0 = 0.3, w_1 = 0.21, w_2 = 0.14, \ldots, w_{30} = 0.01\)
Long-Run - \( MPC = k = 0.9 \)

Short-Run - \( Y_t \downarrow \$1 \Rightarrow Y_{t-1} \downarrow \) by \( w_0 = 0.3 \) \(. \cdot C_t \downarrow \) by \( w_0k = 0.3(0.9) = 0.27 \) ** consumption not affected that much by recessions

Robert Hall (1978) - paper found support for and against permanent income theory

Specific Utility Function - \( U(C_1, C_2, \ldots) = \ln(C_1) + \frac{\ln(C_2)}{1 + r} + \frac{\ln(C_1)}{(1 + r)^2} + \ldots \)

Test Assumptions -
\[
U_i = \frac{dU}{dC_i} = \frac{1}{C_i} > 0, \quad U_{ii} = \frac{d^2U}{dC_i^2} = -\frac{1}{C_i^2} < 0, \quad \text{and} \quad U_{ij} = 0 \quad (\forall \ i \neq j)
\]

Consumption "Discount Rate" (\( \rho \)) - if \( \rho > 0 \), people are impatient; rather consume more now rather than later

First Order Conditions - \( U_1 = (1 + r)U_2 \); \( U_i \) is what you lose for reducing consumption in year 1; \( (1 + r)U_2 \) is what you gain in year 2 for reducing consumption in year 1

General Case - \( U_i = (1 + r)U_{i+1} \)

Year 1 - \[
\frac{1}{C_1} = \frac{1 + r}{(1 + \rho)C_2} \quad \ldots \quad \text{solve for} \ C_2: \quad C_2 = \frac{1 + r}{1 + \rho} C_1
\]

So \( r \uparrow \Rightarrow \) move consumption to year 2; \( \rho \uparrow \Rightarrow \) move consumption to year 1

Year 2 - \[
\frac{1}{(1 + \rho)C_2} = \frac{1 + r}{(1 + \rho)^2C_3} \quad \ldots \quad \text{solve for} \ C_3: \quad C_3 = \frac{1 + r}{1 + \rho} C_2 = \left( \frac{1 + r}{1 + \rho} \right)^2 C_1
\]

General Case - \( C_i = \left( \frac{1 + r}{1 + \rho} \right)^{i-1} C_1 \)

Substitute 1st Order conditions into intertemporal budget constraint:
\[
C_1 + \left( \frac{1 + r}{1 + \rho} \right)^{C_1} + \left( \frac{1 + r}{1 + \rho} \right)^{2C_1} + \ldots = A_0 + Y_1 + \frac{Y_2}{1 + r} + \frac{Y_3}{(1 + r)^2} + \ldots
\]

\[
C_1 \left( 1 + \frac{1}{(1 + \rho)} + \frac{1}{(1 + \rho)^2} + \ldots \right) = A_0 + Y_1 + \frac{Y_2}{1 + r} + \frac{Y_3}{(1 + r)^2} + \ldots
\]

Solve for \( C_1 \):
\[
C_1 = \frac{\rho}{1 + \rho} \left( A_0 + Y_1 + \frac{Y_2}{1 + r} + \frac{Y_3}{(1 + r)^2} + \ldots \right)
\]

Problem - still can’t measure future incomes \( (Y_2, Y_3, \ldots) \)

Expected Future Income \( E_i(Y_j) \) - expectation in year \( i \) of income in year \( j \)

Solution -
\[
C_1 = \frac{\rho}{1 + \rho} \left( A_0 + Y_1 + \frac{E_1(Y_2)}{1 + r} + \frac{E_1(Y_3)}{(1 + r)^2} + \ldots \right)
\]

Year 2:
\[
C_2 = \frac{\rho}{1 + \rho} \left( A_1 + Y_2 + \frac{E_2(Y_3)}{1 + r} + \frac{E_2(Y_4)}{(1 + r)^2} + \ldots \right)
\]

Rational Expectation - will look at optimal forecast; \( E_1(Y_3) \) contains all pertinent information in year 1 regarding income in year 3; \( E_2(Y_3) \) updates this information so
expected value could be different, but the difference should only result from new
information (i.e., no lagged variables should be significant)

**Know Now or Later** - some use 𝑌1, others use 𝐸1(𝑌1), depends on assumption of when
information becomes available; not critical for results

Assume 𝑟 = 0 (for simplicity) and consume same amount each period (permanent
income theory), now:

\[
C_1 = \left[ A_0 + E_1(Y_1) + E_1(Y_2) + E_1(Y_3) + \cdots \right] / T
\]

\[
C_2 = \left[ A_0 + E_2(Y_2) + E_2(Y_3) + \cdots \right] / (T-1)
\]

Substitute \( A_1 = A_0 + Y_1 - C_1 \) and add terms like \( E_i(Y_i) - E_i(Y_i) = 0 \) \( (i = 2, 3, \ldots) \)

\[
C_2 = \left[ A_0 + Y_1 - C_1 + E_2(Y_2) + E_2(Y_3) + \cdots + [E_1(Y_2) - E_i(Y_2)] + [E_1(Y_3) - E_i(Y_3)] + \cdots \right] / (T-1)
\]

Swap the \( E_2(Y_2) \) and \( E_1(Y_1) \) terms

\[
C_2 = \left[ A_0 + Y_1 - C_1 + E_2(Y_2) + E_1(Y_3) + \cdots + [E_2(Y_2) - E_i(Y_2)] + [E_2(Y_3) - E_i(Y_3)] + \cdots \right] / (T-1)
\]

**Forecast Revision** - \( E_2(Y_i) - E_1(Y_i) \)

Add \( E_1(Y_1) - E_1(Y_i) \)

\[
C_2 = \left[ [A_0 + Y_1 - C_1 + E_2(Y_2) + E_1(Y_3) + \cdots + [E_2(Y_2) - E_i(Y_2)] + [E_2(Y_3) - E_i(Y_3)] + \cdots \right] / (T-1)
\]

Rearrange terms and substitute \( E_2(Y_1) = Y_1 \) (perfect info after the fact so year 2’s
expected value of income in year one is the actual income from year 1)

\[
C_2 = \left[ -C_1 + A_0 + E_1(Y_1) + E_2(Y_2) + \cdots + [E_2(Y_2) - E_i(Y_2)] + [E_2(Y_3) - E_i(Y_3)] + \cdots \right] / (T-1)
\]

\[
T-C_1 \quad \text{Revision on all forecasts}
\]

Plug in \( T-C_1 \) and collect terms

\[
C_2 = \left[ C_1(T-1) + [E_2(Y_1) - E_i(Y_1)] + [E_2(Y_2) - E_i(Y_2)] + \cdots \right] / (T-1)
\]

Break up sum to cancel \( (T-1) \) in first term

\[
C_2 = C_1 + \left[ [E_2(Y_1) - E_i(Y_1)] + [E_2(Y_2) - E_i(Y_2)] + \cdots \right] / (T-1)
\]

**Finding** - only change consumption if there’s been some change in expected future
income (i.e., \( C_2 = C_1 \) unless forecasts change)

**What Changes Forecasts** - from rational expectations (& math shown above), only new
information changes forecasts (hence consumption); any lagged terms should be
insignificant: \( C_t = C_{t-1} + a Y_{t-2} \) should yield \( a = 0 \) because \( C_{t-1} \) already incorporates \( Y_{t-2} \)

**Regression** - used quarterly consumption data (1948:1 to 1977:1) for services and non-
durables

**Consumption Categories** - national income and product accounts include 3 types of
consumption: durables, non-durables, and services

**No Durables** - Hall left out durables because he argued they are more like savings or
investment; purchase is done at one time, but consumption is taken over a period of
time; Example: car provides transportation service; NIPA records sale in year 1 as
consumption, but services consumed last longer (10 years in Bomberger’s case); so
consumption of service is smooth over time, but purchases aren’t

**Confirmation** - \( C_t = 1.02C_{t-1} - 0.01Y_{t-1} \) (model used \( Y_1 \) not \( E(Y_1) \))

\[
R^2 = 0.9988
\]

Confirmation of theory because coefficient for \( Y_{t-1} \) is insignificant

**Discredit** - another regression using inputs that shouldn’t affect \( C_t \); discredits the theory

\[
C_t = 1.01C_{t-1} + 0.223S_{t-1} - 0.258S_{t-2} + 1.67S_{t-3} - 0.120S_{t-4} \quad (S = \text{index of stock prices})
\]

\[
R^2 = 0.9985
\]

Evidence against theory; shouldn’t have lagged variables beyond \( t - 1 \) that are
significant in determining \( C_t \); that information should be captured in \( C_{t-1} \)
John Shea (1995) - many tried to find other ways to discredit permanent income theory and rational expectations; Shea focused on fact that Hall used aggregate income so he didn't catch individuals with lower income resulting from job loss or illness; Shea used survey of income dynamics that followed individual households; focused on those with union labor because of the predictable income; 647 observations

**Regression**  
\[ C_t = C_{t-1} + aY_{t-1} \]  
\( Y_{t-1} \) is predictable component of income taken from labor contracts;  
\( a = 0.89 \ (0.46) \), large impact, but not statistically significant

**Salvaging Personal Income Theory** - economists like the theory so there are several explanations for why empirical tests seem to contradict it

**Liquidity Constraints** - borrowing to maintain constant consumption may not be possible because banks don't care about person's expected future income; difficult to borrow against human capital

- **Person A** - \( A_0 \) large and \( E_1(Y_{t0}) \) low; will save a lot now for future consumption
- **Person B** - \( A_0 \) small and \( E_1(Y_{t0}) \) large; would like to borrow now, but banks won't let him
- **Person C** - \( A_0 \) large (inheritance) and \( E_1(Y_{t0}) \) large; will run down assets (equivalent of borrowing)

**Predictions** - life-cycle model and permanent income theory predict person A and C; it's not possible for person B to behave like C which is what theory says; instead B will have lower consumption in early years, but will make up for it with higher consumption in future then he would've otherwise because he won't have to repay debt (see graph)

**Savings**

**Misconception** - many people say U.S. doesn't save much, but our GDP growth is same or higher than other industrialized countries over last 10-15 years; data from 1990s as % GDP:

<table>
<thead>
<tr>
<th>Savings</th>
<th>U.S.</th>
<th>Germany</th>
<th>UK</th>
<th>Canada</th>
</tr>
</thead>
<tbody>
<tr>
<td>National</td>
<td>16.5</td>
<td>21.9</td>
<td>15.7</td>
<td>17.5</td>
</tr>
<tr>
<td>Government</td>
<td>-2.9</td>
<td>-2.6</td>
<td>-3.7</td>
<td>-4.4</td>
</tr>
<tr>
<td>Private</td>
<td>19.4</td>
<td>24.5</td>
<td>19.4</td>
<td>21.9</td>
</tr>
</tbody>
</table>

**What Savings** - data above show U.S. saving isn't much different, but reports on low U.S. savings tend to focus on personal savings

**2002 Data** - savings for U.S. as % of GDP

- **National** 15.1
- **Government** -0.2
- **Business** 12.5
- **Personal** 2.8

**Government Policy**

**Budget Constraint** - if people plan and are forward looking, they plan budget based on after-tax income

\[ C_1 + \frac{C_2}{(1+r)} + \frac{C_3}{(1+r)^2} + \cdots = A_0 + (Y_1 - T_1) + \frac{Y_2 - T_2}{(1+r)} + \frac{Y_3 - T_3}{(1+r)^2} + \cdots \]

**Expected Future Taxes** \((E_t(T_i))\) - taxes people expect to pay in year \( i \) based on information in year 1
\[
C_1 + \frac{C_2}{(1+r)} + \frac{C_3}{(1+r)^2} + \cdots = A_0 + (Y_1 - T_1) + \frac{E_1(Y_2) - E_1(T_2)}{(1+r)} + \frac{E_1(Y_3) - E_1(T_3)}{(1+r)^2} + \cdots
\]

**Rebate vs. Rate Cut** - tax rebate reduces \(T_1\) (or some other specific year); tax rate cut changes all future \(T_i\); \(\therefore\) rate cuts which haven't occurred could still influence current consumption based on life-cycle model and permanent income theory (people smooth consumption)

**Government Budget** - \(B_i = B_0(1+r) + G_1 - T_1\)

**Budget Surplus/Deficit** - revenue minus interest expense minus government purchases \(T_i - rB_0 + G_1\); positive value is a surplus; negative value is a deficit

**Net Taxes \((T_i)\)** - taxes collected minus transfer payments in year \(i\)

\textbf{Note}: \(T^\uparrow\) means tax revenue increase; says nothing about tax rates; if lowering tax rate increases revenue you still have \(T^\uparrow\)

**Government Debt \((B_i)\)** - bonds outstanding in year \(i\); can rewrite equation above to show debt from previous year minus surplus (or plus deficit): \(B_i = B_0 + rB_0 + G_1 - T_1\)

Use same formula for \(B_2\):

\(B_2 = B_1(1+r) + G_2 - T_2\)

Substitute \(B_1\):

\(B_2 = B_0(1+r)^2 + (G_1 - T_1)(1+r) + G_2 - T_2\)

Look at general case:

\(B_N = B_0(1+r)^N + (G_1 - T_1)(1+r)^{N-1} + (G_2 - T_2)(1+r)^{N-2} + \cdots + G_N - T_N\)

**Government Budget Constraint** - put \(G_1, G_2, \ldots, G_N \& B_0\) on left side (similar to intertemporal budget constraint):

\[
\left(\begin{array}{c}
G_1 \\
\frac{G_2}{1+r} \\
\frac{G_3}{(1+r)^2} \\
\vdots \\
\frac{G_N}{(1+r)^{N-1}} \\
B_0
\end{array}\right)
= \left(\begin{array}{c}
T_1 \\
\frac{T_2}{1+r} \\
\frac{T_3}{(1+r)^2} \\
\vdots \\
\frac{T_N}{(1+r)^{N-1}} \\
B_N
\end{array}\right)
\]

**Interpretation** - people view government spending as a liability to them; \(\therefore\) \(C\) is unchanged when \(T^\downarrow\) with \(G\) fixed because people view it as an increase in liability in future (i.e., if \(G\) doesn't change, \(T\) will have to be raised again in future)

**Previous Models** - IS-LM multipliers and permanent income theory suggested that \(T^\downarrow\) with fixed \(G\) would increase consumption

**Ricardian Equivalence** - tax finance and debt finance are equivalent; cutting taxes with fixed \(G\) doesn't change consumption because people save \(\Delta T\) in anticipation of having to pay it back in the future; \(C(T - Y)\) assumption is incorrect

**Criticism** - there's no logical flaw with Ricardian Equivalence, but people argue the assumptions: rather than being forward looking, many economists say people are short-sighted; also some don't like the infinite horizon of the permanent income model; this second criticism was addressed by Barro

**Overlapping Generations Model (Barro 1974)** - Assume each generation lives 2 periods and overlaps the next generation by 1 period; each has a utility function based on their consumption in those two periods AND the utility of their offspring
Generation A: $U^A(C^A_1, C^A_2, U^B)$, Generation B: $U^B(C^B_1, C^B_2, U^C)$, etc.

**Assumptions** - $U^A_1 = \frac{\partial U^A}{\partial C^A_1} > 0$, $U^A_2 = \frac{\partial U^A}{\partial C^A_2} > 0$, $U^A_3 = \frac{\partial U^A}{\partial U^B}$ depends

- **Hate Kids** - leave debt; $U^A_3 < 0$; not allowed by assumption
- **No Bequest** - don’t leave anything for kids; $U^A_3 = 0$
- **Like Kids** - leave something for kids; doesn’t have to be inheritance, could be transfer during their lives (e.g., school, car, etc.); $U^A_3 > 0$

**1st Order for Consumption** - $U^A_1 = (1 + r)U^A_2$

**1st Order for Bequest** - if $U^A_3 > 0$ (i.e., leaving bequest), $U^A_2 = U^B_1U^A_3$ (i.e., marginal utility for consumption in period 2 for generation A equals increased utility of consumption for generation B times how this increase in utility for B increases utility for A)

**Indifference Curves** -

- Budget Constraint - slope of -1; transfer $1$ from generation A means generation B gets $1$ because transfer occurs in same time period; vertical intercept is sum of total income for both generations; interpretation here is questionable since generation A can’t possibly transfer 100% of income to generation B; since generation A can’t leave debt for generation B (by assumption), budget line stops before reaching horizontal axis

- **Tax Cut** - if $T \downarrow$ for generation A with no change in $G$, two cases: (a) generation A spends all of it, they reduce consumption for generation B (because generation B will eventually be faced with $T \uparrow$ to pay for tax cut); (b) benefits of tax cut get passed to generation B; technically there’s a third cases where the tax cut gets split, but it requires a funny shaped indifference curve

**Implication** - $T \downarrow$ doesn’t change consumption (just like Recardian Equivalence suggests, but now we see it with finite lifetimes); if you don’t believe in Recardian Equivalence because of finite lifetimes, then people wouldn’t leave bequests, but in the real world they do; finite lifetime argument against Recardian Equivalence is weak