IS-LM Model

Solow Assumptions - demand irrelevant in long run; assumes economy is operating at potential GDP; concerned with growth

IS-LM Assumptions - supply is irrelevant in short run; assumes economy is operating below potential (i.e., have excess capacity to absorb any increase in demand); concerned with fluctuations in business cycle (based solely on aggregate demand)

Consumption Function - relationship between consumption & those economic variables that determine decision to consume; we only consider it a function of disposable income:

\[ C = C(Y - T(Y)) \]  (poor notation; using \( C \) as consumption and as function for consumption)

Marginal Propensity to Consume (MPC; \( C' \)) - amount of increased consumption that results from an increase in income; derivative of consumption function with respect to income; assume \( 0 < C' < 1 \)

Marginal Propensity to Save (MPS) - \( MPC + MPS = 1 \); i.e., \( MPS = 1 - C' \)

Investment - has multiple meanings, but for economists, it means using productive capacity to build capital goods (vs. consumption goods); for now treat as exogenous (given)

Planned Investment - amount businesses want to spend on capital goods, including amount they want to add to their inventories; decide to buy capital goods because they foresee profits accruing to them from using these capital goods

Unplanned Investment - amount businesses have to add or take away from their inventories to make up for excess supply or demand

Equilibrium - in goods market occurs when unplanned investment doesn't exist

Simple Model - no taxes, no government purchases, closed economy

2 equations, 2 unknowns (\( C \) and \( Y \)): \( C = C(Y) \) and \( Y = C + I \)

Take derivatives: \( dC = C'dY \) and \( dY = dC + dI \)

Sub \( dC \) into \( dY \) equation and solve for investment multiplier:

\[ dY = C'dY + dI \Rightarrow \frac{dY}{dI} = \frac{1}{1 - C'} > 0 \]

Sign - 1 - \( C' \) is > 0 because \( 0 < C' < 1 \) (assumption)

Graph - plots consumption (demand, \( C + I \)) on vertical axis and production (supply, \( Y \)) on horizontal axis; for equilibrium (supply = demand) must be on 45° line; \( Y_2 \) has excess supply; \( Y_3 \) has excess demand

Change in Investment on Graph - \( I \uparrow \) shifts curve up and increase equilibrium \( Y \)

Multiplier - \( I \uparrow \Rightarrow Y \uparrow \) by inverse of MPS... the more people consume (i.e., steeper \( C + I \)), the larger impact an increase in \( I \) will have on output (see smaller graphs)

Adding Fiscal Policy - \( Y = C + I + G \) and \( C = C(Y - T) \)

\( T \) = net taxes (tax revenues minus transfer payments)

\( G \) = government purchases

Disposable Income - \( Y - T \)

Still 2 equations, 2 unknowns
Take derivatives: \( dC = C' \cdot dY - C' \cdotdT \) and \( dY = dC + dI + dG \)

Sub \( dC \) into \( dY \) equation and solve for investment multiplier:

\[
\begin{align*}
\frac{1}{1 - C'} \left( dI \right) + \frac{1}{1 - C'} \left( dG \right) + \frac{-C'}{1 - C'} \left( dT \right) & = 0 \\
\end{align*}
\]

Tax Cut or Government Purchases? looking at multipliers, \( G \) has larger impact on \( Y \) (note that \( dT \) is multiplied by \( C' < 1 \)); that means, dollar for dollar, government purchases are more effective than tax cuts because with cuts, people retain some of the money (determined by MPS) whereas money from \( G \) goes straight into \( Y \)

Balanced Budget? if you have increased taxes to cover increased government purchases (i.e., \( dG = dT \)), there is no change in \( Y \), \( 1/(1 - C') - C'/(1 - C') = (1 - C')/(1 - C') = 1 \)

Purpose - output and employment are very sensitive to changes in investment which is volatile; changes in \( G \) and \( T \) can be used to stabilize output and employment

Taxes as Function of Income - \( Y = C + I + G \) and \( C = C(Y - T(Y)) \)

Realistic - Congress sets tax rates and policies for transfer payments, but actually amount collected and paid depends on \( Y \)

Automatic Stabilizer - net taxes rise as \( Y \) because the government collects more tax dollars and makes fewer transfer payments; if \( Y \), taxes collected automatically go down and transfer payments go up; not good for government's fiscal position, but credited with minimizing fluctuations in business cycle (difference between peak and trough); want fiscal policy to be automatic because political system is too slow to build consensus (e.g., deciding which taxes to cut)

Surplus - \( T - G > 0 \); i.e., transfer payments plus government purchases are less than taxes: \( G + Tr < Tx \); or look at net taxes: \( G - T < 0 \)

Deficit - \( T - G < 0 \); i.e., transfer payments plus government purchases are more than taxes: \( G + Tr > Tx \); or look at net taxes: \( G - T > 0 \)

2002 Deficit - \( G \), \( T \), and \( Y \) at same time

Marginal Tax Rate (\( T' \)) - assume \( 0 < T' < 1 \); marginal rate is higher than you think because it incorporates transfer payments; also called marginal propensity to tax (MPT)

Still 2 equations, 2 unknowns:

Take derivatives: \( dC = C' \cdot (1 - T') \cdot dY - C' \cdot dT \) and \( dY = dC + dI + dG \)

Sub \( dC \) into \( dY \) equation and solve for investment multiplier:

\[
\begin{align*}
\frac{1}{1 - C' \cdot (1 - T')} \left( dI \right) + \frac{1}{1 - C' \cdot (1 - T')} \left( dG \right) + \frac{-C'}{1 - C' \cdot (1 - T')} \left( dT \right) & > 0 \\
\end{align*}
\]

Will get more complicated in a bit

Poor notation (again) This is 1 minus the product of \( C' \) and \( (1 - T') \), not \( C' \) evaluated at \( (1 - T') \)
Impact of Taxes - all multipliers are smaller now because people have less disposable income (effectively reduces $MPC$)

Adding Money - can't talk about fiscal policy without looking at money; will now look at investment as endogenous (explained by model)

Investment Function - relationship between investment demand ($I$) & those economic variables that determine the decision by firms to purchase capital goods; $I = I(i - \pi)$ (poor notation; using $I$ as investment and as function for investment)

Real Interest Rate ($r$) - difference between interest rate ($i$) and inflation rate ($\pi = (dP/dt)/P$); for model, use expected inflation ($\pi_e$) because decisions made before inflation is known

Investment and Interest - assume increased interest rates reduce investment (i.e., $i \uparrow \Rightarrow Y \downarrow$ or $I_i < 0$)

Back to Model - now have $Y = C(Y - T(Y)) + I(i - \pi_e) + G$; $Y$ and $i$ are endogenous so we have 1 equation and 2 unknowns; need to look at all goods-market equilibria (IS curve)

IS Curve - combinations of interest rate ($i$) & income ($Y$) that generate goods-market equilibrium (i.e., [1] aggregate supply = aggregate demand; [2] $Y = C + I + G$; [3] planned investment = savings; [4] unplanned investment = 0); downward sloping because high interest rates discourage investment & therefore reduce equilibrium income; slope of IS curve shows how much equilibrium income will change with change in interest rate; gets name from planned investment ($I_p$) equals saving ($S$)

Saving - part of income that is not used for consumption; $S = Y - C$; condition for goods-market equilibrium is saving equals planned investment ($S = I_p$)

Shifts in IS - IS↑ (i.e., curve shifts to the right) if $T\downarrow (C\uparrow)$, or $\pi_e \uparrow (I\uparrow)$, or $G\uparrow$; results in larger output ($Y$) for given interest rate ($i$)

Money-Market Equilibrium - IS curve doesn’t give a specific equilibrium, but a set of possible goods-market equilibria; to find a specific equilibrium point, you need to find the equilibrium in the money-market

Money Market - where people increase or decrease the amount of money they hold by selling or buying short-term bonds (e.g., T-bills)

Money - has multiple meanings: wealth (stock), income (flow), etc., but for economists, it means liquid portion of wealth (cash, checking balances, etc.)

M1 - purely transaction-based definition; currency plus demand deposits & travelers checks
M2 - purely transaction-based (M1) plus easily transferable savings accounts (e.g., overnight repurchase agreements, US dollar accounts in Europe, money-market mutual funds, savings deposits, small time deposits)

M3 - everything in M2 plus large time deposits & other accounts used less frequently for transactions purposes

Credit Cards - affect how much money people want to hold, but are excluded from definition of money because they’re not assets

Benefit of Money - certainty that asset can be quickly & readily used to purchase goods & services

Cost of Money - holding money costs because it earns no interest or has very low interest rate

Demand for Money ($L$) - portion of our wealth we want to hold in the form of money; use $L$ for "liquidity preference"; function of income (# of transactions) and interest rate (cost of holding money): $L = L(Y, i)$  Note: $i \uparrow \Rightarrow L \downarrow$ and $Y \uparrow \Rightarrow L \uparrow$, so $L_i < 0$ and $L_Y > 0$

Supply of Money ($M$) - determined by central bank (Fed); treat as fixed in short run

Constant Purchasing Power - look at supply of money based on purchasing power by dividing by price level, $P$

Money-Market Equilibrium - if there is less or more money demanded than available, actions of money holders in trying to acquire or get rid of money will bring about a change in interest rate & hence quantity of money demanded; if excess demand for money then interest rate is too low (everyone wants more money than is available; acquire money by selling bonds which drives up interest rate on bonds)

Theater Analogy - only limited number of seats (fixed supply); demand can’t create extra seats, so ticket prices are bid up by those who want to attend until demand is brought into line with supply

LM Curve - combinations of interest rate ($i$) & income ($Y$) that generate money-market equilibrium (i.e., [1] supply of money = demand for money; [2] $L(Y, i) = M/P$); upward-sloping because higher income ($Y \uparrow$) causes higher demand for money ($L \uparrow$) which causes higher interest rate ($i \uparrow$) to bring money demand back down to equilibrium with fixed supply; name comes from $M$ for money supply & $L$ for money demand

Shifts in LM - LM$\uparrow$ (i.e., curve shifts to the right) if $M \uparrow$, $P \downarrow$, or $L \downarrow$; results in larger output ($Y$) for given interest rate ($i$)
IS-LM Framework - point of intersection of IS & LM schedules is one combination of interest rate & income common to both schedules ⇒ point where both goods market & money market are in equilibrium

Multiplier Effect - change in income that would occur following a shift in goods market if there were no change in the interest rate (i.e., ignore asset market)

Interest-Rate Effect - following shift in conditions in goods market, interest-rate effect is change in income resulting from change in interest rate

Back to Model - now have 2 equations, 2 unknowns:

\[ Y = C(Y - T(Y)) + I(i - \pi) + G \] and \[ L(Y, i) = M/P \]

Take derivatives: \[ dY = C'(1 - T')dY + I'di - I'd\pi + dG \] and \[ L_YdY + L_idi = dM/P - (M/P^2)dP \]

Note: \[ I' = dI/dr, r = i - \pi \]

Solve for \[ dY \]

\[ dY = \frac{I'}{1 - C'(1 - T')} di - \frac{I'}{1 - C'(1 - T')} d\pi + \frac{1}{1 - C'(1 - T')} dG \]

Then sub in \[ di \] equation

\[ \frac{I'L_Y}{1 - C'(1 - T')} di - \frac{I'L_Y}{1 - C'(1 - T')} d\pi + \frac{L_Y}{1 - C'(1 - T')} dG + L_idi = dM/P - (M/P^2)dP \]

Solve for \[ di \]

\[ L_i\left[1 - C'(1 - T')\right] + I'L_Y \]

\[ \frac{1}{P} \]

\[ d\pi + \frac{-L_y}{L_i(1 - C'(1 - T')) + I'L_y} dG \]

Plug back into \[ dY \] equation to get \[ dY \] multipliers:

\[ dY = \frac{I'}{L_i(1 - C'(1 - T')) + I'L_y} \]

\[ \frac{1}{P} \]
Note, the terms for \(d\pi\) and \(dG\) have two terms that need to be combined. The math is ugly, but easy. Multipliers above show final result.

**Extra Multipliers** - note that you can get additional multipliers from

\[
dY = C'(1 - T_Y) dY + I'di - I'd\pi + dG
\]

by recognizing that \(dC = C'(1 - T') dY\)

\[
dC = \frac{C'(1 - T') I'}{[L_i(1 - C'(1 - T')) + I'L_Y]} P > 0
\]

\[
dM = \frac{-C'(1 - T') I'M}{[L_i(1 - C'(1 - T')) + I'L_Y]} P^2 < 0
\]

\[
d\pi = \frac{-C'(1 - T') I'L_i}{[L_i(1 - C'(1 - T')) + I'L_Y]} > 0
\]

\[
dG = \frac{C'(1 - T') L_i}{[L_i(1 - C'(1 - T')) + I'L_Y]} > 0
\]

You can also get additional multipliers form the \(di\) equation by recognizing \(dl = I'di - I'd\pi\)

\[
dl = \frac{I'[1 - C'(1 - T')]}{[L_i(1 - C'(1 - T')) + I'L_Y]} P > 0
\]

\[
dM = \frac{-I'[1 - C'(1 - T')] M}{[L_i(1 - C'(1 - T')) + I'L_Y]} P^2 < 0
\]

\[
d\pi = \frac{-I'L_Y}{[L_i(1 - C'(1 - T')) + I'L_Y]} > 0
\]

\[
dG = \frac{-I'L_i}{[L_i(1 - C'(1 - T')) + I'L_Y]} < 0
\]

**Looking at Changes** - take \(dY/dG\)

**No Money:** \(\frac{1}{1 - C'(1 - T')} > \text{With Money:} \frac{L_i}{L_i(1 - C'(1 - T')) + I'L_Y}\)

No money version treats LM curve as flat (horizontal); says demand for money is sensitive to interest rates; with money, multiplier is smaller because \(G\uparrow\) causes \(i\uparrow\) which causes \(Y\downarrow\) so the overall change in \(Y\) is less than it was before considering the money market

**Sensitive to \(L_i\)** - note if \(L_i = 0\) (i.e., demand for money is insensitive to interest rates), LM curve is vertical; increase in \(G\) leaves \(Y\) unchanged and increases \(i\)

**Implication for Fiscal Policy** - have to worry about how sensitive money demand is to interest rate \((L_i)\); larger \(L_i\) means fiscal policy is more effective (i.e., greater change in \(Y\) with less impact on \(i\))
Recap - looked at model three times:

\[
\frac{dY}{dG} = \frac{1}{1-C} > \frac{1}{1-C'(1-T')} > \frac{L_i}{L_i(1-C'(1-T')) + \Gamma L_Y}
\]

(1) T & I exogenous (given)
(2) I exogenous & T endogenous
(3) T & I endogenous

(1) Larger MPC \((C')\Rightarrow dY/dG\) larger
Note: this conclusion is more important than the actual value of the multiplier
(2) Larger marginal tax rate \((T')\Rightarrow dY/dG\) smaller
Note: conclusion form first model is still valid in the second. Start simple (or later purposely make things exogenous) to make conclusions more obvious.
(3) Demand for money more sensitive to interest rate \((|L_i|)\Rightarrow dY/dG\) larger
Note: \(L_i = 0 \Rightarrow dY/dG = 0\) (see graph with vertical LM curve)
Note: conclusions form first and second models still valid in third

More Results - go back to \(T\) being exogenous (i.e., \(T' = 0\)); this simplifies the multipliers to find other conclusions:

\[
\frac{dY}{dG} = \frac{L_i}{L_i(1-C') + \Gamma L_Y} > 0
\]

Change in \(G\) - Short Version:

Long Version - increase in \(G\) causes too much demand for goods (i.e., excess demand for \(Y\)); firms increase output to eliminate excess demand (\(Y\uparrow\)); as firms increase output (1) \(C\) increases (based on \(C'\), marginal propensity to consume or the sensitivity of consumption to income) and (2) demand for money \((L)\) increases (based on \(L_Y\), sensitivity of demand for money to income); increased \(C\) further increases demand for goods (this goes back to the original \(dY/dG\) multiplier) while increased demand for money forces interest rates \((i)\) to climb (based on \(L_i\), sensitivity of demand for money to interest rates); higher rates eliminate excess demand for money (pulls it back to LM curve) and increase borrowing costs for investment so \(I\) decreases (based on \(\Gamma\), sensitivity of investment to interest rate); process continues until decrease in \(I\) soaks up excess demand (“crowding out”)

Analyzing Components -
\(C'\) larger \(\Rightarrow dY/dG\) larger
\(L_Y\) larger \(\Rightarrow dY/dG\) smaller
\(|L_i|\) larger \(\Rightarrow dY/dG\) larger
\(\Gamma\) larger \(\Rightarrow dY/dG\) smaller
\[
\frac{dY}{dM} = \frac{I'}{[L_i(1-C') + I'L_Y]P} > 0
\]

**Change in \( M \) - Short Version:**

\[ M \uparrow \Rightarrow \text{Excess supply of money} \Rightarrow i \downarrow \Rightarrow i' \uparrow \Rightarrow \text{Excess demand for goods} \Rightarrow C \uparrow \Rightarrow Y \uparrow \Rightarrow L_Y \uparrow \]

**Long Version** - increase in \( M \) causes too much supply of money which drops interest rates \((i)\); lower rates increase demand for money \((L \uparrow\), based on \( L \)) and investment \((I \uparrow\), based on \( I \)); increased investment increases demand for goods so output increases; this increases consumption \((C \uparrow\), based on \( C \)) and increases the demand for money \((L \uparrow\), based on \( L \)); eventually the increased demand for money from lower interest rate and increased output will offset the excess supply

**Analyzing Components** -

- \( C \) larger \( \Rightarrow \frac{dY}{dM} \) larger (same as with \( dY/dG \))
- \( L_Y \) larger \( \Rightarrow \frac{dY}{dM} \) smaller (same as with \( dY/dG \))
- \(|L_i| \) larger \( \Rightarrow \frac{dY}{dM} \) smaller (opposite of with \( dY/dG \))
- \( I \) larger \( \Rightarrow \frac{dY}{dM} \) larger (opposite of with \( dY/dG \))

**Monetary or Fiscal Policy?** -

- **Multipliers** - size of multipliers not important unless one of them is zero because you can always get to potential GDP
- **Conventional Wisdom** - monetary is better because it's easier to change \( M \) than \( G \) (less political)
- **Interest Rates** - both policies get to potential GDP, but fiscal policy increases interest rates and monetary policy lowers interest rates
- **Composition of Output** - \( Y = C + I + G < \bar{Y} \); in order to increases \( Y \), one of the components has to increase; fiscal and monetary policy target different components; fiscal policy through government purchases increases \( G \); fiscal policy through taxes increases \( C \); in both cases, fiscal policy results in decreased \( I \); monetary policy increases \( I \)
- **Future Growth** - recall Solow Model said \( I \) (savings) has consequences for growth

**Problem with "Fact" 3?** - said economy would fix itself, but we've only looked at government intervention through monetary or fiscal policy; how does economy fix itself? If we're below potential we know we have unemployment, but we ignored the labor market;

**Prices** - we also kept prices constant throughout; that will change when we add the labor market