Repeated Play

**Prisoners Dilemma** - could coordinate to get to (A,U) (best payoff), but incentives point away from that so it's not rational to expect the coordination to work.

**Finitely Repeated**
- 2 Periods - each period is simultaneous, but players know what happened in first period before playing the second; basically end up with 5 information sets for each player, each with 2 strategies... that's 25 strategies... probably don't want to do that in strategic form; we'll look at subgame perfection.

**Discounting** - note that if we discount payoffs in future periods, say $\delta$ for each period, we can't just add them up like this; for example, (UU,AA) would be $(5 + 5\delta, 5 + 5\delta)$; discounting is critical for infinitely repeated games (in order to have bounded limits of payoffs).

Looking at all the second periods games, they're identical with Nash equilibria of (M,B) (payoff of (1,1)). Since they're all the same, we can ignore them and just look at the first period; in this case the Nash equilibrium is (M,B)... ∴ subgame perfect Nash equilibrium for this game is (MM, BB).

**No Cooperation** - finite repetition doesn't solve the prisoners' dilemma (or any game with unique Nash equilibrium); that unique equilibrium will be the subgame perfect Nash equilibrium for the finitely repeated game.

**Penalties** - if we have another strategy that is very bad all the way around, but gives a pure strategy Nash equilibrium like the one shown here, we can get cooperation because final stage has to be either (B,M) or (D,C), but first stage can be anything as long as combination forms a Nash equilibrium.

**# Strategies** - in this case there are 10 information sets each with 3 strategies: $3^{10} = 59,049$... that's a lot!

**Specific Choice** - since there are so many strategies, let's just consider the case of cooperating in first round, then either going for the (1,1) payoff if the opponent cooperated or the (-4,-4) payoff if he didn't.

- **Strategy for Player 1**: (U, if UA then M, else D)
- **Strategy for Player 2**: (A, if UA then B, else C)

**Credible Threat** - each player is making a credible threat because it leads to a Nash equilibrium in the last round.

**Nash?** - need to check strategy for each player to see if he has an incentive to change.
**Player 1** - changing to D in first round is obviously bad; changing to M gains 1 unit of payoff (based on player 2's strategy), but then player 1 gets -4 instead of 1 (loss of 5) in second round

**With Discounts** - if we were using discounts we'd have to compare the gain of 1 now with a loss of 5 in the second round which is worth -5\(\delta\); \(\therefore\) we'd only switch if \(\delta < 1/5\) (i.e., discounting a lot; payoff now is much more important than payoff later); another way to look at this is to compare (6 - 4\(\delta\)) to (5 + \(\delta\))... get same result, only switch if \(\delta < 1/5\)

**Player 2** - exact same logic since it's symmetric

\(\therefore\) this specific choice we looked at is a subgame perfect Nash equilibrium

**Note:** if the payoff for (D,C) goes more negative to (-6,-6) or worse, this strategy is no longer subgame perfect

**Trigger Strategy** - cooperate, but punish if opponent doesn't cooperate

**When to Penalize** - if there are 20 rounds, and penalty comes at the end, we're looking at a discount of \(\delta^{19}\)... probably too small to make a difference; opposite extreme is grim trigger

**Grim Trigger** - punish forever

**Need "Bad" Equilibrium** - only way to punish in finite game is to have a "bad" equilibrium (because threat must use an equilibrium for strategy to be subgame perfect)

**Infinitely Repeated** - don't need a "bad" equilibrium to punish since we're not worried about subgame perfect (never get to the end); we do have to have discounted payoffs with \(\delta < 1\) to allow payoffs (which are infinite sums) to converge

**Grim Trigger Strategy** - here's an example for infinitely repeated game:
- Player 1: Initial State: U; Every stage after: if previous stage was UA, then U, else M
- Player 2: Initial State: A; Every stage after: if previous stage was UA, then A, else B

If player deviates in round 1, gain 1 (6 instead of 5), but lose 5 (-4 instead of 5) from period 2 to infinity (i.e., \(-5(\delta + \delta^2 + \delta^3\ldots) = -5(\delta/(1 - \delta))\ldots\) again end up with cooperate if \(\delta > 1/5\)

**Too Many Equilibria** - problem with infinitely repeated games is that there are lots of equilibria... all equally compelling so even thought people used infinitely repeated games to show possibility of good outcome (e.g., cooperation), there's also possibility of bad outcome; there's no justification for one over the other (**Folk Theorem**)