Uncertainty

Belief - player’s belief is a probability distribution over the strategies of his opponents ($\mu_i$)
Mixed Strategy - act of selecting a strategy according to a probability distribution ($\sigma_i$)
Pure Strategy - regular strategy (i.e., pick strategy with probability one)
Expected Payoff - "average" payoff that player would get if he played strategy $s_i$ and opponents played according to $\mu_i$)

\[ u_i(s_j, \mu_i) = \sum_{s_{-i} \in S_{-i}} \mu_{-i}(s_{-i}) \cdot u_i(s_i, s_{-i}) \] (probability of $s_{-i}$ times payoff to player $s_i$ and $s_{-i}$)

Example - player 1 believes with probability 1/2 that player 2 will play strategy L, with probability 1/3 that player 2 will play M, and probability 1/4 that player 2 will play R (i.e., $\mu_2(L) = 1/2$, $\mu_2(M) = 1/4$, $\mu_2(R) = 1/4$); if player 1 selects strategy U, then the expected payoff is $u_1(U, \mu_2) = (1/2)8 + (1/3)0 + (1/4)4 = 5$

Maximize Monetary Gain - if this is a player's objective, he is risk neutral; allows us to use monetary amounts instead of utility numbers; can also add a constant or multiply payoffs by a positive number without affecting the player's preferences (explained in more detail below)

Expected Utility Theory

Endogenous Uncertainty - results from way players play the game (e.g., mixed strategy)
Exogenous Uncertainty - not related to player actions (e.g., weather)
Expected Utility Theory - people behave as if to maximize expected utility; some people don't like the technique, but it's still used because (1) no better alternatives, (2) easy to work with; proposed by Von Neumann-Morgenstern
Lotteries - specifies payoffs and probability of receiving each payoff; anything with uncertainty can be viewed as a lottery
Simple Lottery - $L(A_1, A_2, p)$ means you get payoff $A_1$ (either money or a commodity bundle) with probability $p$ and payoff $A_2$ with probability $(1-p)$; Note: If there are $n$ payoffs, there will be $n-1$ probabilities because the last probability is one minus the sum of the others
Compound Lottery - lottery of lotteries; $L(L_1, L_2, p)$; can always write as a simple lottery
Example - $L(L_1, L_2, \frac{1}{3})$ with $L_1(100, 50, \frac{1}{3})$ and $L_2(200, 10, \frac{1}{4})$; to figure out simple lottery work out the probability of getting every possible payoff; the only way to get 200 is to get $L_2$ (probability $\frac{1}{3}$) and then get 200 (probability $\frac{1}{4}$) so probability of 200 is $(1 - \frac{1}{3})(\frac{1}{4}) = 1/6$; continue and you get $L_2(200, 100, 50, 10, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$

Assumptions of VN-M EUT - people still debate the reasonableness of these assumptions
1. Preferences are complete and transitive - this means given any number of options, the player can make a choice; for any two options, the player either prefers one to the other or is indifferent between the two

<table>
<thead>
<tr>
<th>Player 1</th>
<th>L</th>
<th>M</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>8, 1</td>
<td>0, 2</td>
<td>4, 0</td>
</tr>
<tr>
<td>C</td>
<td>3, 3</td>
<td>1, 2</td>
<td>0, 0</td>
</tr>
<tr>
<td>D</td>
<td>5, 0</td>
<td>2, 3</td>
<td>8, 1</td>
</tr>
</tbody>
</table>
2. **Compound Lottery Assumption** - individuals are indifferent between a compound lottery and a simple lottery with the same payoffs and probabilities

   **Contradiction** - people with pure love of gambling prefer compound lottery; people get benefit from watching the wheel spin not just from the payoff

   **Counter** - hard to measure the satisfaction from gambling; economics focuses on outcomes not the process; love of gambling doesn't apply for all scenarios (e.g., gambling with bads... change to lose home or life)

3. **Monotonicity** - (a) if you take a lottery and increase either or both payoffs with the same \( p \), a player will prefer this over the original lottery (i.e., \( L(A_1^1, A_2^2, p) \) \( P \) \( L(A_1, A_2, p) \) if \( A_1^1 \geq A_1 \) and \( A_2^2 \geq A_2 \) and one of these is be strictly \( > \)); (b) if you take a lottery and increase the probability of the higher payoff, a player will prefer it over the original lottery (i.e., \( L(A_1, A_2, p_2) \) \( P \) \( L(A_2, A_2, p_1) \) if \( p_2 > p_1 \))

4. **Substitution** - if \( L_1 \) \( I \) \( L_2 \) then \( L_1 \) can always replace \( L_2 \) in any compound lottery and not change preferences (i.e., \( L(L_2, L_3, p) \) \( I \) \( L(L_1, L_3, p) \) if \( L_1 \) \( I \) \( L_2 \))

   **Complementarities** - could argue this doesn't make sense because of complimentary goods; you may be indifferent between an apple and an orange, but when it comes time to bake an apple pie, you're not indifferent anymore

   **Response** - that type of complementarity is not an issue in this lottery; you get \( L_2 \) or \( L_3 \), not both so you don't consume them together

   **Sure Thing Principle** - more powerful critique; people prefer uncertainty; look at special case were \( A_1 \) \( I \) \( A_2 \), but \( L(A_1, A_1, p) \) \( P \) \( L(A_2, A_1, p) \) because the first one is certain and the second involves uncertainty

5. **Continuity** - if \( A_1 \) \( P \) \( A_2 \) \( P \) \( A_3 \) then there exists a probability \( q \) such that \( L(A_1, A_3, q) \) \( I \) \( A_2 \)

   **Proof**: if \( q = 1 \), \( L(A_1, A_3, 1) = A_1 \) \( P \) \( A_2 \); if \( q = 0 \), \( A_2 \) \( P \) \( L(A_1, A_3, 0) = A_3 \); since the preference between the lottery and the certain outcome switch, somewhere in between they must be indifferent

   **WW II Example** - two options: (1) fly 100 planes and lost 10, (2) fly 10 planes and lost all of them; same outcome either way (mission is accomplished and 10 planes lost); pilots voted and showed preference for 100 planes; in this case, people preferred uncertainty :. extreme example (life/death) could violate continuity

**Theorem (\( U \) Exists)** - there exists a utility function over certain outcomes (e.g., \( A_1, A_2, \) etc.) such that the utility of lotteries (\( L(A_1, A_2, p) \)) is the expected value of \( U \):

\[
U(L) = pU(A_1) + (1 - p)U(A_2)
\]

**Fair Gamble** - expectation of gains and losses = 0 (e.g., win $1 or lose $1 with \( p = 0.5 \))

**Risk Neutral** - indifferent between amount with certainty and a lottery who's expected payoff is equal to the certain amount (i.e., \( \frac{1}{2}(I_1 + I_2) \) \( I \) \( \frac{1}{2}(U(I_1) + U(I_2)) \)); utility function is linear

**Risk Averse** - would prefer amount with certainty over lottery; utility function is concave

**Risk Premium** - difference between certain payoff and expected value of a lottery such that risk averse person is indifferent between the lottery and certain payoff

**Risk Seeking** - would prefer the lottery; utility function is convex
**Theorem (Many U’s)** - the utility function will be unique up to linear transformation

\[ V = aU + b \text{ with } a > 0 \]

**Normalizing** - can use linear transformation to get \( U(A_1) = 1 \) and \( U(A_2) = 0 \) in a lottery

\[ L(A_1, A_2, p) \text{ (assuming } A_1 > A_2) \]

**Proof**: \( U(A_1) = u_1 > U(A_2) = u_2 \)

\[ v_1 = au_1 + b = 1 \text{ and } v_2 = au_2 + b = 0 \ldots 2 \text{ equations & 2 unknowns (} a \text{ & } b \)**

**Zero vs. Constant Sum Game** - same thing because you can transform a constant sum game to a zero sum game

**Allais’ Critique of VN-M EUT** - Allais was Nobel laureate who opposed Von Neumann-Morgenstern theory; conducted experiment

1. choose between \( L_1 = $50K \) and \( L_2($250K, $50K, $0, 0.89, 0.1) \)
2. choose between \( L_3($50K, $0, 0.9) \) and \( L_4($250K, 0, 0.89) \)

**Rational?** - what happens if people fall in box 2 (similar result for 3)?

\begin{align*}
L_1 \land L_2 & \Rightarrow U(50) > 0.89U(250) + 0.1U(50) + 0.01U(0) \Rightarrow \\
0.9U(50) > 0.89U(250) + 0.01U(0) \\
L_4 \land L_3 & \Rightarrow 0.89U(250) + 0.11U(0) > 0.9U(50) + 0.1U(0) \Rightarrow \\
0.9U(50) < 0.89U(250) + 0.01U(0)
\end{align*}

Opposite results! Von Neumann & Morgenstern would say these people are unreasonable

**Allais’ Explanation** - it’s OK to have \( L_1 \land L_2 \) and \( L_4 \land L_3 \); in the first case, the person is risk averse to the point that certainty is better (even with a much lower expected payoff); in the second case, there’s not much difference in the probabilities so it’s not a big deal to go for the higher payoff

**Savage’s Rebuttal** - people make decisions based on beliefs of probability because real probabilities aren’t known; said people in boxes 2 and 3 are irrational and he only needed a couple of minutes to talk with them to set them straight; rewrite lotteries and present them with a dart board; choice should only depend on where the lotteries are different (black and green areas only)

**??? Test** - went out and performed Allais’ experiment; then gave a card with Allais’ explanation to the people who picked boxes 1 and 4 and a card with Savage’s explanation to the people who picked boxes 2 and 3; had more people switch from Allais’ argument rather than Savage’s