Describing Games

Economics - two areas
  Optimization
  Equilibrium - two areas
    Competitive - don't care about what competitors are doing; participants only need to know their own technology (firms) or preferences (consumers) and the market price
    Interaction - have to worry about other players; e.g., Coke vs. Pepsi, schools competing for graduate students
Optimization - maximize objective subject to constraint(s); one time event
Game - set of simultaneous, interrelated optimizations; choice of 1 player affects optimization of the other

Background Info

Interdependence - one person's behavior affects another person's well-being, either positively or negatively
Strategic Setting - situations of interdependence; in order for one person to decide how best to behave, he must consider how others around him choose their actions

Purpose of Game Theory - 2 views
  Normative - help participants know what to do; "here's how you should play this game"
  Positive - develop an understanding of how people actually behave; predictive theory; this view is used more for economics and social sciences
    Limited Rationality - trying new models with limitations on rationality to get model predictions to better reflect real world outcomes (mainly focused on limited memory)

Game - situation in which 2 or more adversaries match wits; inherently entail interdependence; usually have sets of rules that must be followed by the players
  Constant (Zero) Sum Game - if one participant gains, the other loses by same amount; will always have "efficient" outcome because sum of payoffs is always the same; unrealistic; introduced by Von Neumann & Morgenstern
  Non-Constant Sum Game - possible for both parties to gain (or lose); e.g., in labor strike both sides lose; brings up question of efficiency
  Non-cooperative Game - participants don't work together; each player decides on his own, independent of the other people present in the strategic environment; Nash focused on non-constant sum, non-cooperative games
  Cooperative Game - look at what a coalition can do, how it will form, and how it will divide profits; no all that useful because there are too many equilibria
    Coalition - 2 or more players join to improve their payoffs at the expense of other players
  Complete Information - participants know everything there is to know about the game (who makes what decisions and when); focuses on structure of the game
  Incomplete (Private) Information - player knows more about something in the game than another player
    Perfect Information - players knows everything that happened before (i.e., aware of previous decisions by other players); equivalent to saying all information sets have only 1 node or saying it's a sequential game; focuses on decisions in the game
    Perfect Recall - player remembers his own choices
  Imperfect Information - player doesn't know what choice opponent made; equivalent to having a simultaneous choice or saying at least one information set has 2 or more nodes
Common Knowledge - each player knows the other has complete info
1-Shot vs. Infinitely Repeated - results depend on whether there is an infinite time horizon
Chess Example - chess is a 1-shot constant sum, non-cooperative game with complete,
perfect information (except for limits on skill, calculation, and mistakes)
Information - note that what’s available to the modeler may not be the same as the players;
psychology of players or technical aspects of firms may not be known to the modeler, but
may be somewhat known by players
Elements of a Game -
Players - need a list of everyone involved; 2 types
 Strategic Player - makes choices
 Nature - no objectives or payoffs; makes random moves
Possible Actions - complete description of what players can do; usually conditioned on
where they are in the game
Information - description of what players know at each decision point (perfect vs. imperfect
info, perfect vs. imperfect recall, beliefs about other players, etc.)
Outcomes - results of every possible combination of player actions
Preferences - players preferences over outcomes
 Ordinal - just shows order of preference
 Cardinal - shows order of preference and assigns numerical values to know how much
 more one outcome is preferred; required for using nature because there will be
 probability distributions
Strategy - set of instructions on how to play the game

Extensive Form

Game Tree - graphically portrays 5 elements of a game

Node - represents a place where something happens in the game
 Decision Node - a player makes a decision at that place in the game
 Initial Node - every extensive-form game has exactly one initial node
 Terminal Node - places where the game ends; represent outcomes of the game; each
terminal node corresponds to a unique path through the tree
Payoffs - listed as a vector at each terminal node; entries correspond to player order
(e.g., from node n above, K gets 35 and E gets 100); could also use utilities
**Label Them** - each node is assigned to a player by putting the player number (or name) next to the node

- **Player 0** - nature; other players assigned numbers (1, 2, etc.)

**Branch** - indicates various actions that players can choose at a node

- **Label Them** - write out description of action taken; if you want to abbreviate it make sure you use a unique identifier; note in the example Eisner (player E) has N and N’ to distinguish between his two "Not" alternatives; Katzenberg (player K) however, has the same N because nodes c and d are in the same information set (simultaneous move)

**Information Set** - what a player knows at a decision node; every node is in one information set, although one information set can contain multiple nodes; only one decision is made at each information set

- **Sequential Move** - player knows what opponent did prior to making his decision

- **Simultaneous Move** - player doesn't know what opponent did prior to making his decision so the decision nodes are in the same information set; represented by connecting nodes with a dotted line; **Note**: nodes representing a simultaneous decision must have the same possible actions (see nodes c & d above)

**Infinite Number of Actions** - represented as range; example: ultimatum bargaining; player 1 offers one time take-it-or-leave-it offer of anything from 0 to p dollars to sell a painting; player 2 gets a chance to accept or reject the offer; the painting is worth nothing to player 1 and $100 to player 2

**Strategy** - complete contingent plan for a player in the game; full specification of a player's behavior which describes the actions that the player would take at each of his possible decision points; entries in brackets denote which decision the player should make based on the opponent's previous decision (described in more detail in next section)

**Example** - player 1 has 8 strategies: {(i,[iii,v]), (i,[iii,v]), (i,[iv,v]), (i,[iv,v]), (i,[iv,v])}, (i,[iv,v]), (i,[vi,v]), (i,[vi,v]), (ii,[vii,v]), (ii,[viii,v])}; player 2 has 4 strategies: {{a,c}, {a,d}, {b,c}, {b,d}}

**Book Version** - doesn't eliminate strategies that can be ruled out such as (i,[vii,v]) so player 1 has 16 strategies

**Not Observable** - we observe single iteration of a game at a time; that only reveals part of a player's strategy; can't observe the complete plan

"**Simple Game**" - tic-tac-toe; # of strategies for player 1 is between $9(7^8)(5^{65})$ and $9(7^8)(5^{65})(3^{64})$; for player 2 it's between $8^8(6^{57})$ and $8^8(6^{57})(4^{5755})$; these numbers can be reduced if you take advantage of the symmetry of the game, but the point is it's not a complicated game to play, but it's definitely complicated to model; a person can quickly figure out how to play to a tie every time without using extensive form
Normal (Strategic) Form
Lists all strategies available to player; shows payoffs for each combination of strategies

Set Notation - normal form consists of set of players, strategy spaces for the players, and payoff functions for the players; only use set notation if too many players (or infinite strategies)

\( I \) (or \( S \) in text) - set of players (1, 2, ..., \( n \))

\( S \) (or \( T \) in text)- set of strategies \( S^1 \times S^2 \times ... \times S^n \) (Cartesian product; e.g., \( S^1 = \{A, B\} \) and \( S^2 = \{X, Y\}, S = S^1 \times S^2 = \{(A, X), (A, Y), (B, X), (B, Y)\})

Strategy Space, \( S^i \) - the set of all possible strategies for player \( i \) (e.g., \( S^1 = \{A, B\} \))

Specific Strategy, \( s^i \) - a specific strategy for player \( i \), \( s^i \in S^i \) (e.g., \( s^i = A \))

Opponents' Strategies, \( s_{\sim i} \) - strategies played by player \( i \)'s opponents (e.g., \( s_{\sim i} = (X, D) \), where \( S^2 = \{X, Y\} \) and \( S^3 = \{C, D\} \)); Note: can use \( -i \) or \( ~i \)

Strategy Profile - vector of strategies, one for each player that fully describes how the game is played and is associated with a payoff vector (e.g., \( (A, X, D) \) means player 1 plays A, player 2 plays X, and player 3 plays D)

Independence - \( s^i \) is assumed to be independent of what other players do (i.e., \( s_{\sim i} \)); dependencies are already built in to the game (e.g., cattleman vs. farmer; cattleman has several options when there is a fence or there isn’t a fence [the farmer’s strategies]; the actual strategy the cattleman picks may depend on the farmer’s strategy, but the strategy space is unchanged)

\( U \) (or \( P \) in text)- set of payoff functions \( (u_1, u_2, ..., u_n) \)

Payoff Function, \( u^i \) - function with domain in set of strategy profiles (\( S \)) and whose range is the real numbers \( (u^i : S \rightarrow \mathbb{R}) \); sometimes written as function of strategies \( u^i(s^1, s^2, ..., s^i) \) or \( u^i(s^i, s_{\sim i}) \)

Bimatrix Game - for two player game with finite number of strategies, use matrix to list 1 player’s strategies by row and the other player’s by column; cells contain payoffs for each player resulting from strategies (hence name bimatrix for pairs of numbers)

Link to Extensive Form - strategic form models players that simultaneously and independently selecting complete contingent plans (strategies) for an extensive form game; there is only one strategic form for an extensive form game, but reverse isn’t true (see below)

Difference? - if game only has simultaneous and independent moves, strategic and extensive forms are identical; some theorists argue there is a difference, but others say there are a series of transformations that can link all the different extensive forms that create a single strategic form; debate centers on whether these transformations change the way the game is played (i.e., are the extensive forms equivalent)

Redundant Strategy - alternatives that have same payoffs; these can be added or deleted as one of the transformations discussed above; in example shown here \( e \& f \) are redundant and so are \( g \& h \)
3 Player - write a matrix for each of player 3’s strategies; player 3 picks the matrix to be played on and players 1 & 2 play on that matrix
4 Player - write a page of matrices; one page for each of player 4’s strategies; gets difficult to visualize and not very useful

**Classic Normal-Form Games**
Can gain great insights from simple 2x2 games

**Prisoners’ Dilemma** - two suspects are suspected of having committed a major crime, but the prosecutor only has enough evidence to convict on a lesser offense (1 year max); prosecutor needs confession (C) in order to convict for longer sentence; if one prisoner confesses, he gets a "good deal" (either 0 times or 4 years if both confess); note that payoffs equal jail time (a bad) so objective is to minimize the payoff; from perspective of an individual prisoner, it's always best to confess (dominant strategy), but if both prisoners don’t confess they’re better off; there's an inefficient outcome from prisoner’s point of view

**Mechanism Design** - try to set up payoffs to induce people to behave a certain way

**Powerful Payoffs** - doesn’t matter if prisoners are in separate rooms or even if they talk to each other; basic problem still exists: even if they agree to not confess, their incentives will be contrary to the agreement and they are more likely to confess than not

**Other Examples** - donations to common-use good (free-rider problem); firms colluding

**Solving Dilemma** - organized crime essentially changes the payoffs by punishing those who confess; trying to negate the mechanism design

**Fundamental Insights in Economics** - only 2

**Invisible Hand** - in surprising ways, individuals looking for own best interest (maximizing own utility), creates efficient outcome (Adam Smith)

**Opposite Result** - circumstances like prisoner's dilemma where inefficiency results when people look at own best interest

**Coordination Game** - both players obtain same positive payoff if they select the same strategy, otherwise they get nothing; have multiple equilibria in which neither player has an incentive to change strategies

**Island Example** - two drivers on opposite ends of same road have choice to drive on left or right side of road

**Problem** - how do you get to the equilibria?

**Communication** - players can discuss which side they will drive on; talk only works if it coincides with incentives (which is why it doesn't work in prisoners' dilemma)

**Role of Government** - (one of many) solves coordination problem by supplying communication (tells people what side of the road to drive on)

**Pareto Coordination** - same thing by both players prefer to coordinate on a particular strategy

**Battle of the Sexes** - two friends prefer to do something together but each likes one activity more than the other (here player 1 prefers boxing and...
player 2 prefers ballet); players have to make decision independently and simultaneously because they can't communicate

**Distributional Consideration** - just like coordination game, there are two equilibria, but this solution isn't as simple as arbitrarily picking one of them because payoffs are different

**Matching Pennies** - two players simultaneously and independently select heads (H) or tails (T) by uncovering a penny in his hand; if selections match, player 2 gives his penny to player 1; otherwise, player 1 gives his penny to player 2

**Two Representations** - can look at change in pennies help (top matrix) or total pennies at end of round (bottom matrix); result is the same

**No Equilibrium** - there isn't a cell where both players are content (at least 1 has an incentive to move to another cell)

**Result** - best strategy is mixed strategy; players must randomize decision so opponent won't know what the other is doing

**Another Example** - rock, paper, scissors game