Nonlinear Dynamic Systems
Homework 4
Due: 28Mar05

1. Figure 1 shows a turning process that can be described by a delay differential equation. Assume Eq. 1 will adequately describe this process

\[ m\dot{y} + c\dot{y} + ky = -bK_n \left[ h_o + y(t) - y(t - \tau) \right]. \]  

(1)

You are asked to determine the stability boundaries and to plot a stability chart for the following range of rpm’s 1,000 – 30,000 [rpm]. You must show your work! You should also use the following parameters \( K_n = 2 \times 10^8 \text{ [N/m}^2\text{]}, m = 0.5 \text{ [Kg]}, c = 1 \text{ [Ns/m]}, k = 1 \times 10^6 \text{ [N/m]}, h_o = 0.001 \text{ [m]}.\)

2. In order to characterize the behavior of a system near the bifurcation point, the dynamics of the system are often reduced via center manifold reduction and the use of the normal forms. Use center manifold reduction and normal forms to reduce the dynamics of Eq. 2 to a single dimensional expression for the stability. Use this expression to construct a bifurcation diagram.

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
x_2 \\
\beta x_1 - x_1^2 - \frac{\beta}{10} x_2
\end{bmatrix}.
\]  

(2)

3. Use Harmonic Balance to examine the equation below, which is representative of a clamped-
clamped buckling beam subjected to harmonic excitation, to obtain expressions for the amplitude and phase response of the fundamental harmonic. Plot the amplitude and phase response of the system for the following parameters: $c = 10$, $\omega = 1$, $\alpha = 0.5$, $\beta = 0.3$, and $f = 1$.

$$\ddot{x} + \omega^2 x = -c \dot{x} - \alpha x^2 - \beta x^3 + f \cos \Omega t.$$  (3)

4. Compare the Harmonic Balance results from problem 3 to a multiple scales analysis. It is obviously important to show your multiple scales analysis and plot the amplitude and phase response of the system for comparison. Also, show the stable and unstable solutions only for the multiple scales results.

References


