PROBLEM 6-20

Statement: A ±100 N·m torque is applied to a 1-m-long, solid round steel shaft. Design it to limit its angular deflection to 2 deg and select a steel alloy to have a fatigue safety factor of 2 for infinite life.

Units:

<table>
<thead>
<tr>
<th>Unit</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newton</td>
<td>N</td>
<td>100 N·m</td>
</tr>
<tr>
<td>Kilonewton</td>
<td>kN</td>
<td>100 N·m</td>
</tr>
<tr>
<td>Pascal</td>
<td>Pa</td>
<td>10^6 Pa</td>
</tr>
<tr>
<td>Megapascal</td>
<td>MPa</td>
<td>10^9 Pa</td>
</tr>
<tr>
<td>Gigapascal</td>
<td>GPa</td>
<td>10^9 Pa</td>
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</tbody>
</table>

Given:

- Applied torque: \( T_a \) = 100 N·m
- Shaft length: \( L \) = 1000 mm
- Max deflection: \( \theta_{\text{max}} \) = 2 deg
- Design safety factor: \( N_{\text{fd}} \) = 2
- Modulus of rigidity: \( G \) = 80.8 GPa

Assumptions: There are no stress-concentrations anywhere on the shaft. The shaft is machined, reliability is 99.9%, and the it is at room temperature.

Solution:

1. This is a case of fully reversed torsion. We will use the von Mises effective stress so the load factor will be 1.

   The maximum torque is \( T_{\text{max}} = T_a + T_m \).

2. The diameter of the shaft can be found from equations 4.24 and 4.25 with \( \theta_{\text{max}} \).

   \[
   \max = \frac{T_{\text{max}} L}{J G} = \frac{32 T_{\text{max}} L}{d^4 G}
   \]

   Solving for \( d \),
   \[
   d = \frac{32 T_{\text{max}} L}{\max G}
   \]

   Rounding, let \( d = 24.5 \text{ mm} \).

3. Now, we can solve for the stress in the shaft.

   Polar moment of inertia
   \[
   J = \frac{d^4}{32}
   \]

   Torsional stress
   \[
   \sigma_a = \frac{T_a d}{2 J}
   \]

   The corresponding von Mises normal stress is
   \[
   \sigma_a' = \sqrt{3} \sigma_a = 59.984 \text{ MPa}
   \]

4. Using the factor of safety equation for reversed loading, calculate the required endurance limit

   \[
   N_f = \frac{S_e}{\sigma_a'} = S_e \cdot N_{\text{fd}} \cdot \sigma_a' = 119.967 \text{ MPa}
   \]

5. This endurance limit is a function of the unknown ultimate tensile strength. Use the endurance limit modification equation to determine the required \( S_{\text{ut}} \):

   \[
   S_e = C_{\text{load}} \cdot C_{\text{size}} \cdot C_{\text{surf}} \cdot C_{\text{temp}} \cdot C_{\text{reliab}} \cdot S_{\text{e}}
   \]
6. Calculate the endurance limit modification factors for a solid, round steel shaft.

Load

\[
C_{\text{load}} = 1
\]

Size

\[
C_{\text{size}} = 1.189 \frac{d}{\text{mm}}
\]

Size

\[
C_{\text{size}} = 0.872
\]

Surface

\[
A = 4.51 \quad b = 0.265 \quad \text{(machined)}
\]

\[
C_{\text{surf}} = A \left(\frac{S_{\text{ut}}}{\text{MPa}}\right)^b
\]

Temperature

\[
C_{\text{temp}} = 1
\]

Reliability

\[
C_{\text{reliab}} = 0.753 \quad (R = 99.9\%)
\]

Uncorrected

endurance strength

\[
S'e = 0.5 S_{\text{ut}}
\]

7. Substituting these into the equation above and solving for \(S_{\text{ut}}\),

\[
S_{\text{ut}} = \frac{S_e}{0.5 A C_{\text{size}} C_{\text{reliab}} \text{MPa}}
\]

\[
S_{\text{ut}} = 395 \text{ MPa}
\]

Based on this requirement, choose AISI 1020 cold-rolled steel that will be machined to size.

8. Check the actual factor of safety based on the material chosen. For this material, \(S_{\text{ut}} = 469 \text{ MPa}\)

Surface factor

\[
C_{\text{surf}} = 0.884
\]

Uncorrected

endurance strength

\[
S'e = 0.5 S_{\text{ut}}
\]

\[
S'e = 234.5 \text{ MPa}
\]

Corrected

endurance strength

\[
S_e = C_{\text{load}} C_{\text{size}} C_{\text{surf}} C_{\text{temp}} C_{\text{reliab}} S'e
\]

\[
S_e = 136.046 \text{ MPa}
\]

Factor of safety

\[
N_f = \frac{S_e}{\sigma'}
\]

\[
N_f = 2.3
\]