Spread Spectrum and Code Division Multiple Access Project Report

Simulation Comparison of Multiuser Receivers in DS/CDMA Systems

Yong Liu, EEL6503— Spread Spectrum and Code Division Multiple Access, Nov. 30th

I. INTRODUCTION

The near-far problem is an important factor that affects the user capacity and performance of DS/CDMA systems. In conventional CDMA system, the matched filter is used to demodulate the signal of the desired user. When spreading sequences among different users are not orthogonal, the MAI (multiple access interference) will be introduced. Consequently, the interferers with larger powers will cause high performance degradation for the desired user.

Tight power control has been used to eliminate the near-far effect in IS-95. Another important approach, multiuser detection technique, has been developed in the recent years to solve the near-far problem and consequently increase the user capacity of the CDMA system. The optimal multiuser detector, which yields the minimum achievable probability of error (and optimum asymptotic multiuser efficiency, as well as optimum near-far resistance) in CDMA channels, was presented and analyzed in [3]. But the complexity of the optimal multiuser detector increases exponentially with the number of users which makes it difficult to implement in the practical system. It triggered the new research effort on suboptimal multiuser detectors and serves as a baseline of comparison for suboptimal multiuser detectors. The first suboptimal multiuser receiver is the decorrelating detector (decorrelator), which was developed in [4] and [5]. In [6], another linear multiuser receiver, the MMSE receiver, was presented. Suboptimal nonlinear detectors employing successive cancellation was consindered in [7] and [8]. These multiuser receivers, including decorrelating detector, the minimum mean-square error (MMSE) receiver and the successive interference cancellation (SIC) receiver have received considerable attention.

This project has made an attempt to study and implement some kinds of multiuser receivers to achieve better performance than the conventional matched filter receiver. In project, the DS/CDMA systems with four kinds of receivers (the matched filter, the decorrelator, the MMSE receiver and the successive interference canceller) are simulated by using Matlab language. The bit error rate plots are drawn and the BERs of different receivers in different channels (AWGN channels and fading channels) and in different situations (equal power and unequal power) are compared. Then we use the Gaussian approximation to analyze the performance of these receivers.

The rest of the project report is organized in the following way. A brief overview of multiuser receivers is provided in Section II. In Section III, we describe the theoretical analysis of these multiuser receiver techniques in AWGN channels. Computational complexity of multiuser detectors is analyzed in Section IV. In Section V, simulation results are given. Conclusions are drawn in Section VI.

II. DESCRIPTION of MULTIUSER DETECTORS

In this section, we describe the structure of the conventional matched filter receiver and three kinds of multiuser receivers.

1. Synchronous CDMA model

In project, the synchronous CDMA model is constructed for simulation. A CDMA channel with K users sharing the same bandwidth is shown in Fig.1.

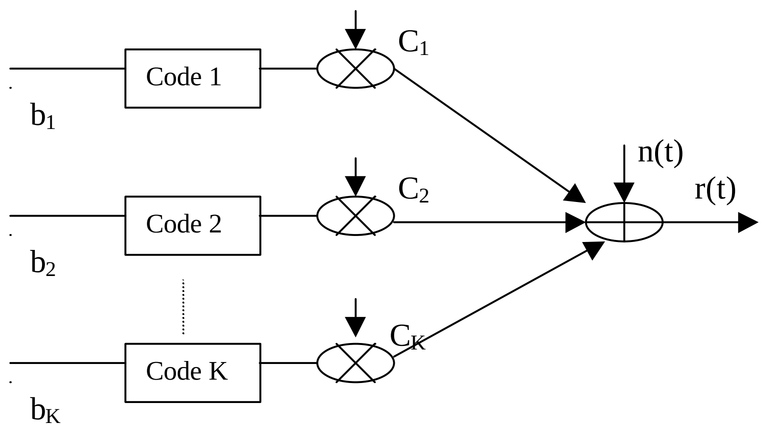

Figure 1. *The CDMA channel model*

Then the received signal is:

$$r(t) = \sum_{k=1}^{K} A_k C_k b_k S_k(t) + n(t), t \in [0, T] \qquad (1)$$

- r(t) is received signal
- K: total number of users
- A_k: the amplitude of the kth user's signal. A_k^2 is referred to as the energy of the kth user.
- C_k: the channel attenuation. $b_k \in \{-1,1\}$, BPSK modulation
- $S_k(t)$: spreading sequence of the kth user
- n(t): additive white Gaussian noise with power spectrum density σ^2.

In simulation, the information bits and chips are rectangular. The information bits are i.i.d. random variables with probability 0.5 of 1 and –1.

2.Conventional matched filter receiver:

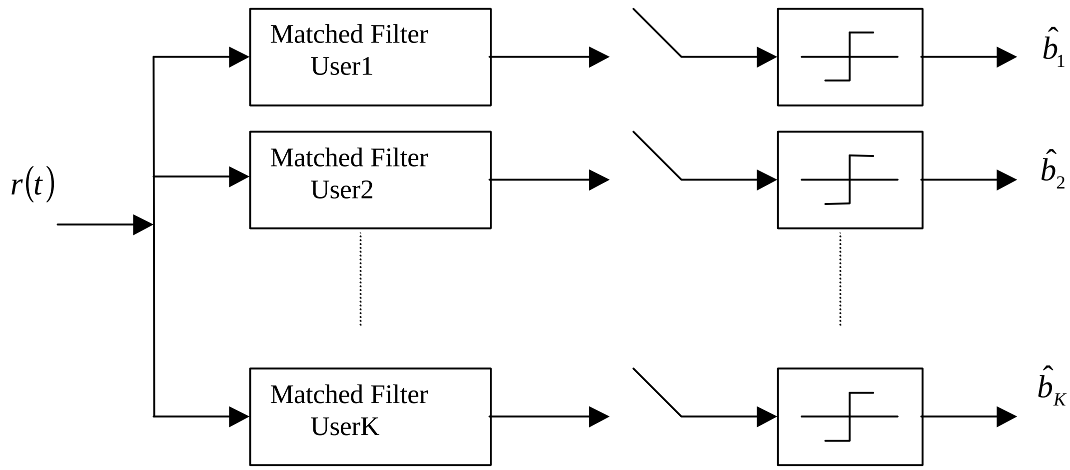

Figure 2. *Conventional Single-User matched filter receiver for multiple user detection*

In conventional CDMA system, the single-user matched filter receiver is used to demodulate the received signal.

The sampled output of the k-th matched filter is:

$$y_k = \int_0^T r(t)S_k(t)dt = \int_0^T S_k(t)[\sum_{j=1}^{K} A_j b_j S_j(t) + n(t)]dt$$

$$= A_k b_k + \sum_{j \neq k}^{K} A_j b_j \int_0^T S_k(t)S_j(t)dt + \int_0^T S_k(t)n(t)dt \qquad (2)$$

Where,

$$r_{kj} = \int_0^T S_k(t)S_j(t)dt \qquad (3)$$

ρ_{kj} is the crosscorrelation of the spreading sequences between the kth and the jth user.

The decision is made by:

$$\hat{b}_k = \mathrm{sgn}(y_k) \qquad (4)$$

y_k consists of three terms. The first is the desired information which gives the sign of the information bit b_k. The second term is the result of the multiple access interference (MAI), and the last is due to noise. The single-user matched filter receiver takes the MAI as noise and can't suppress MAI.

In matrix form, we present the outputs of the match filters as

$$\mathbf{y=RAb+n} \quad (5)$$

where **R** is the normalized crosscorrelation matrix whose diagonal elements are equal to 1 and whose (i,j) elements is equal to the cross-correlation ρ_{ij}, **A**=diag{A_1, …, A_k}, and **y**=$[y_1, \ldots, y_k]^T$. **b**=$[b_1, \ldots, b_k]^T$ and **n** is a Gaussian random vector with zero mean and covariance matrix $\sigma^2\mathbf{R}$.

3.Decorrelator:

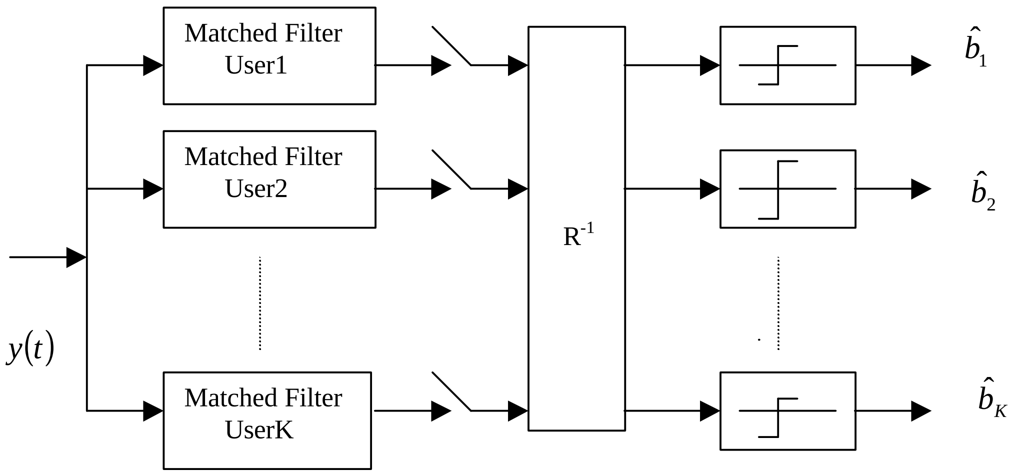

Figure 3. *Decorrelating detector*

Decorrelator is a kind of linear multiuser receivers where $T=R^{-1}$. It's optimal according to three different criteria: least-squares, near-far resistant and maximum-likelihood when the receive powers are unknown.

So the decision for the kth user is made based on

$$\begin{aligned}\hat{b}_k &= \mathrm{sgn}((\mathbf{R^{-1}y})_k) \\ &= \mathrm{sgn}((\mathbf{R^{-1}(RAb+n})_k) \\ &= \mathrm{sgn}((\mathbf{Ab+R^{-1}n})_k) \end{aligned} \quad (6)$$

The decorrelating detector eliminates MAI completely, thus providing unbiased estimates. However, the receiver considerably enhances the noise in the system.

4.MMSE receiver

The MMSE receiver is another kind of linear multiuser receivers where $T=(R+\sigma^2A^{-2})^{-1}$. For MMSE receiver, we choose the linear transformation that minimizes the mean-square error between its outputs and the data.

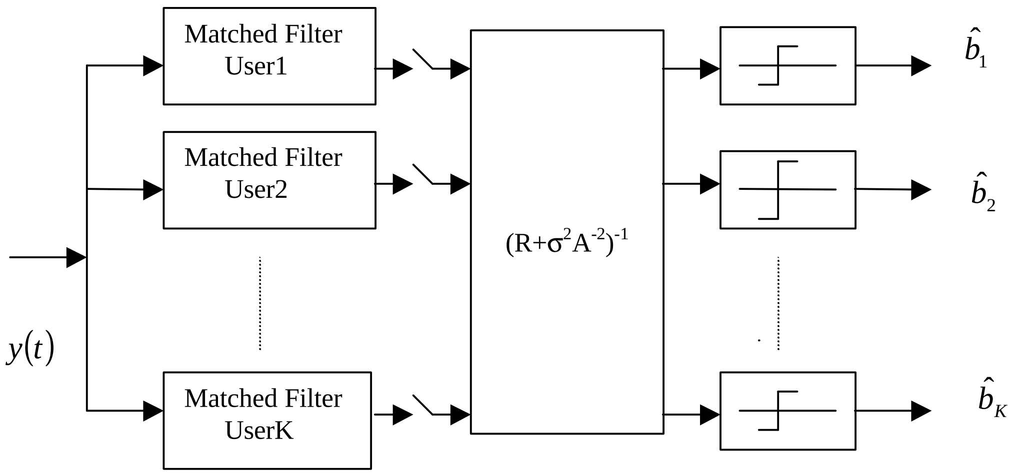

Figure 4. *the MMSE receiver*

So the decision for the kth user is made based on

$$\begin{aligned}\hat{b}_k &= \operatorname{sgn}((\mathbf{(R+\sigma^2A^{-2})^{-1}y})_k) \\ &= \operatorname{sgn}((\mathbf{(R+\sigma^2A^{-2})^{-1}(RAb+n)})_k)\end{aligned} \quad (7)$$

As the noise goes to zero, the MMSE receiver approaches the decorrelator. As the noise grows large, it is reduced to the matched filter receiver. The MMSE receiver attemps to balance the removal of the multiple access interference (MAI) and the reduction of the noise enhancement. [6] studied the adaptive implementation of the MMSE receiver requiring no prior knowledge of received signal-to-noise ratios or interfering signature waveforms. Prior to data transmission it is assumed that the receiver has been trained by a known training sequence, while an adaptive algorithm constantly adjusts it during data transmission.

5.Successive Interference Cancellation Scheme

The successive interference canceller is a kind of nonlinear multiuser receivers which estimates and cancels multiple accesss interference successively using feedback. In the scheme, the received signal $r(t)=\sum_{k=1}^{K}A_k b_k S_k(t)+n(t), t\in[0,T]$ is first passed through a bank of matched filters. Then the user with the strongest correlation value (the correlations of each of the users' spreading sequence $S_k(t)$ with the received signal r(t)) is selected for decoding. The signal of the user can be regenerated and subtracted from the received waveform.

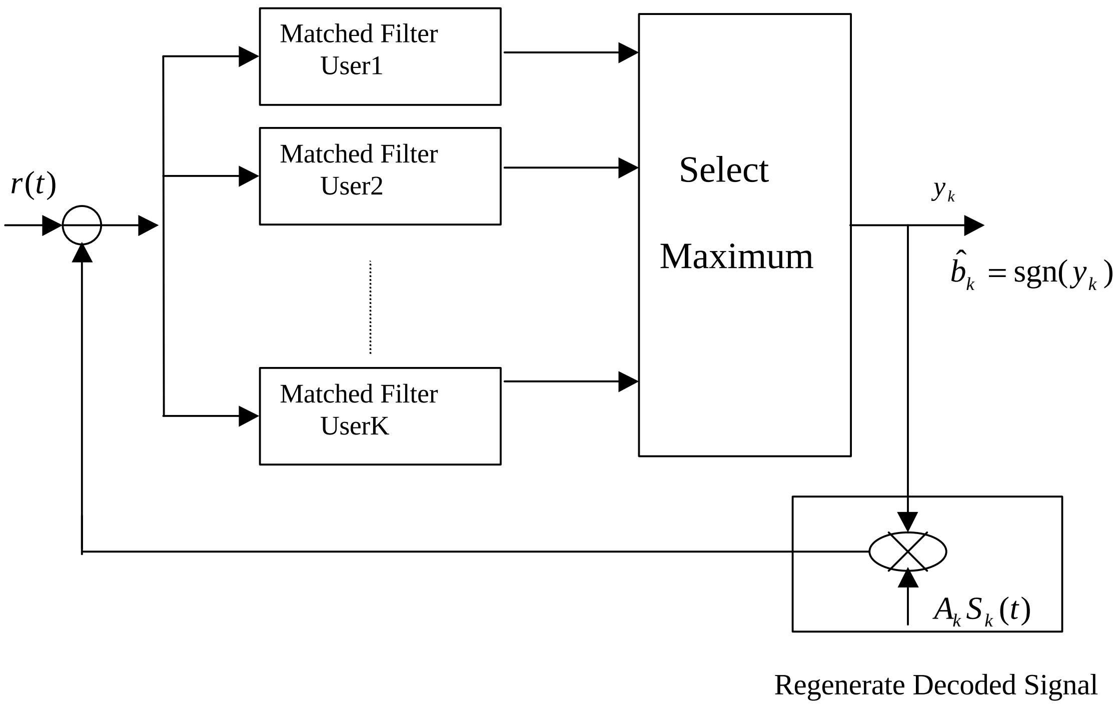

Figure 5. *Successive Interference Cancellation Scheme*

Assume the kth user has the strongest correlation value,

$$\hat{r}(t) = r(t) - A_k b_k S_k(t) = \sum_{j=1, j\neq k}^{K} A_j b_j S_j(t) + A_k(b_k - \hat{b}_k) S_k(t) + n(t) \qquad (8)$$

So this will cancel the interfering signal provided that the decision was correct. The process is repeated until the weakest user is decoded.

III. Performance Analysis in AWGN Channels

In this section, we use the Gaussian approximation method to approximate the probability of bit error for multiuser receivers in an AWGN channel. These theoretical results are used to verify the correctness of Monte-Carlo simulation.

1.Conventional matched filter receiver:

In CDMA Gaussian channel, the bit-error-rate of the kth user of the mathed filter can be calculated by:

$$P_k^c(\sigma) = P[b_k = +1]P[y_k < 0 \mid b_k = +1] + P[b_k = -1]P[y_k > 0 \mid b_k = -1]$$

$$= \frac{1}{2^{K-1}} \sum_{e1\in\{-1,1\}} \cdots \sum_{\substack{e_j\in\{-1,1\}\\ j\neq k}} \cdots \sum_{e_K\in\{-1,1\}} Q\left(\frac{A_k}{\sigma} + \sum_{j\neq k} e_j \frac{Aj}{\sigma}\rho_{jk}\right) \quad (9)$$

We can observe that the error probability of the matched filter depends on spreading sequences through their crosscorrelations. It's due to both the nature of the receiver and the fact that the background noise is white and Gaussian.

The computational complexity for (9) grows exponentially in the number of users. The bit- error-rate can be approximated as:

$$\tilde{P}_k^c(\sigma) = Q\left(\frac{A_k}{\sqrt{\sigma^2 + \sum_{j\neq k} A_j^2 \rho_{jk}^2}}\right) \quad (10)$$

Formula (10) is used to compute the bit-error-rates that will be compared with the results from Monte-Carlo simulation.

2.Decorrelator:

The probability of symbol error of the kth user for the decorrelator can be represented as [4]

$$\tilde{P}_k^d(\sigma) = Q\left(\frac{A_k}{\sqrt{\sigma^2 R_{kk}^{-1}}}\right) \quad (11)$$

So the performance of the decorrelator is identical to the single user case with the exception of the noise enhancement factor $(R^{-1})_{kk}$. Since all the elements of R are less than or equal to one, we note that $(R^{-1})_{kk}>1$. In the simulation, we obtain an estimate of the performance by calculating the average of the elements along the diagonal of the inverse of the crosscorrelation matrix R.

3.Successive Interference Cancellation Scheme

In the Guassian approximation for SIC, we assume random sequences are used in CDMA system (each chip bit is i.i.d. random variable with probability 0.5 of 1 and –1) and the number of the users K approach infinite.

The bit-error rate of the successive interference cancellation can be approximated by the recursive formula: (In deduction, we assume the noise (from other users and channel) is Gaussian.)

$$P_k^{SC}(s) \approx Q\left(\frac{A_k}{\sqrt{s^2 + \frac{1}{N}\sum_{j=1}^{k-1} A_j^2 + \frac{4}{N}\sum_{j=k+1}^{K} A_j^2 P_j^{sc}(s)}}\right) \qquad (12)$$

For a system with the finite number of users, the bit-error rate can be approximated [9] in the following way:

$$P_k^{SC}(s) \approx Q\left(\sqrt{\frac{E[y_k]^2}{\mathrm{var}[y_k]}}\right) \qquad (13)$$

where

$$E[y_k]^2 = A_k \qquad (14)$$

$$\mathrm{var}[y_k] = [s^2 + \frac{1}{N}\sum_{i=2}^{K} A_i^2](1+\frac{1}{N})^{k-1} - \frac{1}{N}\sum_{i=2}^{k}(1+\frac{1}{N})^{k-i} A_i^2 \qquad (15)$$

IV. Computation Complexity

The computational complextity of multiuser detectors is an important factor for practical implementation. High computational complexity would prevent the practical use of multiuser receivers. Here we define the computational complexity as the number of real single floating point operations required per bit decision and express it in terms of the number of users K, the frame length N_b, the number of paths tracked L, the spreading factor N and the number of samples per chip Ns.

1. Decorrelator:

The implementation of the decorrelator needs four-step operations:

a. Calculating the matched filter outputs.
b. Multiplication of the matched filter output by R^{-1}.
c. Creating the decision metrics.
d. Computing the inverse of the correlation matrix.

So the computational complexity per bit decision of the decorrelator is:

$$C_{dec}(b) = LK(2NN_s + N_b LK + 5) + \frac{2LK(KL-1)NN_s}{N_b} + \frac{2}{3}N_b^2(LK)^3 \quad (16)$$

2. Successive Interference Cancellation:

We consider two cases here: wideband case and narrowband case.
For the wideband case, the computational complexity per bit decision can be expressed as

$$L[K(8NN_s + 12) - 6NN_s - 7] + \frac{K}{N_b}(2N_b + \log_2(K) + 1) \quad (17)$$

For the narrowband case, the computational complexity per bit decision is

$$\begin{aligned} &L[2NN_s K + 5K + 8\sum_{k=1}^{K-1} k + (K-1)(5 + 2NN_s)] \\ &+ \frac{2KLNN_s(KL-1) + K + K\log_2(K)}{N_b} + 2K \end{aligned} \quad (18)$$

V. Simulation Results

Monte-Carlo simulation is used to calculate bit-error-rate. Then we use Gaussian approximation to verify the correctness of Monte-Carlo simulation results. The program is written in Matlab. Random spreading sequences are generated for all users in the CDMA system. The performance of multiuser receivers in both AWGN channels and flat Rayleigh fading channels is simulated. We consider two scenarios here: equal powers (perfect power control) and unequal powers (near-far scenario). Finally, the computational complexity of multiuser detectors is compared through simulation.

Simulation 1: performance in AWGN channels (perfect power control)

In this simulation a synchronous CDMA channel is considered. We assume perfect power control and choose the signal-to-noise ratio (SNR) Eb/No=8dB, spreading gain N=15. The Monte-Carlo simulation results of the BER performance versus capacity (the number of users in the system) are plotted along with the corresponding theoretical results in the following figure.

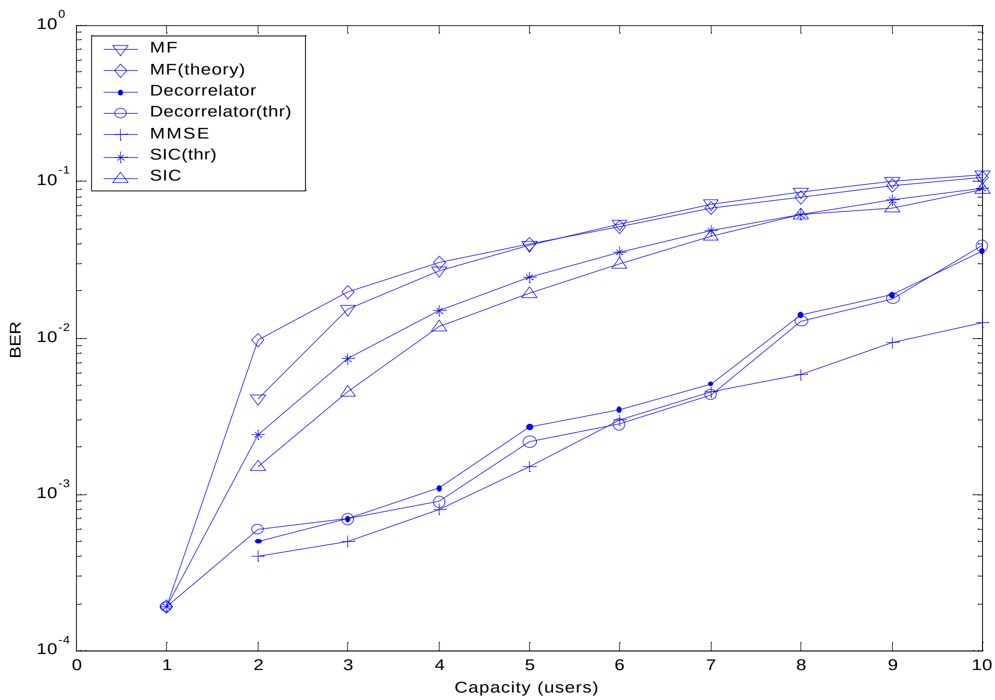

Figure 6. *Capacity curves in AWGN channels with perfect power control*

The Monte-Carlo simulation results show excellent agreement with the theoretical results. The theoretical result for the MMSE receiver is not plotted because of the complexity of Caussian approximation for the MMSE receiver. The performance of the MMSE receiver is upper bounded by that of the decorrelator. We observe that the decorrelator and the MMSE receiver can achieve similar performance and provide an obvious capacity improvement with equal powers. However, the performance of the successive interference canceller is poorer due to the lack of variance of the received powers of all users.

Simulation 2: performance in AWGN channels (perfect power control)

The simulation results of the BER performance versus SNR are shown in the following figure. We also assume perfect power control and choose the spreading gain N to be 15 and the number of users in the system K to be 5.

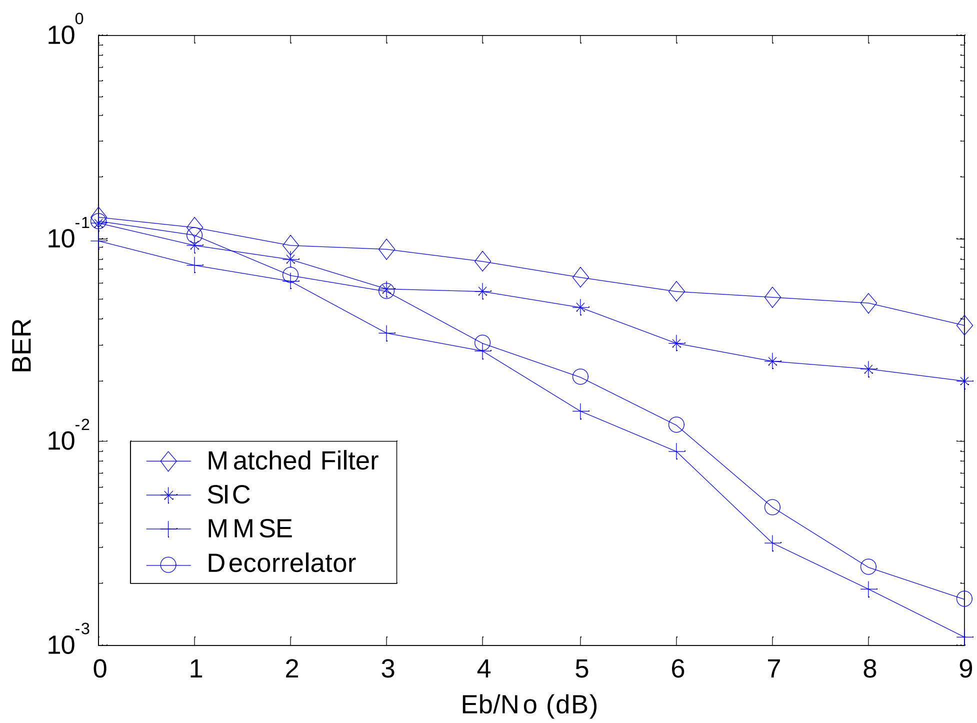

Figure 7. *BER versus Eb/No in AWGN channels with perfect power control*

We observe that the decorrelator and the MMSE receiver can achieve significant performance improvement while the successive interference canceller provides a small improvement.

Simulation 3: performance in AWGN channels (near-far scenario)

The performance of multiuser detectors in near-far scenario is studied. We assume there are two users in the system: one is the desired user with the SNR 5dB, the other is the interferer with a power which varies from 10-dB below the desired user to 30-dB above the desired user. The spreading gain N is 15. The simulation results are shown in the following figure.

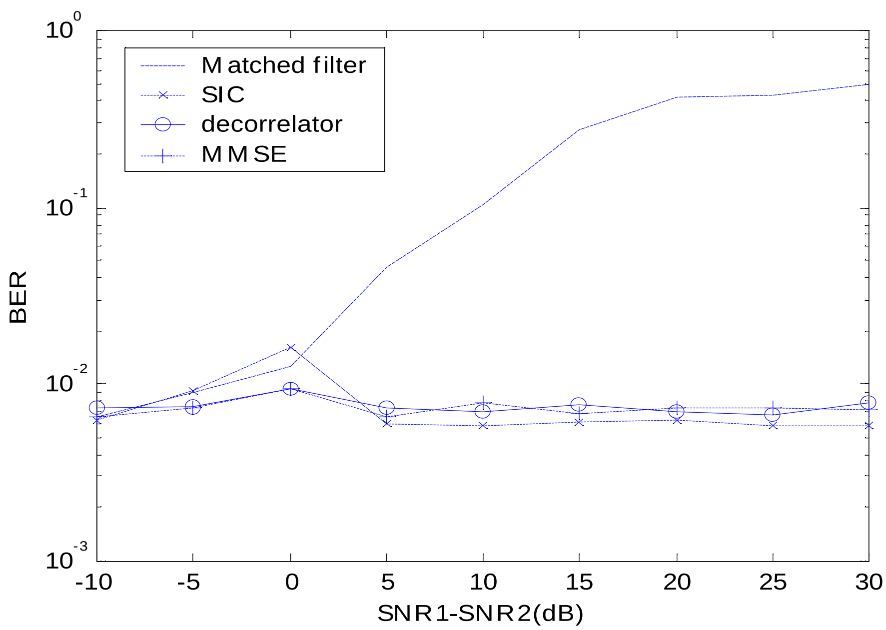

Figure 8. *Performance in near-far scenario*

The matched filter receiver degrades quickly with the increase of the interferer power. Unlike that in the perfect power control scenario, the successive interference canceller (SIC) can achieve an obvious performance improvement due to the disparity of the received powers. The decorrelator and the MMSE receiver provide the similar performance improvement. All the multiuser detectors simulated are robust to the near-far problem.

Simulation 4: performance in flat Rayleigh fading channels

The performance of multiuser receivers in flat Rayleigh fading is studied. We assume perfect channel estimation and the channel is flat with a single path experiencing Rayleigh fading. We choose the spreading gain N to be 15 and the number of users K in the system to be 5. The Monte-Carlo simulation results of the BER performance versus the average SNR are plotted in the following figure.

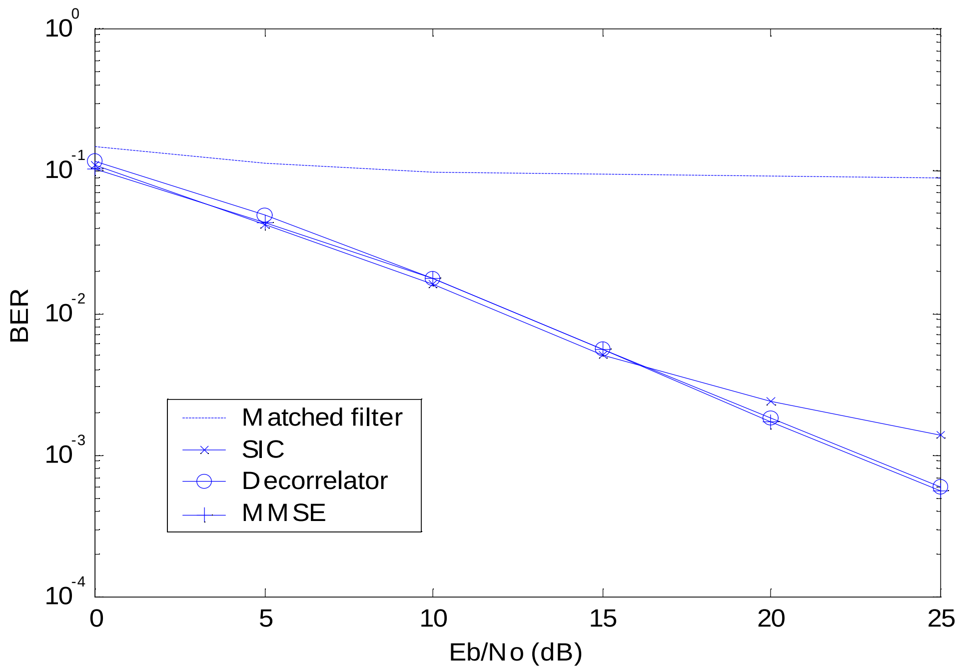

Figure 9. *Performance in flat Rayleigh fading channels with perfect channel estimation*

We can see that all the three kinds of multiuser receivers considered provide significant performance improvement over the conventional matched filter in flat Rayleigh fading and they achieve nearly equivalent performance.

Simulation 5: computational complexity of multiuser detectors

The computational complexity of multiuser receivers is studied. The preceding equations (16), (17) and (18) are used to compute the computational complexity of the different multiuser receivers. The computational complexity of the MMSE receiver is omitted here because it would be similar to the decorrelator. We choose N=15, N_s=4, L=2 and N_b=100.

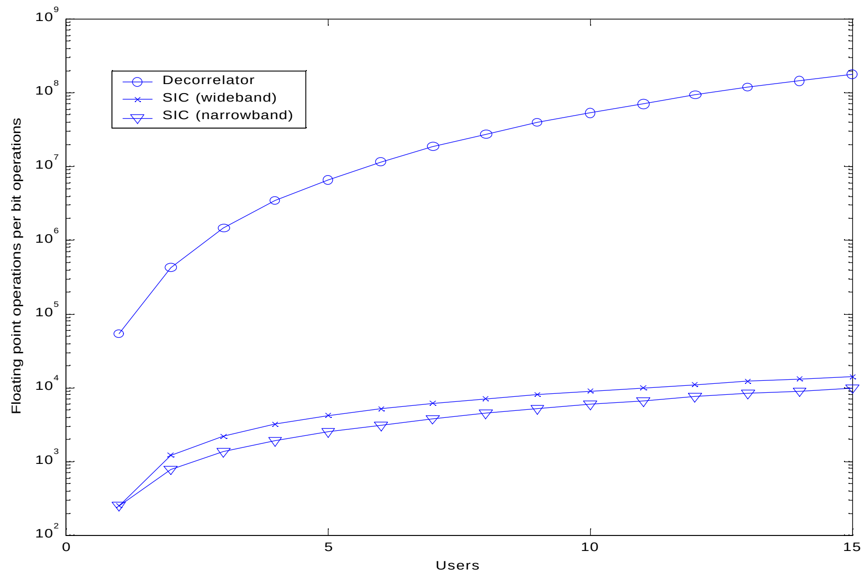

Figure 10. *Computational complexity of multiuser detectors*

We see that the decorrelator is extremely computational complicated due to the matrix inversion. So the iterative approaches without the direct matrix inversion for the decorrelator have been studied in [11], which have a computational complexity of order $N(KL)^2$. For the successive interference cancellation, the narrowband approach is less complicated than the wide one.

VI. CONCLUSIONS

In this project, we analyzed and simulated the performance of the conventional matched filter receiver and three kinds of multiuser receivers (the decorrelator, the MMSE receiver, the successive interference canceller) in a synchronous CDMA system. In AWGN channels, the decorrelator and the MMSE receiver provide a significant performance improvement relative to the conventional matched filter receiver when all the users in the system have the same received powers (perfect power control). However, the performance of the successive interference canceller is poorer because this multiuser detection scheme needs to take advantage of the variance of the received powers of all users to suppress multiple access interference. In the near-far scenario (unequal powers), all the multiuser receivers considered including the successive interference canceller can achieve better performance than the conventional match filter. In flat Rayleigh fading channels , these multiuser detectors provide significant performance improvement over the conventional matched filter.

During the project, I learned how to construct the DS/CDMA system, how to use Gaussian approximation method to approximate the bit-error-rates of different multiuser detection schemes in the AWGN channel, and how to use Monte-Carlo simulation method to calculate them.

REFERENCES

[1] S. Verdu, *Multiuser detection,* Cambridge, U.K.: Cambridge Univ. Press, 1998.

[2] Tan F. Wong, Spread Spectrum & CDMA, course notes.

[3] S. Verdu, "Minimum probability of error for asynchronous Gaussian multiple access channels," *IEEE Trans. Inform. Theory, vol. IT-32, pp. 85-96, Jan. 1986.*

[4] R. Lupas and S. Verdu, "Linear multiuser detectors for synchronous code-division multiple access channels," *IEEE Trans. Inform. Theory, vol. 35, pp. 123-136, Jan. 1989.*

[5] R. Lupas and S. Verdu, "Near-far resistance of multiuser detectors in asynchronous channels," *IEEE Trans. Commun., vol. 38, pp. 496-508, Apr. 1990.*

[6] U. Madhow and M. Honig, "MMSE interference suppression for direct-sequence spread-spectrum CDMA," *IEEE Trans. Commun., vol. 42, pp.3178-3188, Dec. 1994.*

[7] P. Patel and J. Holtzman, "Analysis of a simple successive interference cancellation scheme in DS/CDMA system." *IEEE JSAC, 12:796-807, Jun. 1994.*

[8] A. J. Viterbi, "Very low rate convolutional codes for maximum theoretical performance of spread-spectrum multiple-access channels," *IEEE J. Select. Areas Commun., vol. 8, pp. 641-649, May 1990.*

[9] R. M. Buehrer, "Equal BER performance of linear successive interference cancellation," *IEEE Trans. Commun., vol.49, pp. 1250-1258, Jul 2001.*

[10] R.M.Buehrer, N.S.Correal-Mendoza and B.D.Woerner, "A simulation comparison of multiuser receivers for cellular CDMA." *IEEE Trans. Veh. Technol., vol. 49, pp. 1065-1085, Jul. 2000.*

[11] M.J.Juntti, B.Aazhang and J.O.Lilleberg, "Iterative implementation of linear multiuser detection for dynamic asynchronous CDMA systems," *IEEE Trans. Commun., vol.46, pp. 503-508, Apr. 1998.*

[12] Y. Liu, "Simulation and analysis of successive interference cancellation scheme in DS/CDMA system", *course project for EEL 6509 spring 2001*

Comment: part of the description of the matched filter and SIC is borrowed from the author's previous project [12].