EQUAL TREATMENT AS A MEANS OF EVALUATING
PUBLIC SCHOOL ASSIGNMENT MECHANISMS

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This paper proposes a new criterion to assess public school assignment mechanisms based on a standard 14th Amendment equal protection requirement. Evaluating the three prominent assignment mechanisms discussed in the recent literature, findings reveal that neither the “Boston Mechanism” nor the two strategy-proof alternatives proposed as replacements satisfy this ‘equal treatment’ criterion. These findings surface a serious cause for concern about the public school assignment procedures used in major school districts.

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1. Introduction

Open enrollment programs such as inter-district and intra-district school choice, which allow parents to send their children to public schools outside of the neighborhood they reside, have become increasingly popular in the United States during the last two decades. As of 2005, 27 states had passed legislation mandating school districts to implement intra-district school choice, and 20 states had mandated the school districts within their boundaries to participate in the inter-district choice program of the state (ECS, 2005). There is also an increasing trend in the percentage of households participating in open enrollment programs. Between 1993 and 2003, the percentage of students attending a public school other than their ‘assigned’ neighborhood schools increased from 11 percent to 15.4 percent in the United States (NCES, 2006).

In the ideal setting, absent frictions, open enrollment programs allow parents to send their children to any public school within the boundaries of a region that contains, but is not limited to, the household’s neighborhood. In this scenario, public school assignments are trivial; each student is assigned to the public school of her choice within these boundaries. However, in practice, parents are typically limited in their public school choices by non-boundary constraints, especially public school capacities. The presence of such constraints necessitates other parents’ public school preferences to be taken into account in order to determine the public school assignment of a given student, which turns public school assignments into a complicated problem and obligates school districts to employ various priority categories to classify applicants at schools along with
centralized assignment mechanisms. These assignment mechanisms have so far been evaluated in the economics literature along three major dimensions:

1. **Strategy-proofness**: A preferred public school assignment mechanism avoids creating incentives for parents to play complicated games. Hence, truthful parental ranking of schools should be a dominant strategy. Strategy-proofness of the assignment mechanism is then desirable.

2. **Stability**: An assignment set is defined to be *stable* if there is no school-student pair \((i,s)\) such that student \(i\) prefers school \(s\) to her current assignment and *either* school \(s\) prefers student \(i\) to at least one of the students assigned to it *or* school \(s\) has at least one empty seat. Absent stability, there exists ‘justified envy’ in the assignments, providing incentives for parents to seek legal action to overturn assignment decisions.

3. **Efficiency**: For the public school assignment problem in the context of this paper, only the welfare of students is considered for Pareto efficiency, since schools are regarded as objects to be consumed by students.\(^1\) Pareto efficient assignments are obviously desirable.

One of the most commonly used student assignment mechanisms is the Boston mechanism, so named because of its use until recently in Boston. This mechanism is still being used in other major school districts including Cambridge (MA), Charlotte (NC), Denver (CO), Hillsborough (Tampa, FL), Miami-Dade (FL), Minneapolis (MN) and

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\(^1\) For the public school assignment problem discussed in this paper, priority categories mandated by school districts are employed along with student preferences to determine public school assignments. Since these rankings do not necessarily correspond to schools’ preferences, only students’ preferences are considered for efficiency. On the contrary, there are cases such as the high school assignments in NYC where schools determine their own priority rankings. In that case, school preferences as well as student preferences might be taken into account for welfare considerations.
Pinellas (St.Petersburg, FL). Despite its common use, ironically, previous literature has shown that the Boston mechanism fails to satisfy any of the aforementioned properties of a ‘well-behaving’ assignment mechanism in practice.\textsuperscript{2} Given these results, two alternatives have been proposed to replace the Boston mechanism: the Gale-Shapley Deferred Acceptance (GS-DA) mechanism, which, in fact, replaced the Boston mechanism in Boston Public Schools in 2006, and the Top-trading Cycles (TTC) mechanism. Both of these alternatives have been shown to dominate the Boston mechanism on important dimensions such as strategy-proofness.\textsuperscript{3}

This paper introduces a new dimension of merit to evaluate public school assignment mechanisms based on the Equal Protection Clause of the 14th Amendment, which, in public school assignment context, seems to require that students with the same public school preferences and in public schools’ same priority categories must be treated equally.\textsuperscript{4} In the presence of binding public school capacity constraints, a weaker application would require that if two students who are in the same priority category for a given school have the identical true strict preference ranking of schools with that school as their first choices, the assignment mechanism imply an equal probability of assignment to the ‘school of interest’. Generalizing this weak application of equal protection, this paper presents a new criterion, which I refer to as equal treatment, to evaluate public school assignment mechanisms.

\textsuperscript{2} See Abdulkadiroglu and Sonmez (2003) and Ozek (2009).
\textsuperscript{3} While both alternatives achieve strategy-proofness, there is a trade-off between stability and Pareto efficiency when different schools have different priority rankings of students. The GS-DA mechanism attempts to assign each student to her highest possible public school choice for which she has high enough priority to be assigned. Thus, this mechanism produces stable assignments, though Pareto efficiency is not guaranteed. On the other hand, the TTC mechanism trades priorities of students among themselves starting with the students with highest priorities, producing Pareto efficient, but not necessarily stable assignments. See Abdulkadiroglu and Sonmez (2003), Abdulkadiroglu et. al. (2005), Abdulkadiroglu et. al. (2006).
\textsuperscript{4} The legal precedent is further discussed below.
Evaluating the Boston mechanism and the two alternatives proposed in the recent literature as replacements for the Boston mechanism along this new dimension, findings reveal that none of these mechanisms satisfy the equal treatment criterion. While these findings surface a serious cause for concern, subsequent sections of this paper indicate that, under certain procedural constraints (e.g. completely eliminating priority categories), it is possible for school districts to benefit from the desirable properties of the aforementioned alternatives without facing the possible legal challenges.

2. Public School Assignment Problem and the Assignment Mechanisms

In a public school assignment problem, there are $n$ students ($i_1, i_2, \ldots, i_n$) and $k$ public schools ($s_1, s_2, \ldots, s_k$) each of which has a certain number of seats available ($c_1, c_2, \ldots, c_k$). Public school assignments depend on students’ reported preferences, schools’ priorities over students, and the assignment mechanism. It is assumed that each student has a utility function over the $k$ public schools with strict preferences. Students first submit their preferences, i.e., a strict ranking of the public schools. Public school assignments are then determined based on the set of submitted (ordinal) rankings. Schools have priority rankings of students, based on broad priority categories mandated by the school district (e.g., residing in a walk zone). A single random lottery is conducted to break the ties between the students in the same priority categories after students submit their preferences.\(^5\)

\(^5\) School districts differ in the ways they use priority categories along with the lottery outcome to rank applicants. In Boston, applicants for a given school are first ranked with respect to the priority categories and then the outcome of the lottery is used to rank those within the same priority category. In Miami-Dade, on the other hand, a weighted lottery is conducted where more random numbers are generated for those in higher priority categories. The rankings are then constructed using the best random number for each applicant. In this paper, I focus on the former noting that the results also apply to the latter.
There are two types of students: strategic (sophisticated) students who play best response to the other students, and sincere (unsophisticated) students who always reveal their public school preferences truthfully in equilibrium. I assume that strategic students, who are the only active players of the games of school choice, have perfect information about students’ true preferences over schools, pre-lottery priority rankings of students at schools and rules of the assignment mechanism, yet do not know the outcome of the lottery while making their public school choices. In other words, strategic students choose the strategy (public school ranking) that yields the highest expected utility among different possible rankings of public schools, some of which, if submitted, may yield different outcomes (assignments) with known probabilities depending on the lottery result. How the submitted equilibrium preferences and school priorities interact to yield assignments depends on the assignment mechanism, which is applied after the random lottery is conducted. I illustrate three of these mechanisms using the following example:

*Example 1:* Let $n = k = 3$ and $(c_1, c_2, c_3) = (1, 1, 1)$. In other words, suppose that there are three students $(i_1, i_2, i_3)$, each of whom is strategic, and three schools $(s_1, s_2, s_3)$, each of which has only one seat available. Public school preferences of students and pre-lottery priority rankings of students at each school are given as:

- $i_1$: $s_1 > s_2 > s_3$
- $i_2$: $s_1 > s_2 > s_3$
- $i_3$: $s_2 > s_1 > s_3$

where ‘$>$’ indicates strict preference for students and higher priority category for schools whereas ‘$=$’ indicates that the two students are in the same priority category for the given school.
2.1. The Boston Mechanism

Under the Boston mechanism, a student who is not assigned to his first choice is considered for his second choice only after the students who ranked that student’s second choice as their first choices. Formally, the algorithm is as follows:

- In the first step, only the first choices of students are considered. Based on schools’ post-lottery priority rankings of students, seats at each school are assigned one at a time until either there are no seats left or there is no student left who has listed it as her first choice.

- In the \( n^{th} \) step, only the \( n^{th} \) choices of the students who could not be placed in the \((n-1)^{th}\) round are considered. Based on schools’ post-lottery priority rankings of students, seats at each remaining school are assigned one at a time until either there are no seats left or there is no student left who has listed it as her \( n^{th} \) choice.

Applying this algorithm to Example 1, given that each student reveals her public school preference truthfully to illustrate, the Boston mechanism results in the assignments \((i_1, s_1), (i_2, s_3)\) and \((i_3, s_2)\) in the state of nature where the tie between \(i_1\) and \(i_2\) for school \(s_1\) is broken in favor of \(i_1\) or the assignments \((i_1, s_3), (i_2, s_1)\) and \((i_3, s_2)\) otherwise.\(^6\)

Recent literature has shown that truthful revelation of public school preferences is not necessarily the weakly dominant strategy for each parent under the Boston mechanism; strategy-proofness fails. In fact, truthful revelation for each student might not be a Nash equilibrium strategy set under the Boston mechanism in Example 1: \(i_2\) might find it beneficial to reveal \(s_2\) as her first choice provided that others reveal

\(^6\)When applied to Example 1, the Boston mechanism works as follows. First step: Only the first choices are considered. Given the priorities, \(i_3\) is assigned to \(s_2\). If the tie between \(i_1\) and \(i_2\) for school \(s_1\) is broken in favor of \(i_1\), \(i_1\) is assigned to \(s_1\). Otherwise; \(i_2\) is assigned to school \(s_1\). Second step: Depending on the outcome of the random lottery, the student who gets rejected from \(s_1\) is assigned to \(s_3\), since the only available seat in \(s_2\) is occupied by \(i_2\). The algorithm terminates.
truthfully. Boston mechanism thus induces parents to play a complicated preference revelation game where payoffs are determined by student preferences over schools, school priority rankings over students, and the rules of the mechanism. Furthermore, the Nash equilibrium assignments of this game guarantee neither Pareto efficiency nor stability.

2.2. Top-trading Cycles Mechanism

Abdulkadiroglu and Sonmez (2003) show that the TTC mechanism is strategy-proof and Pareto efficient; however, stability is not guaranteed. The formal algorithm works as follows:

- Step 1: Each student points to her favorite school and each school points to the highest priority-ranked student. Assign all students in a cycle to the schools they point to and remove them from the cycle. Also remove a school from the available schools list if its capacity becomes full.
- Step k: Apply the same algorithm to the remaining students and schools. The process terminates when there are no remaining cycles.

When applied to Example 1, for the true preferences which are expressed in equilibrium, the TTC mechanism results in the assignments \((i_1, s_3), (i_2, s_1), (i_3, s_2)\) for both outcomes of the lottery.

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7 This possibility is further discussed in the following section.
8 See Abdulkadiroglu and Sonmez (2003). While the findings of Ergin and Sonmez (2006) suggest that the Boston mechanism produces stable assignments in equilibrium, Ozek (2009) shows that this result relies on complete information assumption, which is not satisfied in practice due to the timing of lotteries.
9 A cycle is an ordering of distinct students and schools \((s_1, i_1, s_2, \ldots, s_k, i_k)\) where \(s_1\) points to \(i_1\), \(i_1\) points to \(s_2\), \(s_2\), \(s_{k-1}\) points to \(i_k\), \(i_k\) points to \(s_1\). In a public school assignment problem, we know that there is at least one cycle, since the number of students and schools are finite.
10 When applied to Example 1, the TTC mechanism determines the public school assignments as follows. First step: There is only one cycle: \(i_2 \rightarrow s_1 \rightarrow i_3 \rightarrow s_2 \rightarrow i_2\). \(i_2\) is assigned to \(s_1\) and \(i_3\) is assigned to \(s_2\). Students \(i_2\) and \(i_3\) are removed from the algorithm as well as schools \(s_1\) and \(s_2\), which became full. Second
2.3. Gale-Shapley Deferred Acceptance (GS-DA) Mechanism

Unlike the previous two mechanisms, none of the assignments are guaranteed until the assignment algorithm terminates under this mechanism. The algorithm works as follows:

- Step 1: Each student’s first choice is considered. Each school places all applicants into its queue unless the number of applicants is higher than the number of seats available at the school. Otherwise, each school rejects the applicants ranked lower than its number of empty seats using its post-lottery priority ranking, while placing the rest of the applicants in its queue.

- Step k: The rejected applicants’ next choices are considered. Comparing the new applicants with the applicants already in the queue, each school replaces the students on its queue based on its priority rankings. The process terminates when no student is rejected and each student is assigned to the school whose queue she belongs to when the algorithm terminates.

Applying to Example 1, again using the true preferences, the GS-DA mechanism yields the assignments \((i_1, s_3), (i_2, s_2)\) and \((i_3, s_1)\) if the tie is broken in favor of \(i_1\) or produces \((i_1, s_3), (i_2, s_1)\) and \((i_3, s_2)\) otherwise.\(^{11}\) Even though it preserves strategy-proofness and guarantees stable assignments, the GS-DA mechanism does not necessarily

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\(^{step}\): The only cycle is between \(i_1\) and the only remaining school, \(s_3\), \(i_1\) is assigned to school \(s_3\) and the algorithm terminates.

\(^{11}\): When applied to Example 1, if the tie is broken in favor of \(i_1\), the GS-DA mechanism works as follows. 
First step: \(i_2\) is in the queue of \(s_1\), \(i_3\) is in the queue of \(s_2\), and \(i_2\) gets rejected from school \(s_1\). Second step: \(i_2\) proposes to school \(s_2\) and \(i_2\) gets rejected from school \(s_2\). Third step: \(i_1\) proposes to school \(s_1\) and \(i_1\) gets rejected from school \(s_1\). Fourth step: \(i_1\) proposes to school \(s_2\) and gets rejected from school \(s_2\). Fifth step: \(i_1\) proposes to school \(s_3\) and the algorithm terminates. If the tie is broken in favor of \(i_2\), first step: \(i_2\) is in the queue of \(s_1\), \(i_3\) is in the queue of \(s_2\), and \(i_2\) gets rejected from school \(s_1\). Second step: \(i_2\) proposes to school \(s_2\) and gets rejected from school \(s_2\). Third step: \(i_1\) proposes to school \(s_3\) and the algorithm terminates.
result in Pareto efficient assignments\textsuperscript{12}. Notice that in the state of nature where \( i_1 \) wins the lottery, the resulting assignment is Pareto dominated by \((i_1, s_3), (i_2, s_1), \) and \((i_3, s_2), \) which is also stable under students’ true preferences and pre-lottery priority rankings at schools.

So far, assignment mechanisms have been evaluated in the literature along the three traditional norms of merit. In what follows, I introduce a new criterion to evaluate assignment mechanisms and examine the three prominent assignment mechanisms mentioned earlier in the light of this equal treatment criterion.

3. The Equal Treatment Criterion

Absent frictions, public school assignments under an open enrollment regime are trivial; each student is assigned to the public school of her choice within the boundaries of a region that contains, but is not limited to, the neighborhood where the student resides. When capacity constraints result in a scarcity of slots at public schools, school districts need to implement centralized assignment mechanisms as well as criteria to classify the students in order to determine the assignments in public schools where the number of applicants is greater than the number of seats available. For this purpose, broad priority categories (e.g. sibling currently attending the school) are commonly used and the ties between students in the same priority categories are broken using an equal-probability random lottery, which preserves the \textit{ex ante equivalence} of such students, before the assignment algorithm can be applied.\textsuperscript{13} Based on these priority categories and

\textsuperscript{12}See Dubins and Freeman (1981) and Roth (1982).

\textsuperscript{13}For instance, in Boston, the following priority categories are used: (1) Students who have siblings currently attending that school \textit{and} who live in the ‘walk zone’ of the school; (2) Students who have siblings currently attending that school; (3) Students who live in the ‘walk zone’ of the school; (4) Students who do not fall into the three categories above. Furthermore, each applicant is assigned a random number, which is used to break the ties between students in the same priority categories when necessary.
the true preferences of students, I introduce a criterion called *equal treatment* to evaluate public school assignment mechanisms.

For an arbitrary set of strict true student preferences over schools \( T = \{T_1, T_2, \ldots, T_n\} \) and pre-lottery school priority rankings over students \( P = \{P_1, P_2, \ldots, P_k\} \), let \( I_m(T, r, p) \) denote the set of students who are in a given priority category \( p \) for school \( s_m \) and have the same true public school ranking \( (T_e \in T) \) with \( s_m \) being their \( r^\text{th} \) choices. Under these student preferences, pre-lottery school priority rankings, and a given assignment mechanism, let \( S = \{S_1, S_2, \ldots, S_n\} \) denote the set of submitted strict student preferences over schools in equilibrium and \( \Pr_i(j | S, P) \) represent the conditional probability of being assigned to her \( j^\text{th} \) preferred school for an arbitrary student \( i, \in I_m(T_e, r, p) \).

**Definition:** For all values of \( T, r \) and \( p \), the members of the set \( I_m(T_e, r, p) \) are *equal* for school \( s_m \) and should be *treated equally* for an assignment in that school.\(^{14}\) For an arbitrary pair of students \( i, i, \in I_m(T_e, r, p) \), a public school assignment mechanism violates the *equal treatment of the equal* if and only if

a. \( \Pr_i(r | S, P) \neq \Pr_{i'}(r | S, P) \) if \( r = 1 \).

b. \( \Pr_i(r | S, P) \neq \Pr_{i'}(r | S, P) \) given that \( \Pr_i(j | S, P) = \Pr_{i'}(j | S, P) \) for all \( j < r \) and \( r > 1 \).

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\(^{14}\) It is worth noting that the equal treatment criterion does not take students’ priorities at other schools into account to determine the ‘identical’ students at a given public school. This follows since under state and local laws (e.g. Florida Statute 1002.31), priority categories are intended to provide certain subgroups of students higher probability of assignment *only* to a specific school and are not meant to impact the assignment probabilities of these students to other schools. For instance, as noted in Abdulkadiroglu and Sonmez (2003), some school districts (e.g. Boston Public Schools) require siblings of students already attending a given school to be provided higher priority for a seat in that school. However, from a fairness standpoint, given two ‘identical’ students for school A, having a sibling at school B should not provide one of the students advantage over the other for a seat in school A.
First consider the simple case demonstrated by Example 1 where students \( i_1 \) and \( i_2 \), who are in the same priority category for \( s_1 \) have the same public school preferences with \( s_1 \) being their first choices. These two students are equivalent from the point of view of the school district for a seat in school \( s_1 \) and the equal treatment criterion guarantees that \( i_1 \) and \( i_2 \) are provided equal chance of assignment to that school. Absent equal treatment, the assignment mechanism discriminates against one of the two equivalent students, making her assignment to school \( s_1 \) less likely.

On the other hand, when \( r > 1 \), \( \Pr_I (r | S, P) > \Pr_R (r | S, P) \) does not necessarily imply that the equal treatment criterion is violated; the lower probability of assignment to school \( s_m \) might be the result of that student being assigned to a public school that she prefers to \( s_m \) for all outcomes of the lottery (i.e. \( \Pr_J (j | S, P) = 1 \) for some \( j < r \), which implies that \( \Pr_R (r | S, P) = 0 \)). The latter condition in the equal treatment criterion rules out this possibility by imposing that all members of the set \( I_m (T_r, r, p) \) have equal probability of assignment to each public school they prefer to \( s_m \). Hence, \( \Pr_R (j | S, P) = 1 \) if and only if \( \Pr_I (j | S, P) = 1 \) for some \( j < r \), which in turn implies that \( \Pr_I (r | S, P) = \Pr_R (r | S, P) = 0 \).

**Proposition 1:** None of the three aforementioned public school assignment mechanisms satisfy the equal treatment requirement.\(^{15}\)

Example 1 is sufficient to show that neither of the two strategy-proof alternatives mentioned earlier satisfies the equal treatment criterion.\(^{16}\) In this case, in order to comply

\(^{15}\) The composition of the student body is inconsequential for the main finding of this study as long as there is at least one strategic student. If all students are sincere, among the three mechanisms discussed in this paper, the Boston mechanism becomes the only one that satisfies the equal treatment criterion. Proof of the latter statement is available upon request.
with this requirement, an assignment mechanism needs to provide students $i_1$ and $i_2$ equal probability of assignment to school $s_1$, since they have identical preferences and are in the same priority category for that school. Table 1 presents these probabilities as well as the public school assignments for the two possible outcomes of the lottery under the two mechanisms.\(^{17}\)

Under both of these alternatives, $i_1$ has no chance of being assigned to $s_1$ whereas the equivalent student $i_2$ is guaranteed a seat in $s_1$ under the TTC mechanism and has 0.5 chance of being assigned to $s_1$ under the GS-DA mechanism. Furthermore, between the two alternatives, the TTC mechanism is worse yet in the following sense. The latter mechanism results in assignments where even though $i_1$ wins the lottery over $i_2$ for $s_1$, $i_2$ is assigned to $s_1$. The GS-DA mechanism, on the other hand, avoids such cases by assuring stable assignments.\(^{18}\)

In order to show that the Boston mechanism violates equal treatment as well, consider a slightly modified case where the true preference ranking of schools for $i_3$ is now $s_2 > s_3 > s_1$. Provided that all students reveal truthfully, Boston mechanism results in the assignments $(i_1, s_1)$, $(i_2, s_3)$ and $(i_3, s_2)$ in the state of nature where the tie between $i_1$ and $i_2$ for school $s_1$ is broken in favor of $i_1$ or the assignments $(i_1, s_3)$, $(i_2, s_1)$ and $(i_3, s_2)$ otherwise, providing equal probability of assignment to $s_1$ for $i_1$ and $i_2$. However, notice that this mechanism creates an incentive for $i_2$ to misrepresent her preferences: if she

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\(^{16}\) Example 1 illustrates a case where the number of public schools equals the number of choices each parent can make, which is typically pre-determined by the school district. It is worth noting that the analysis extends to cases where the number of public schools exceeds the ‘allowed’ number of choices.

\(^{17}\) How the two alternative mechanisms work when applied to Example 1 is explained in footnotes 12 and 13.

\(^{18}\) Since the GS-DA mechanism guarantees stable assignments with respect to post-lottery priority rankings, there can not exist a school-student pair such as $(i_3, s_3)$ where $i_3$ strictly prefers $s_3$ to her current assignment and $i_3$ has higher priority than at least one student assigned to $s_3 (i_2)$ with respect to post-lottery priority rankings.
reveals $s_2$ as her first choice, $i_2$ will be guaranteed a seat in $s_2$ under the Boston mechanism. Therefore, assuming that all three students are strategic and that $i_1$ and $i_3$ reveal truthfully, $i_2$ might find it beneficial to misreport her preferences and play this ‘safe’ strategy rather than risking to be assigned to her least favorite school by revealing truthfully.\textsuperscript{19} If this is the case, $i_1$ and $i_3$ revealing truthfully, and $i_2$ revealing $s_2$ as her first choice is a Nash equilibrium under the Boston mechanism, yielding the assignment set $(i_1, s_1), (i_2, s_2) \text{ and } (i_3, s_3)$ for both outcomes of the lottery and violating the equal treatment requirement.\textsuperscript{20} As illustrated in this example, the undesired incentives created by the Boston mechanism for students to behave strategically lead to violation of the equal treatment criterion, even though this mechanism treats equivalent students fairly if all members of $I_m(T_e, r, p)$ reveal $s_m$ and all other schools they prefer to $s_m$ in their true preference rankings in equilibrium.\textsuperscript{21}

4. Equal Treatment and Alternative Assignment Procedures

Recent literature has indicated several important aspects along which the Boston mechanism is dominated by the proposed alternatives. Following such findings, Boston Public Schools (BPS) abandoned the Boston mechanism in favor of the GS-DA mechanism in 2005. Among the desirable features of the GS-DA mechanism, strategy-proofness played a key role in the decision of BPS as suggested by the following memorandum of Superintendent Thomas W. Payzant on May 25, 2005:\textsuperscript{22}

\begin{quote}
\textsuperscript{19} $i_2$ will prefer to rank $s_2$ as her first choice if the utility she obtains from being assigned to $s_2$ exceeds the average of her utilities from being assigned to $s_1$ and $s_3$.
\textsuperscript{20} Given that $i_2$ reveals $s_2$ as her first choice and $i_3$ reveals truthfully, $i_1$ revealing truthfully is a weakly dominant strategy, since she will be assigned to her most preferred school by doing so. Likewise, given that $i_2$ reveals $s_2$ as her first choice, $i_1$ has no chance of being assigned to her most preferred school. Therefore, given that $i_2$ reveals $s_2$ as her first choice, revealing truthfully and being assigned to her second choice is a weakly dominant strategy for $i_3$.
\textsuperscript{21} Formal proof of the latter statement is available upon request.
\textsuperscript{22} The memorandum, in its entirety, is available upon request.
\end{quote}
“…The most compelling argument for moving to a new algorithm is to enable families to list their true choices of schools without jeopardizing their chances of being assigned to any school by doing so. We know that the current algorithm (Boston mechanism), because it prioritizes families’ first choices, requires families to be strategic in their selection of school preferences…”

The findings presented in the previous section, on the other hand, suggest that both of these alternatives, just like the Boston mechanism, might create arbitrary distinctions between equivalent students, possibly inducing legal challenges. This finding raises the question about the existence of a solution that would enable school districts to benefit from the desirable properties of the alternatives without suffering the possible legal ramifications.

The use of broad priority categories along with random lottery results to rank students is the main reason behind this artificial classification between identical students. Therefore, there are two paths school districts can pursue in order to benefit from the desirable features of the alternative mechanisms while avoiding such classifications.

In one extreme, school districts can choose criteria that strictly rank students at each school and thus eliminate the random lotteries. Then, trivially, equal treatment is satisfied since no two students can be ranked the same by any school. A common example of this practice is the use of proximity-based measures such as ‘distance from the applicant’s primary address to the school’, which reduces the likelihood of two students falling into the same priority category and thus minimizes the need to implement random lotteries, as evidenced in Seattle Public Schools (WA) and Pinellas County

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23 Abdulkadiroglu and Sonmez (2003) indicate that if school districts use priority categories that lead to strict rankings of students, the strategy-proofness, stability and/or efficiency properties of these mechanisms will still prevail.
If ranking students on a measured ability scale is socially acceptable, with the same ranking at each school, another example of this approach arises.

At the other extreme, school districts can avoid the aforementioned classification by completely eliminating priority categories and conducting a single random lottery, which breaks the ties between all students in the same way for each school, to determine the public school assignments.

**Proposition 2:** Given that each school has the same priority ranking of students as determined by a single random lottery, the two alternative mechanisms satisfy the equal treatment condition.

In the case where each school has the same priority ranking, the two alternative mechanisms produce the same assignments as the *random serial dictatorship (RSD) mechanism*, which is strategy-proof and works as follows: order all students with a random lottery and assign the first student to her first choice, the next student to her top choice among the remaining slots, and so on. However, notice that this is a symmetric problem for any \( i \in I_m(T_e, r, p) \), since, *ex ante*, all students have the same priority at each school and all possible priority rankings of students are equally-likely. Thus, the probability of assignment to school \( s_m \) for \( i \), depends on factors none of which is student-specific. Therefore, each member of \( I_m(T_e, r, p) \) has equal probability of assignment to school \( s_m \) under the RSD mechanism.

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24 Pinellas County School Board uses the ‘shortest driving distance from the applicant’s primary address to the public school computed to the nearest hundredth of a mile’ whereas Seattle Public Schools employ the ‘straight-line distance from the primary address to the public school’ as a criterion to determine public school assignments. In both cases, the student living closer to the public school is given higher priority.  
25 A detailed explanation is provided in the Appendix. We also know that the RSD mechanism is stable and Pareto efficient (Abdulkadiroglu and Sonmez, 2003). Therefore, employing a single random lottery along with the alternative mechanisms to determine the public school assignments preserves the appealing features of the alternatives.
While not ‘problem free’, these assignment procedures suggest that school districts might benefit from the appealing features of the alternative assignment mechanisms without violating equal treatment. The equilibrium implications differ markedly across the alternatives however. Using proximity as a criterion to determine public school assignments counteracts the main objective of open enrollment programs by implicitly reducing the number of ‘feasible’ school choices available to parents. Consider the extreme case where parents share a common perceived quality hierarchy of schools. Then, with the proximity criterion, housing prices would ultimately conform to the hierarchy, households would sort by income and preference for school quality, and a neighborhood schooling system would effectively emerge. If schools instead rank students by a standardized ability measure, again assuming a common ranking of school qualities, then ability stratification across the hierarchy can be predicted. In the alternative with school rankings based solely on a lottery, perhaps the purest form of school choice, one can predict a representative cross section of students across a given school quality hierarchy. Hence, the choice of the assignment procedure has profound implications for access to schools along the quality hierarchy so that the preferred approach requires an expression of social preferences. The results, on the other hand, suggest that all three assignment mechanisms might induce legal challenges in a hybrid assignment process where broad priority categories along with random lottery results are used.

5. Concluding Remarks

One of the most commonly exercised forms of school choice is the open enrollment program, which allows parents to send their children to public schools outside
of the neighborhoods they reside. When capacity constraints result in a scarcity of seats at public schools, such programs necessitate the implementation of centralized public school assignment mechanisms, among which the Boston mechanism is commonly implemented by school districts. Despite its common use, previous literature has shown that the Boston mechanism is dominated by clever alternatives along almost all of the desirable features of a ‘well-behaving’ assignment mechanism. Consequently, Boston Public Schools abandoned the Boston mechanism in favor of one the alternatives in 2006.

This paper introduces a new criterion, which I refer to as the equal treatment requirement, to evaluate public school assignment mechanisms based on a direct implication of the Equal Protection Clause of the 14th Amendment. Evaluating the Boston mechanism along with the two alternatives proposed in the literature as replacements, findings indicate that none of these mechanisms satisfy the equal treatment criterion under the assignment procedures commonly employed in major school districts. I propose two alternative procedures that would enable school districts to benefit from the desirable properties of the two alternatives while avoiding the probable undesired legal ramifications.

The first section of the 14th Amendment, which is commonly known as the Equal Protection Clause, reads in part as follows:

“…No State shall …deny to any person within its jurisdiction the equal protection of the laws.”

Over the recent decades, the Supreme Court has developed a three-tiered-scrutiny approach to analysis under the Equal Protection Clause. Even though education is typically not considered as a fundamental right at the federal-level as evidenced in ‘San
Antonio School District v. Rodriguez’, there have been numerous state-level cases over the last four decades where the Court ruled that education is a fundamental right and classifications burdening one’s right of education falls under the strict scrutiny category, which requires the government (i.e. the school district in this case) to show that the challenged classification serves a compelling state interest and that the classification is necessary to serve that interest. 26 However, from the discussion in the preceding sections, we already know that the artificial classification between identical students is not necessary to achieve the desirable features of the aforementioned assignment mechanisms.

Previous literature has convincingly indicated that, given the established criteria to evaluate public school assignment mechanisms in the economics literature, it is socially beneficial to abandon the Boston mechanism in favor of one of the proposed alternatives in major school districts. An important policy implication of the findings presented in this paper is that these alternatives, just like the Boston mechanism, might initiate serious legal consequences.

Finally, it is worth noting an important practical limitation of the equal treatment criterion when evaluating assignment mechanisms under which truthful revelation of preferences is not a dominant strategy. Under assignment mechanisms that induce gaming (e.g. the Boston mechanism), the equal treatment criterion fails to sustain its legal

significance entirely, since under mechanisms that induce gaming, it is impossible to identify which students are actually ‘identical’ and hence improbable for parents to seek legal action based on their ‘claimed’ true preferences, which may be misreported in equilibrium.
Appendix. The two alternatives when each school has the same priority ranking

Suppose that there are \( n \) students \((i_1, i_2, \ldots, i_n)\) and \( k \) public schools \((s_1, s_2, \ldots, s_k)\) each of which has at least one seat available. Assume further that the students are ranked the following way at each school:

\[ i_1 - i_2 - i_3 - \ldots - i_n \]

1. **TTC mechanism**

   In the first step of the algorithm, all schools point to student \( i_1 \), and student \( i_1 \) points to her first choice. Therefore, the only cycle is between student \( i_1 \) and her first choice; \( i_1 \) is assigned to her first choice. In the second step, all remaining schools point to student \( i_2 \) and \( i_2 \) points to her top choice among the remaining schools. The only cycle is between \( i_2 \) and her top choice among the remaining schools; \( i_2 \) is assigned to her top choice among the remaining schools and so on. Notice that this is exactly the same as the random serial dictatorship mechanism.

2. **GS-DA mechanism**

   In the first step of the algorithm, each student proposes to her first choice. Let’s examine the assignments of each student with respect to their priority rankings:

   1. Since each school has at least one seat available, we know that student \( i_1 \) will be in the queue of her first choice at the end of the first step. Furthermore, since she has the highest priority ranking at each school, \( i_1 \) will not be rejected from her first choice at any step of the algorithm, and thus will be assigned to her first choice when the algorithm terminates.

   2. There are two cases to consider for the assignment of student \( i_2 \). First, if both \( i_1 \) and \( i_2 \) rank the same school, which has only one seat available, as their first
choices, $i_2$ will be rejected from her first choice at the end of the first step of the algorithm. If this is the case, in the second step, she will be placed in the queue of her second choice. Otherwise, she will be placed in the queue of her first choice at the end of the first step. In both cases, she will not be rejected in the following steps of the algorithm, and thus will be assigned to either her first choice or second choice when the algorithm terminates. Therefore, under the GS-DA mechanism, $i_2$ will be assigned to her top choice among the remaining schools after $i_1$ is assigned to her first choice.

Using the same analogy, one can show that the students ($i_3$, $i_4$, …, $i_n$) will be assigned to their top choice among the remaining schools. Therefore, the GS-DA mechanism produces the same assignments as the random serial dictatorship mechanism.

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References


Table 1
Public School Assignments and Assignment Probabilities in Example 1

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<thead>
<tr>
<th>Lottery Winner</th>
<th>Assignment Probability to $s_1$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$i_1$</td>
</tr>
<tr>
<td>TTC</td>
<td>$(i_1, s_3), (i_2, s_1), (i_3, s_2)$</td>
</tr>
<tr>
<td>GS-DA</td>
<td>$(i_1, s_3), (i_2, s_2), (i_3, s_1)$</td>
</tr>
</tbody>
</table>