Introduction to Neutron Transport

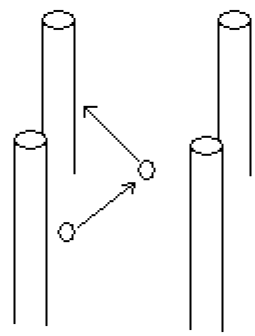
Determination of the neutron distribution in the reactor system, leading to analysis of the fission power in the reactor.

The Neutron Transport equation describes the precise behavior of ${}^{1}_{0}n$ in system (Boltzmann transport Eqn (BTE)). In some reactor systems, the diffusion equation offers an approximation to the transport equations.

$\rightarrow$"Diffusion" from high density to low density
$\rightarrow$Limited validity.

mfp for materials $\left(1/\Sigma_t \right)$ are 0(cm)

also, PWR fuel pins <0(cm)



Neutron Transport equation fully describes neutron population elegantly.

But it is difficult to solve by hand!

$\therefore$ we define a number of simplifying assumptions to make it treatable.

Transport $\Leftrightarrow$ common sense...

Neutron diffusion has been used for years to solve for neutron population; but it is inaccurate in places where the flux can change radically,

e.g.

i) where two different materials meet
ii) strong absorbers (control rods)
iii) boundary of system

Many times diffusion cross sections are "tuned" to give "transport-corrected" fluxes, rxn rates.

Why? Diffusion equation is easier (cheaper) to solve.

Neutron Density

$$\frac{\#}{cm^3} \rightarrow N(\vec{r},t)\overset{cm^3}{\overset{\downarrow}{d^3r}} \equiv \text{expected \# neutrons in differential volume } d^3r \text{ about } \vec{r} \text{ at time } t.$$

Distribution function

Consider the case where all neutrons in the system have speed v…

Recall the interaction frequency

$$(v\Sigma) \rightarrow \left(\frac{cm}{s}\frac{1}{cm}\right) \rightarrow \frac{1}{s}$$

Consider reaction rate density $\underline{F(\vec{r},t)}$ # Rxns in d^3r about $\vec{r}$ at time t

$$F(\vec{r},t)d^3r = v\Sigma N(\vec{r},t)\,d^3r = \phi(\vec{r},t)\Sigma d^3r \qquad \text{(D\&H p.105, Eq 4-3)}$$

$$vN(\vec{r},t) \Rightarrow \phi(\vec{r},t)$$

$$\underbrace{\frac{cm}{s}\;\frac{\#}{cm^3}}_{\frac{\#}{cm^2 s}}$$

Ref Fig 4-1 p.106

$$\phi(\vec{r},t)\Sigma \Big\} \frac{\#\,\text{Rxns}}{cm^3 s}$$

Consider a neutron traveling with speed v
Recall that the direction is $\hat{\Omega}$

$$\frac{1}{2}m_n v^2 = E \qquad \text{so} \qquad v = \left(\frac{2E}{m}\right)^{1/2}$$

Angular Neutron Density

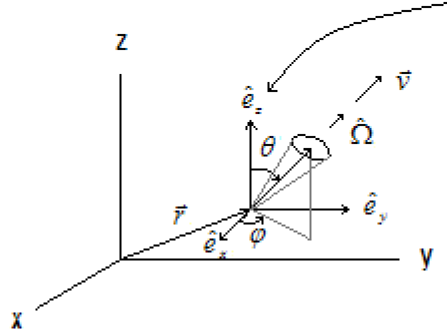
$n(\vec{r}, E, \hat{\Omega}, t)\, d^3r\, dE\, d\hat{\Omega} \;\equiv$ expected # of ${}^1_0 n$'s in d^3r about $\vec{r}$ with energy dE about E with direction $d\hat{\Omega}$ about $\hat{\Omega}$ at time t.

recognize that this is a distribution function

Ref Fig 4-2

$\hat{\Omega} = \{\theta, \varphi\}$

"script φ" vs ϕ

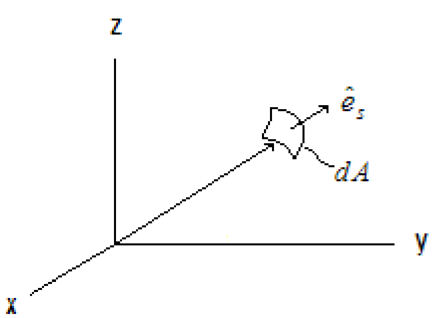


D&H $\hat{e}_x, \hat{e}_y, \hat{e}_z$ to represent unit vectors

"Vector" Angular current density:
$$\left[\begin{aligned} \vec{j}(\vec{r}, E, \hat{\Omega}, t) &= v\hat{\Omega} n(\vec{r}, E, \hat{\Omega}, t) \\ &= \hat{\Omega}\psi(\vec{r}, E, \hat{\Omega}, t) \end{aligned}\right]$$

Now, Considering these definitions, Eq (4-11) D&H States:

$$|\vec{j}| = |\hat{\Omega}|\psi = 1 \cdot \psi = \psi$$



Magnitude of are of $d\vec{A}$

Unit vector in dir of dA
Surface normal vector

Consider $d\vec{A} = dA\,\hat{e}_s$

$\left(\left|\vec{j}\left(\vec{r},E,\hat{\Omega},t\right)\cdot d\vec{A}\right|dE\,d\hat{\Omega}\right)$ = expected # of $\frac{1}{0}n$ passing an area dA per unit time with energy dE about E direction $d\hat{\Omega}$ about $\hat{\Omega}$, at time t

Angular Reaction Rate

$$f\left(\vec{r},E,\hat{\Omega},t\right) = v\Sigma(\vec{r},E)n\left(\vec{r},E,\hat{\Omega},t\right) = \Sigma(\vec{r},E)\psi\left(\vec{r},E,\hat{\Omega},t\right)$$

Case of Isotropic Flux

Isotropic $\equiv$ uniformly distributed in all directions

Recall

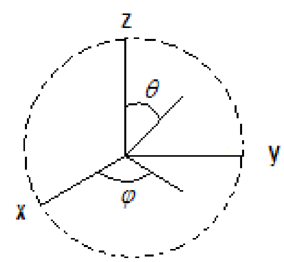
$$\int_{\Omega} d\hat{\Omega} = \int_0^{2\pi}\int_0^{\pi}\sin\theta\,d\theta d\varphi = 4\pi$$

$$= \int_0^{2\pi}\int_{-1}^{1}d\mu\,d\varphi = 4\pi$$

$\mu = \cos\theta$

$d\mu = -\sin\theta\,d\theta$

$\therefore$ changes limits to $[-1,+1]$



So $\quad \int_{\Omega} d\hat{\Omega} = \int_0^{2\pi}\int_{-1}^{1}d\mu\,d\varphi = 4\pi$

If the Angular Neutron Density is isotropic, then

$$\left[n\left(\vec{r},E,\hat{\Omega},t\right) = \frac{1}{4\pi}N(\vec{r},E,t)\right] \qquad \text{D\&H (Eq 4-16)}$$

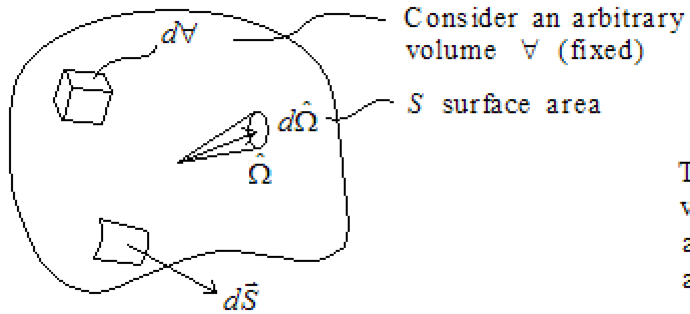
accounts for all directions

Usually, $n\left(\vec{r},E,\hat{\Omega},t\right)$ is <u>not</u> the same in all directions, especially near interfaces, localized absorbers, etc...

$$\left|\phi(\vec{r},E,t) = \int_{4\pi} d\hat{\Omega}\,\psi\left(\vec{r},E,\hat{\Omega},t\right)\right| \qquad \text{D\&H (Eq 4-17)}$$

$$\left[\phi(\vec{r},t) = \int_0^{\infty}\int_{4\pi} d\hat{\Omega}dE\,\psi\left(\vec{r},E,\hat{\Omega},t\right)\right] \qquad \text{D\&H (Eq 4-18)}$$

The Formalism is now established for defining the Neutron Transport Equation



The expected # of ${}_0^1n$ in volume $\forall$ with energies dE about E and directions $d\hat{\Omega}$ and about $\hat{\Omega}$ in the volume

Recall angular neutron density $\left[\int_{\forall} n(\vec{r}, E, \hat{\Omega}, t) d^3r\right] dE\, d\hat{\Omega}$ (8.1)

The time differential applied to this gives us a time rate of change...

$$\underbrace{\frac{\partial}{\partial t}\left[\int_{\forall} n(\vec{r}, E, \hat{\Omega}, t) d^3r\right] dE\, d\hat{\Omega}} = [\overset{G}{gain} - \overset{L}{loss}]_{\forall}$$

evaluated over the volume

Because $\forall$ is fixed,

$$\left[\int_{\forall} \frac{\partial n}{\partial t} d^3r\, dE\, d\hat{\Omega}\right]$$

<u>Now consider G (gain) in</u> ${}_0^1n$ <u>population due to</u>:

D&H
(1) G1) Neutron sources in $\forall$ (<u>fission</u> & <u>independent</u>)
(2) G2) Neutrons crossing S <u>into</u> $\forall$ (leaking <u>in</u>)
(3) G3) Collisions in $\underbrace{E',\Omega'}_{\text{some "other" energy \& angle}} \xrightarrow[dE^1 d\Omega^1]{} E,\hat{\Omega} \quad dEd\hat{\Omega}$ energy & angle of "interest"

<u>Consider L (loss) of</u> ${}_0^1n$ <u>population due to</u>:

D&H
(4) L1) Neutrons crossing S out of $\forall$ (leaking <u>out</u>)
(5) L2) Any collision in ($E,\hat{\Omega}$) removing a neutron. (Absorption or scatter) out of energy, angle of interest

Independent Sources

$$q(\vec{r}, E, \hat{\Omega}, t) d^3r\, dE\, d\hat{\Omega} \equiv \text{rate of appearance of source neutrons in } d^3r \text{ about } \vec{r} \text{ in } dE \text{ about } E \text{ in } d\hat{\Omega} \text{ about } \hat{\Omega}$$

q: $\frac{\#}{cm^3 s}$ $\qquad$ d^3r: cm^3

$$(1) \quad = \left[\int_{\forall} q(\vec{r}, E, \hat{\Omega}, t) d^3r\, dE\, d\hat{\Omega} \right] \qquad \text{over volume } \forall$$

Collisions

$$\int_{\forall} \left[n(\vec{r}, E, \hat{\Omega}, t) v \Sigma_t(\vec{r}, E) \right] d^3r\, dE\, d\hat{\Omega} \tag{8.2}$$

$$(5) \quad = \int_{\forall} \left[\psi(\vec{r}, E, \hat{\Omega}, t) \Sigma_t(\vec{r}, E) \right] d^3r\, dE\, d\hat{\Omega}$$

Scatter into dE about E $d\hat{\Omega}$ about $\hat{\Omega}$

"In scattering" into dE about E, $d\hat{\Omega}$ about $\hat{\Omega}$

$$\left[\int_{\forall} \left[\int_{4\pi} \left[\int_0^{\infty} \psi(\vec{r}, E', \hat{\Omega}', t) \Sigma_s(E' \to E, \hat{\Omega}' \to \hat{\Omega}) dE' \right] d\Omega' \right] d^3r \right] dE\, d\hat{\Omega}$$

double differential
scattering cross section

Leakage into or out of volume $\forall$.

Consider a differential surface $d\vec{S}$

$$\vec{j}(\vec{r}, E, \hat{\Omega}, t) \cdot d\vec{S} = \psi(\vec{r}, E, \hat{\Omega}, t) \hat{\Omega} \cdot d\vec{S} \tag{8.3}$$

Net leaking is (4) – (2) or...

Consider just surface & volume for a moment...

$$\left\{ \int_{\delta} \psi(\vec{r}, E, \Omega, t) \hat{\Omega} \cdot d\vec{S} = \int_{\forall} \nabla \cdot \hat{\Omega}\, \psi(\vec{r}, E, \hat{\Omega}, t) d\forall \right\}$$

Aside, Recall Gauss' Divergence Theorem

$$\int_{\delta} \vec{A} \cdot d\vec{S} = \int_{\forall} \nabla \cdot d\vec{A}\, d\forall \qquad d\forall = d^3 r$$

Surface integral   volume integral

So that Net Leakage is...

$$\left[\int_{4\pi} \int_0^{\infty} \int_{\delta} \psi(\vec{r}, E, \hat{\Omega}, t) \hat{\Omega} \cdot d\vec{S}\, dE\, d\hat{\Omega} \right] = \left[\int_{4\pi} \int_0^{\infty} \int_{\forall} \nabla \cdot \hat{\Omega} \psi(\vec{r}, E, \hat{\Omega}, t) d\forall\, dE\, d\hat{\Omega} \right] \tag{8.4}$$

Combining all terms:

Time rate of change = (+ sources)   (+in-scatter)

$$\int_{4\pi} \int_0^{\infty} \int_{\forall} \left(\frac{\partial n}{\partial t} = q(\vec{r}, E, \hat{\Omega}, t) + \int_0^{\infty} dE' \int_{4\pi} d\hat{\Omega}' \psi(\vec{r}, E', \hat{\Omega}', t) \Sigma_s (E' \to E, \hat{\Omega}' \to \hat{\Omega}) \right.$$

(-Net leakage)   (-Collisions)

$$\left. - \hat{\Omega} \cdot \nabla \psi(\vec{r}, E, \hat{\Omega}, t) - \Sigma_t(\vec{r}, E) \psi(\vec{r}, E, \hat{\Omega}, t) \right) d^3 r\, dE\, d\hat{\Omega}$$

Since our chosen volume was completely arbitrary, the expression inside the integral (the "integrand") must hold true at each point in $\forall$.

Also, recall

$$n(\vec{r}, E, \hat{\Omega}, t) v = \psi(\vec{r}, E, \hat{\Omega}, t) \tag{8.5a}$$

$$n(\vec{r}, E, \hat{\Omega}, t) = \frac{1}{v} \psi(\vec{r}, E, \hat{\Omega}, t) \tag{8.5b}$$

So, we have

$$\left[\frac{1}{v} \frac{\partial \psi}{\partial t} + \hat{\Omega} \cdot \nabla \psi(\vec{r}, E, \hat{\Omega}, t) + \Sigma_t(\vec{r}, E) \psi(\vec{r}, E, \hat{\Omega}, t) \right.$$
$$\left. = \int_0^{\infty} dE' \int_{4\pi} d\hat{\Omega}' \psi(\vec{r}, E', \hat{\Omega}', t) \Sigma_s (E' \to E, \hat{\Omega}' \to \hat{\Omega}) + q(\vec{r}, E, \hat{\Omega}, t) \right] \tag{8.6}$$

Then, the Integro-differential form of the neutron transport equation is

$$\left[\frac{1}{v}\frac{\partial \psi}{\partial t} + \hat{\Omega}\cdot\nabla\,\psi\left(\vec{r},E,\hat{\Omega},t\right) + \Sigma_t\left(\vec{r},E\right)\psi\left(\vec{r},E,\hat{\Omega},t\right)\right.$$

$$\left. = \int_0^\infty dE' \int_{4\pi} d\hat{\Omega}'\,\psi\left(\vec{r},E',\hat{\Omega}',t\right)\Sigma_s\left(E' \to E, \hat{\Omega}' \to \hat{\Omega}\right) + q\left(\vec{r},E,\hat{\Omega},t\right)\right]$$

Recall $$\phi\left(\vec{r},E,t\right) = \int_{4\pi} d\hat{\Omega}\,\psi\left(\vec{r},E,\hat{\Omega},t\right) \qquad (8.7)$$

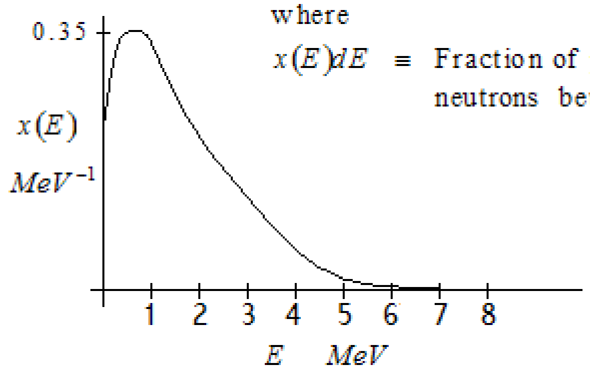
But wait...

Let us expand the "source" term $q\left(\vec{r},E,\hat{\Omega},t\right)$

For independent sources and fission:

$$q\left(\vec{r},E,\hat{\Omega},t\right) = \left[q_{ind}\left(\vec{r},E,\hat{\Omega},t\right) + \frac{x(E)}{4\pi}\int_0^\infty dE' \int_{4\pi} d\hat{\Omega}\,\underbrace{v(E')\Sigma_f\left(\vec{r},E'\right)}_{v\Sigma_f\left(\vec{r},E'\right)}\psi\left(\vec{r},E',\hat{\Omega},t\right)\right]$$

The "Delayed" ${}_0^1n$ discussion is delayed



where

$x(E)dE \equiv$ Fraction of prompt fission neutrons between E and $E + dE$

$$\int_0^\infty x(E)dE = 1$$

(Normalization)

Solving the NTE

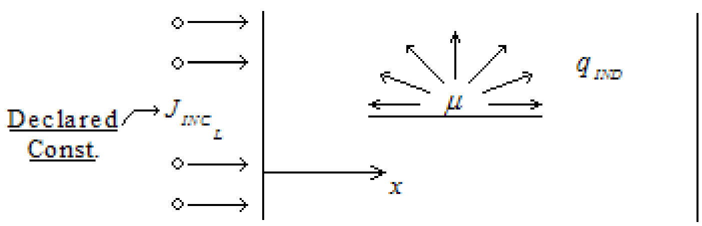
Assuming steady state write the one energy transport equation with no scattering, no fission q_{IND} is a uniform constant angular src.

$$\hat{\Omega} \cdot \nabla \psi + \sigma_t \psi = \frac{q_{IND}}{4\pi} \tag{9.1}$$

In SLAB (x) geometry $\hat{\Omega} = \langle \mu, \overset{0}{\cancel{\eta}}, \overset{0}{\cancel{\xi}} \rangle$

$$\mu \frac{\partial \psi}{\partial x} + \sigma_t \psi = \frac{q_{IND}}{4\pi} \Rightarrow \left(\frac{\partial \psi}{\partial x} + \frac{\sigma_t}{\mu} \psi = \frac{q_{IND}}{4\pi\mu} \right)$$

We can solve $\left[\frac{\partial \psi}{\partial x} + \frac{\sigma_t}{\mu} \psi = \frac{q_{IND}}{4\pi\mu} \right]$ by integrating factor.



Let $\psi(0) = \psi_L$ for B.C.

Given (A const) $\left[J_{INC_L} = \int_0^{2\pi} \int_0^1 \mu \psi_L \, d\mu \, d\varphi \right]$

Usually, we must consider scattering.
For "Real" applications...

Ref p.121 D & H

Ref p.121 D & H	S_N approximation "Discrete ordinates"	Consider a finite # of directions on the unit sphere and discretize angular variable, other variables...
	P_N approximations	Expand angular variable into spherical harmonics (orthogonal) moments function.

The integro-differential form of the transport equation is

$$\left\{\overset{(1)}{\frac{1}{v}\frac{\partial \psi}{\partial t}} = \overset{(2)}{q_{ind}\left(\vec{r}, E, \hat{\Omega}, t\right)} + \overset{(3)}{\frac{x(E)}{4\pi}\int_0^\infty dE' \int_{4\pi} d\hat{\Omega}' \nu\Sigma_f\left(\vec{r}, E'\right)\psi\left(\vec{r}, E', \hat{\Omega}', t\right)}\right.$$

$$+ \overset{(4)}{\int_0^\infty dE' \int_{4\pi} d\hat{\Omega}' \Sigma_s\left(\vec{r}, E' \rightarrow E, \hat{\Omega}' \rightarrow \hat{\Omega}\right)\psi\left(\vec{r}, E, \hat{\Omega}, t\right)}$$

$$\left. - \overset{(5)}{\Sigma_t\left(\vec{r}, E\right)\psi\left(\vec{r}, E, \hat{\Omega}, t\right)} - \overset{(6)}{\hat{\Omega}\cdot\nabla\psi\left(\vec{r}, E, \hat{\Omega}, t\right)}\right\}$$

Again...

To get scalar flux $\phi(\vec{r}, E, t)$ we must integrate angular flux over 4π steradians $(4\pi\ st)$

$$\text{So} \qquad \phi(\vec{r}, E, t) = \int_{4\pi} \psi\left(\vec{r}, E, \hat{\Omega}, t\right) d\hat{\Omega} \qquad (9.2)$$

If we consider a 1-speed diffusion theory model...

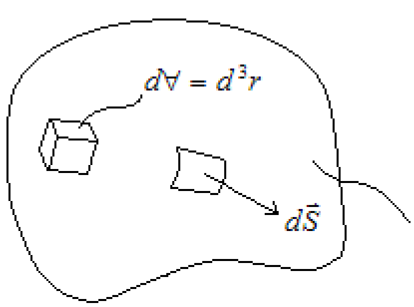
Really, again we are using equations to define where neutrons in a probability dist go...

$\therefore$ any equation describes average behavior...

$N(\vec{r}, t)$ is neutron density in the reactor, related to flux by the velocity term

$$\int_{\forall} d^3r\ N(\vec{r}, t) = \int_{\forall} d^3r\ \phi(\vec{r}, t) \qquad (11.1)$$

Again we assemble the terms relating to neutron balance in a control volume



$$\begin{pmatrix} Time\ Rate \\ of \\ change \end{pmatrix} = \begin{pmatrix} prod \\ in \\ \forall \end{pmatrix} - \begin{pmatrix} Abs \\ in \\ \forall \end{pmatrix} - \begin{pmatrix} Leak \\ from \\ \forall \end{pmatrix}$$

Vol $\forall$ Surf Area S

$$\int_{\forall} dr^3 \frac{1}{v}\frac{\partial \phi}{\partial t} = \left[\int_{\forall} dr^3\, Q(\vec{r},t) - \int_{\forall} dr^3\, \Sigma_a(\vec{r})\phi(r,t) - \int_{S} d\vec{S}\cdot \vec{J}(\vec{r},t) \right] \qquad (11.2)$$

Treating the leakage term using Gauss' Divergence Theorem...

$$\int_{S} d\vec{S}\cdot\vec{J}(\vec{r},t) \Rightarrow \int_{\forall} d^3 r \nabla\cdot\vec{J}(r,t) \qquad (11.3)$$

Writing the balance equation as

$$\int_{\forall} d^3 r \underbrace{\left(\frac{1}{v}\frac{\partial \phi}{\partial t} - Q(\vec{r},t) + \Sigma_a(\vec{r})\phi(\vec{r},t) + \nabla\cdot\vec{J}(\vec{r},t) \right)}_{\text{Integrand must hold at every point in } \forall} = 0 \qquad (11.4a)$$

$$\frac{1}{v}\frac{\partial \phi}{\partial t} - Q(\vec{r},t) + \Sigma_a(\vec{r})\phi(\vec{r},t) + \nabla\cdot\vec{J}(\vec{r},t) = 0 \qquad (11.4b)$$

1-speed balance equation is (omitting $\vec{r},t$ refs)

$$\left[\frac{1}{v}\frac{\partial \phi}{\partial t} = -\nabla\cdot J - \Sigma_a \phi + Q \right]$$

what are units ?

$$\frac{1}{\frac{cm}{s}}\frac{n}{cm^2 s^2} \qquad \left(\frac{1}{cm}\cdot\frac{n}{cm^2 s}\right)\left(\frac{\text{int}}{n}cm^{-1}\frac{n}{cm^2 s}\right) \qquad \frac{n}{cm^3 s}$$

Before moving, on we should address steady state radiation leaking and multiplying from a Pu mass in space...

Consider a $4\ kg$ ball of Pu in space

- ✓ It is sub-critical, $m = 4\ kg$
- ✓ It has a $(k_{eff} \cong 0.75 \quad P_{Leak} \cong 0.76)$ From transport theory calc
- ✓ This Pu generates $60{,}000 \dfrac{n}{s\ kg}$ due to S.F.

$$\text{So } (Q_{IND}\forall) = \left(60{,}000 \frac{n}{s\ kg}\right)(4\ kg) = \underbrace{240{,}000 \frac{n}{s}}$$

Note $Q_{IND} \equiv \dfrac{\#}{cm^3 s}$ isotropic indep. Source density

Intrinsic n source per unit time

If we consider the ball is stationary a long time...Then (without time dependent term)

$$-\nabla\cdot\vec{J} - \Sigma_a\phi + \nu\Sigma_f\phi + Q_{IND} = 0 \qquad (11.4c)$$

Note I have not assumed anything about the leakage term (e.g. diffusion app).

Divide above eqn by $\nu\Sigma_f \phi \neq 0$

ratio of ${}_0^1 n$ loss to production

$$\frac{\left(-\nabla\cdot\vec{J}-\Sigma_a\phi\right)}{\left(\nu\Sigma_f\phi\right)}+1+\frac{Q_{IND}}{\nu\Sigma_f\phi} = 0$$

$$-\frac{1}{k}\left\{-\frac{\left(\nabla\cdot\vec{J}+\Sigma_a\phi\right)}{\left(\nu\Sigma_f\phi\right)}\right.+1+\frac{Q_{IND}}{\nu\Sigma_f\phi} = 0$$

<u>So we have</u>

$$-\frac{1}{k}+1+\frac{Q_{IND}}{\nu\Sigma_f\phi} = 0$$

rewrite as k/k, rearrange

$$\frac{k-1}{k} = \frac{-Q_{IND}}{\nu\Sigma_f\phi}$$

reactivity $\rho \equiv \left(\frac{k-1}{k}\right)$ $\qquad$ $\rho \equiv \frac{-Q_{IND}}{\nu\Sigma_f\phi}$

$$\text{or} \qquad \nu\Sigma_f\phi = \frac{-Q_{IND}}{\rho}$$

$$\frac{\#\ Fiss\ \ {}_0^1 n's}{cm^3 s}$$

Multiply both sides by <u>volume</u>.

$$\nu\Sigma_f\phi\forall = \frac{-(Q_{IND}\forall)}{\rho}$$

total S.F. & IND source

fiss produced per sec in sphere

reactivity $\rho \equiv \left(\frac{k-1}{k}\right)$

This only is valid for subcritical assembly (note "-" sign)

If ${}_0^1 n$'s from Q_{IND} are pumped into a critical, supercritical assembly, there is no steady state solution...

So <u>returning to our problem</u>

$$\left(\nu\Sigma_f \phi \forall\right) = \left(\frac{Q_{IND}\forall}{-\rho}\right)$$

For our $4\ kg\ \ Pu$ ball, $k = 0.75$, $P_{Leak} = 0.75$

$$\rho = \frac{k-1}{k} = \frac{0.75-1}{0.75} \Rightarrow -0.333$$

$(1-P_{NL})$

$-\dfrac{1}{\rho} = 3.0$ This is the amount of multiplication induced in the assembly (4 kg sphere)

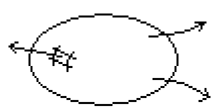
$$-\frac{1}{\rho} = 3.0 \quad \text{and} \quad (Q_{IND}\forall) = 240{,}000\ {}^{n}\!/_{s}$$

$$\left(\nu\Sigma_f \phi \forall\right) = \underbrace{(3.0)(240{,}000)}_{\frac{Q_{IND}\forall}{-\rho}}\ {}^{n}\!/_{s} = \underline{720{,}000}\ {}^{n}\!/_{s}$$

these ${}_0^1n$ <u>induced</u> from S.F. driven <u>multiplication</u>...

How many are available for detection?

Only those that leak out are ${}_0^1n$'s that can be detected...



$\left\{ \begin{array}{l} \text{intrinsic (S.F. etc)} \\ \quad\quad \& \\ \text{induced (mult)} \end{array} \right.$

But we <u>must account for all neutrons</u>!

Since we must account for the $\underbrace{\underline{\text{induced source}}}_{\substack{\text{Multiplication} \\ \text{of intrinsic source} \\ -\frac{Q_{IND}\forall}{\rho}}}$ and the $\underbrace{\underline{\text{intrinsic source}}}_{\substack{\text{S.F. etc} \\ (\alpha,n) \\ Q_{IND}\forall}}$

$$\text{(Subcrit) total src: } \left(\frac{-Q_{IND}\forall}{\rho} + Q_{IND}\forall\right) = \left(\frac{-Q_{IND}\forall}{\frac{k-1}{k}} + Q_{IND}\forall\frac{(k-1)}{(k-1)}\right)$$

$$\Rightarrow \left(\frac{-Q_{IND}k + Q_{IND}k - Q_{IND}}{k-1} \right) \forall$$

$$\Rightarrow \frac{-Q_{IND}\forall}{k-1} = \left[\frac{Q_{IND}\forall}{(1-k)} \right] \text{ total } {}^1_0 n\text{'s available}$$

$\therefore$ accounting for prob of leakage $\equiv P_L$

Then leakage multiplication $M_L = \left(\frac{Q_{IND}\forall}{(1-k)} \right) \cdot P_L = (M)P_L$

Where $M \equiv$ "*Multiplication*" $= \frac{Q_{IND}}{(1-k)} = Q_{IND}\left(1 + k + k^2 + k^3 + \ldots + k^N\right)$

Geometric series if convergent if $k < 1$

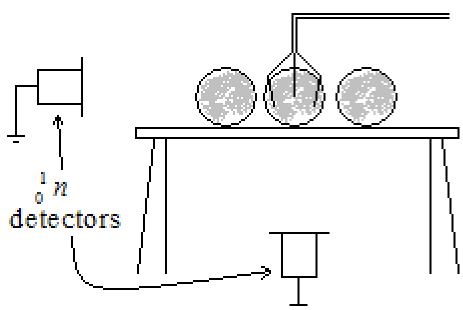
From previous examples $M_L = 729{,}600\ n/s$

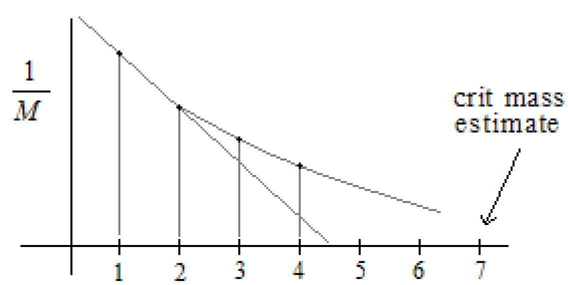
is the leakage multiplication.

<u>Aside</u>

Example: assemble a collection of subcritical masses

Stack sub-critical masses together…





This is the experimental way to monitor for a critical mass estimate.