MODELING THERMAL DYNAMICS IN MULTI-ZONE BUILDINGS

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***Abstract*— This document describes a MATLAB-based simulation environment being developed for simulating dynamics of temperatures inside a multi-zone building based on simplified lumped parameter models. The purpose of the model is to be able to test various feedback control laws and assess the resulting HVAC energy consumption. Interaction among zones are represented using a graph, and inter-zone conductive heat transfer is modeled using lumped parameter resistor-capacitor models developed by Gouda et. al.**

The model is a set of coupled ODEs, which is solved by numerical integration in MATLAB. Mass flow rates into and out of the zones, sensible loads due to occupants and appliance use, outside temperatures, solar radiation, and building construction are inputs to the model that must be specified. Material properties (e.g., thermal resistance and capacitance) that are needed for computing the parameters in the thermal dynamic equations are computed from the architectural design of the building, which is first computed using the CARRIER-HAP software. Once these parameters are entered into MATLAB, the temperature history within each room and HVAC energy consumption can be found for a specified time span.

To mimic real HVAC operation, the control sampling period is taken as 15 minutes, and integration is done for a 15 minute period for a given control action that is specified at the beginning of the sampling period. The simulation environment is designed to be modular, various occupancy and weather inputs can be specified in the accompanying m-files and various control laws can be implemented as desired.

I. INTRODUCTION

Recently, the internal thermal dynamics of a building have become a significant interest. In traditional HVAC work, units were designed, installed, and calibrated according to approximate rules of thumb and trial and error. Additionally, these HVAC systems are used in applications in which the units are ran for the majority of the work day according to a pre-programmed schedule. While this is sufficient, it is not optimal in occupant comfort or power consumption. In efforts to reduce energy consumption and improve occupant comfort, HVAC systems with adaptive control have become an area of focus.

One of the most important aspects of HVAC design is the cooling loads of a building. A cooling load, usually in units of watts (SI) or Btu/hr (English), refers to anything that produces a heat load that must be compensated with and cooled by the HVAC system. This load can be produced by heat conduction through walls and windows, solar radiation, occupants, appliances, or lighting.

To create a computational simulation, in which time steps on the order of minutes rather than hours is required, the internal thermal dynamics becomes a necessary aspect. Internal thermal dynamics of a building refers to how heat travels through (resistance) and is held (capacitance) by the surfaces (walls, ceiling, and floor) in a building. Furthermore, it includes the resistance and capacitance of the air encapsulated by the building.

The movement of air, primarily controlled by the HVAC system, is another important component. Enthalpy, the energy in a mass of air (kJ/kg), is critical in this aspect since room temperature air can contain more than twice the energy per unit mass if it is saturated as opposed to dry. Therefore, the moisture content of air entering and leaving a room is of extreme importance when determining the heat being removed by the HVAC system. Enthalpy also provides a means of calculating the energy being removed from air traveling through the chiller. Air entering each room is assumed to mix nearly instantly, since the fluid movement within each room is not within the scope of this project.

Most commercial and industrial buildings are divided into zones. Each zone usually contains one or more rooms, with similar rooms being grouped into the same zone. Variable Air Volume (VAV) boxes, which are a focus of this report, control the air flow into each zone and vents distribute the air from the VAV box into each room. VAV boxes can be adjusted using a control algorithm according to occupancy, temperature, CO_2 levels, and other parameters. This allows the room parameters to be regulated with higher accuracy, potentially using less energy, while improving occupant comfort.

II. BUILDING THERMAL MODEL

There are three major components to the temperature response model described here:

- Internal dynamics (heat transfer through internal surfaces as well as the capacitance of each surface)
- Cooling loads (heat produced by people, equipment, sunlight, and external environment)

- HVAC inputs (cold air supplied into zones and warm air removed through ventillation)

The first is constructed by the lumped resistance-capacitance model, the second is an external input specified a priori, and the third is generated based on the simplified characteristics of the HVAC system. All these components are interconnected, and result in a system of coupled Ordinary Differential Equations (ODEs) that describe the dynamic response of the zone-level temperatures.

The schematic of a four zone building is shown in Figure 1. It is assumed that interaction among the zones occurs only through conduction.

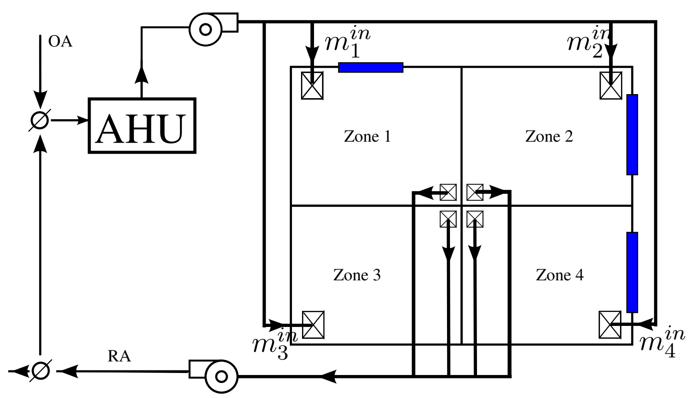

Fig. 1. A schematic of a 4-zone building HVAC system

As shown in Figure 1, conditioned air comes from air handling unit (AHU) through the ducts and goes into each zone. Humid and more warm air, which is called return air (RA), from each zone goes out through vents. Part of return air is mixed with outside air to maintain the CO_2 level and mixed air is sent to AHU to get the conditioned air. Mass flow rate of air going into i^{th} zone is represented by m_i^{in}, where i = 1,2,...N. We ignore the dynamics of ducts and AHUs. Assuming m^{in} can be measured, we want to construct a model that predicts the evolution of the temperatures as m^{in}, Q^p and Q^s change.

List of inputs and outputs for a building with N zones is given below

$$\begin{aligned} \text{Inputs:} \quad & m_1^{in}, ...m_N^{in}, W_1^{in}, ...W_N^{in}, T_1^{in}, ...T_N^{in}, Q_1^p, ...Q_N^p \\ & Q_1^s, ...Q_N^s, T_0, W_{OA} \\ \text{Outputs:} \quad & T_1, ...T_N, W_1, ...W_N \\ \text{States:} \quad & T_1, ...T_{N_T}, W_1, ...W_N \end{aligned} \tag{1}$$

where W_i^{in} is the humidity ratio of conditioned air entering in the i^{th} zone, T_i^{in} is the temperature of conditioned air entering into i^{th} zone, Q_i^p is the rate of heat generated by people in the i^{th} zone, Q_i^s is the solar radiation entering in the i^{th} zone, T_0 and W_{OA} are the temperature and humidity ratio of outside air respectively, W_i is the humidity ratio in the i^{th} zone, where i = 1, 2,...N, T_i is reindexed in such a way that first N components corresponds to the space air temperature for each zone, and remaining states corresponds to the wall internal node temperatures. N_T is the total number of temperature states which is calculated using 2nd order lumped parameter resistor-capacitor model [1] that is described in detail in next section. It is true that the temperature inside a zone is not uniform, but modeling an average zone temperature is usually enough for HVAC control purposes.

Note that theory discussed in this report is applicable to any general N zone building. However, we define a four zone building as an example for more clarifications which is shown in Figure 1. All four zones have an equal size of $25\,m^2$, but zones 1 and 3 are labs with a higher expected occupancy (4 people a majority of the day) and zones 2 and 4 are offices with a minimal expected occupancy (1 person a majority of the day). Zone 1 has a small window ($5\,m^2$) on the north facing wall, whereas zones 2 and 4 have a larger window ($7\,m^2$ each) on the east facing wall. Zone 3 does not have a window. Each wall is 5 meters wide by 3 meters tall. This provides a volumetric area of $75\,m^3$ for each zone. To simplify this model, a return air duct was placed in each zone making it possible to assume virtually no air exchange between zones. The HVAC components for the building were designed using the Carrier software Carrier-HAP [2]. The resulting system used in the four zone model is capable of a maximum flow rate of 0.25 kg/s per zone at $12.78 \circ C$. Table I displays the efficiencies used for the HVAC system.

TABLE I

EFFICIENCIES USED FOR THE HVAC SYSTEM USED IN THE 4 ZONE MODEL. TESH IS THE PRESSURE DROP THROUGH THE DUCT SYSTEM.

Component	Efficiency
Chiller	60
Fan Motor	87
Fan	70
Fan Belt	88
TESH	6 (inches of water)

III. Details

A. Building configuration parameters

This section defines the parameters required to specify the building configuration. Building configuration means how zones, walls and windows are connected in terms of nodes. For a building with N zones and considering each zone as a node, define

$$\begin{aligned} R^c &= I_{N_z \times N_z} + R_R \\ W^c &= I_{N_z \times N_z} + R_W \end{aligned} \tag{2}$$

where R^c, W^c are $N \times N$ matrices, I is an identity matrix and R_R is $N \times N$ adjacency matrix which defines how zones are connected through internal wall. Note thatR_W is $N \times N$ adjacency matrix which defines how zones are connected through internal windows.

Zone orientation also plays an important role in determining thermal loads. For example, external east facing wall will have different outside air temperature than the west facing wall. It is possible that the external wall of each zones may be exposed to different outside air temperature. Therefore, it is assumed that there are N_o outside nodes, which has different outside temperature profile. Define matrices R^O, W^O of dimension $N \times N_o$ as

$$\begin{aligned} &\text{if } i^{th} \text{ zone is connected to } j^{th} \text{ node via external wall} \\ &R^O_{ij} = 1, \text{where } i = 1...N, j = 1...N_o \\ &\text{else } R^O_{ij} = 0, \text{where } i = 1...N, j = 1...N_o \end{aligned} \tag{3}$$

Similarly, matrix W^O of dimension $N \times N_o$ can also be constructed as

$$\begin{aligned} &\text{if } i^{th} \text{ zone is connected to } j^{th} \text{ node via external window} \\ &W^O_{ij} = 1, \text{where } i = 1...N, j = 1...N_o \\ &\text{else } W^O_{ij} = 0, \text{where } i = 1...N, j = 1...N_o \end{aligned} \tag{4}$$

where subscript ij decribes the i^{th}, j^{th} element of corresponding matrix. Matrices in (2), (5) and (4) completely defines the configuration of a building. For a 4 zone building described in above section, above matrices can be written as

$$R^c = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}, \quad W^c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$R^O = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}^T, \quad W^O = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{5}$$

To specify the thermal resistance and thermal capacitance of internal/external wall and windows, define C, R_S, R_M, R_W_R as $N \times N$ matrices, R_O as $3N \times N_o$ matrix, C_O as $2N \times N_o$ matrix, R_W_O as $N \times N_o$ matrix. Note that R_M is a symmetric matrix here. Procedure to calculate the resistance and capacitance matrices are described in next subsection.

B. Lumped parameter resistor-capacitor model for inter-zone thermal conduction

Traditional heat transfer textbooks provide instruction on how to compute the heat transfer through a wall of a single known resistance and a capacitance for that wall. However, when modeling the time response of composite surfaces which are made up of different materials, this approximation degrades. This is because the layers of the wall can be at different temperatures, thereby affecting the heat transfer according to newton's law of cooling. This is illustrated in Figure 2.

The solution is to assign temperature nodes to the surfaces themselves. Intuition would lead one to believe that assigning more nodes would provide more accuracy. Work by Gouda [1] shows that the increase in accuracy for heat transfer through a wall plateaus at approximately 20 nodes. Furthermore, it was shown that nonlinear constrained optimization of a 2 node model can yield nearly identical results to the 20 node model. Therefore, accurate thermal modeling requires only a 2^{nd} order lumped capacitance model for each surface (wall, floor, or ceiling). This 2^{nd} order lumped capacitance model assigns only two nodes to each surface. These

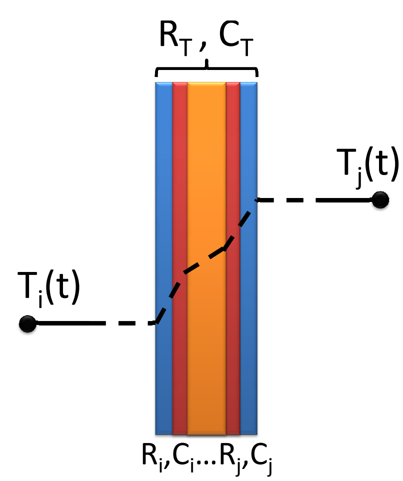

Fig. 2. Heat transfer between nodes. Thermally, composite walls behave differently than homogeneous walls. R_T is the total resistance and C_T is the total capacitance.

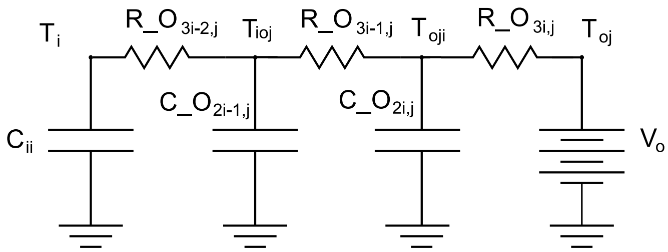

Fig. 3. 2^{nd} order lumped capacitance model for a single surface with one side exposed to the outside and the other surface exposed to an internal air space.

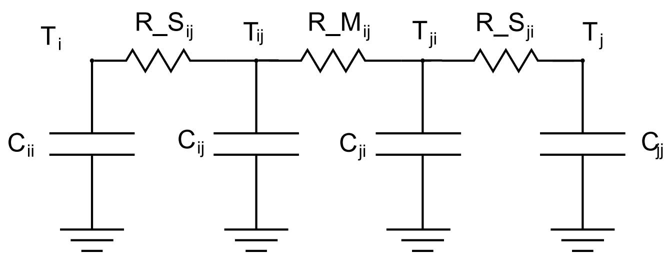

Fig. 4. 2^{nd} order lumped capacitance model for a single surface with each surface exposed to an internal volume of air.

internal nodes have a capacitance assigned to them and resistances interconnecting them to other nodes. The temperature of each zone is also assigned a node with a capacitance. Since the outside air has a relatively infinite heat capacitance, the outside air can be modeled as a voltage source that changes with time. This setup is demonstrated in Figure 3. If studying the heat transfer between two zones, each zone is assigned a capacitance. This scenario is presented in Figure 4.

Contrary to the notation used by Gouda, a different nomenclature was adopted to number each node. Given zone i and j, whose nodes are designated T_i and T_j, the node adjacent to zone i connecting to zone j, would be designated T_{ij}. Similarly, the node adjacent to zone j connecting to zone i would be designated T_{ji}. The nomenclature used for outside node is different. Outside temperature of node i is assigned as T_{oi}. Hence, the temperature adjacent to i^{th} zone connecting to j^{th} outside node is T_{ioj}. For now, it is assumed the celling and floor can be considered as two separate nodes. It means that celling above each zone has a same temperature profile, and floor has also got the same temperature profile below each zone. As we can see in Figure 4, there are four capacitors from node i to node j. Capacitor attached to node T_i is the zone air thermal capacitance, which is the i, i^{th} element of matrix C. Similarly, C_{ij} is the capacitor attached to node T_{ij}, C_{ji} is the capacitor attached to node T_{ji}. In the same way, resistance connected between T_i and T_{ij} is denoted by i, j^{th} element of matrix R_S, resistance connected between T_{ij} and T_{ji} is denoted by i, j_{th} element of a symmetric matrix R_M. Window thermal resistance between i^{th} and j^{th} zone can be represented by an element $R\_W\_R_{ij}$. Resistance of window connected between i^{th} zone and j^{th} outside node is given be $R_W_O_{ij}$Capacitance and resistances for outside node are defined in a slightly different way. Resistance between node T_i and T_{ioj} is defined by $3(i-1)+1, j^{th}$ element of matrix R_O, resistance between node T_{ioj} and T_{oji} is defined by $3(i-1)+2, j^{th}$ element of matrix R_O, resistance between node T_{oji} and T_{oj} is defined by $3(i-1)+3, j^{th}$ element of matrix R_O. In a same way, capacitance connected to node T_{ioj} is given by $2*(i-1)+1, j^{th}$ element of matrix C_O and capacitance connected to node T_{oji} is given by $2*(i-1)+2, j^{th}$ element of matrix C_O. The 2^{nd} order lumped capacitance model, displayed in Figure 3, provides a straightforward way of creating a thermal model for a building. The complexity arises when a more sophisticated model is necessary, such as going from a single zone with 7 nodes (2 for the entire outside wall, 2 for the ceiling, 2 for the floor, and 1 for the zone node) to a 4 zone model with 36 nodes. This large increase is due to 2 nodes being assigned to each independent surface within the building. However, MATLAB is able to convert all the input parameters, such as zone size and wall types, into the proper coefficients for the differential system and solve this differential system relatively quickly.

A diagram showing the lumped capacitance model for the 4 zone model is displayed in Figure 5. Windows are modeled as single resistors since they have relatively little capacitance. In addition, zone 3 does not have a window, therefore there is no resistor connecting it to the outside node. T_C and T_F represent the ceiling and floor temperatures, respectively. The ceiling node, for the four zone model, is the same as the outside air temperature.

Gouda obtained the 2^{nd} order lumped capacitance model by simplifying a complex 20^{th} order model and then determining the optimum values for resistance and capacitance. Gouda then made some generic approximations for all walls with similar construction. In this approach, the total resistance (R_T) and total capacitance (C_T) of each wall is calculated first.

Resistances and capacitances are calculated using a combination of Gouda [1] and Carrier software [2]. Carrier software allows for the total resistance and capacitance of the surface to be calculated based on the individual components of the surface, such as brick type, insulation, and air space. The individual resistances and

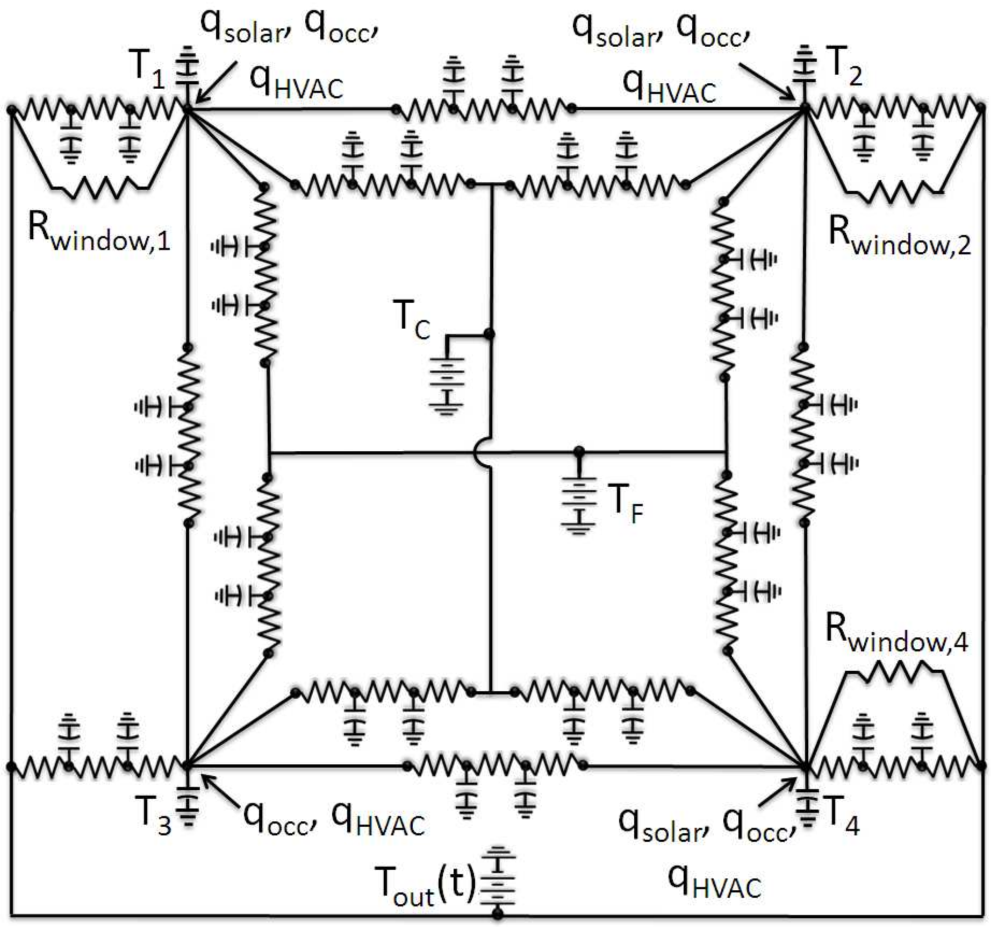

Fig. 5. Four zone lumped capacitance model

capacitances are then based on percentages determined by Gouda and described in detail in Section III. A system of differential equations is created using the coefficients created by these resistances and capacitances using the same technique as Gouda [3]. Equations 6 and 7 show how the total resistance and capacitance of a surface is calculated.

$$R_T = \left(\sum_{i=1}^{i=n} \frac{x_i}{k_i} \right) / A \tag{6}$$

$$C_T = A \sum_{i=1}^{i=n} (x_i \rho_i c_{p,i}) \tag{7}$$

Where i represents each layer of the wall, x is the layer thickness, k is the layer R-value, A is the area, ρ is the layer density, and $c_{p,i}$ is the layer's specific heat.

Using the approximate values provided by Gouda [1], the properties of each resistor and capacitor can be found. Multiplying the total resistance and capacitance, as shown below, with the coefficients in Table II provides the values for each resistor and capacitor in the 2^{nd} order model.

TABLE II

RESISTOR AND CAPACITOR COEFFICIENTS USED TO CALCULATE VALUES FOR THE RESISTORS AND CAPACITORS WITHIN THE LUMPED CAPACITANCE MODEL

Surface	f_1	f_2	f_3	g_1	g_2
External Wall	0.4	0.5	0.1	0.85	0.15
Interal Wall	0.2	0.6	0.2	0.50	0.50
External Floor	0.6	0.3	0.1	0.85	0.15
External Ceiling	0.3	0.4	0.3	0.40	0.60
Internal Floor (reversed for internal ceiling)	0.6	0.3	0.1	0.87	0.13

$$R_k = f_k R_T \tag{8}$$

$$C_l = g_l C_T \tag{9}$$

where $k = 1, 2, 3; 1 =$ outermost and $l = 1, 2; 1 =$ outermost.

Using the results of Equations 6, 7, 8, and 9 and generating the differential equations for the circuit in the same manner as Gouda [3] provides the following equations.

$$C_{ij}\dot{T}_{ij} = \frac{T_i}{R\_S_{ij}} - \frac{T_{ij}(R\_S_{ij} + R\_M_{ij})}{R\_M_{ij}R\_S_{ij}} + \frac{T_{ji}}{R\_M_{ij}} \tag{10}$$

$$C_{ji}\dot{T}_{ji} = \frac{T_j}{R\_S_{ji}} - \frac{T_{ji}(R\_S_{ji} + R\_M_{ji})}{R\_M_{ji}R\_S_{ji}} + \frac{T_{ij}}{R\_M_{ji}} \tag{11}$$

For external surfaces, differential equations for $\dot{T}_{ioj}$ and $\dot{T}_{oji}$ can be written as

$$\begin{aligned} C_{2i-1,j}\dot{T}_{ioj} = &-\frac{T_{ioj}(R\_O_{3i-2,j} + R\_O_{3i-1,j})}{R\_O_{3i-1,j}R\_O_{3i-2,j}} \\ &+\frac{T_i}{\mathbb{R}\_O_{3i-2,j}} + \frac{T_{oji}}{R\_O_{3i-1,j}} \end{aligned} \tag{12}$$

$$\begin{aligned} C\_O_{2i,j}\dot{T}_{oji} = &-\frac{T_{oji}(R\_O_{3i,j} + R\_O_{3i-1,j})}{R\_O_{3i-1,j}R\_O_{3i,j}} \\ &+\frac{T_o j}{R\_O_{3i,j}} + \frac{T_{ioj}}{R\_O_{3i-1,j}} \end{aligned} \tag{13}$$

It is assumed that air density remains constant and uniform in the zone. With Equations (10) - (12) a system of differential equations can be generated for a single surface separating two volumes of air. Using the same technique for all of the surfaces, a system of differential equations defining the internal thermal dynamics can be created for a multi-zone building.

C. Temperature interconnections among zones

The differential system for the 4 zone model has a total of 44 rows. The first 4 rows represent the temperatures within the zones. The next 32 rows represent the temperature of the nodes within the surfaces. The following 4 rows represent the moisture content within each zone. The final set of rows is attributed to pollutants and contaminants. In this preliminary model, only CO_2 was investigated. Figure 6 illustrates the breakdown of the differential system.

In general, there will be:

$$\begin{aligned} N_T = N(\text{zone nodes}) + N_{int} + 2N_oN_z(\text{outside nodes}) \\ + N(\text{humidity}) + n_\psi N(\text{pollutants/contaminants}) \end{aligned} \tag{14}$$

where N_T is the total number of temperature nodes, N_z is the number of zones (including hallways) and N_{int}

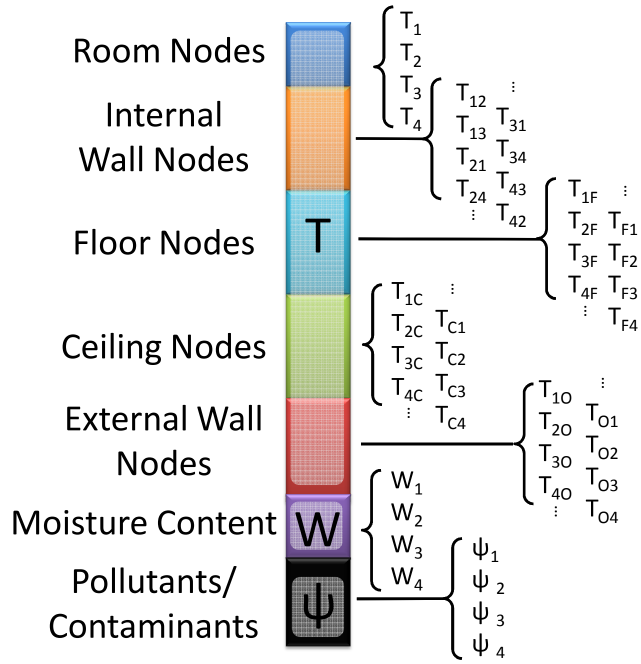

Fig. 6. The matrix layout for the 4 zone model

is the number of internal number of temperature nodes. The uncertainty in the exact amount arises because of the layout of the building. In the 4 zone model, each zone is adjacent to only 2 zones. However, in other models, a zone can be adjacent to 4 zones or more. The number of ceiling nodes, floor nodes, and external wall nodes will always be $2N_z$. If a zone is not adjacent to outside, the external wall area will be specified as zero, which will correct for the external nodes it is assigned. Additionally, pollutants and contaminants can be added to the simulation. These are allotted for by the variable n_ψ and will be a multiple of the number of zones, R. For this preliminary simulation, CO_2 was the only pollutant examined, therefore there was a total of 44 rows with 4 being attributed to CO_2.

It is assumed that air density remains constant and uniform in the zone. With Equations (12) and (13), a system of differential equations can be generated for a single surface separating two volumes of air. Using the same technique for all of the surfaces, a system of coupled ordinary differential equations defining the internal thermal dynamics for conduction in the following form can be created for a multi-zone building

$$C\dot{T} = A^aT + B^aU \tag{15}$$

where $U = [T_0^T \;\; Q^{sT} \;\; n^{pT}]^T$, n^p is the number of people and $Q^p = Q^k n^p$, Q^k is average amount of heat rate generated by a person. However, to accommodate the air and moisture, additional terms need to be added. It is assumed that conditioned air entering into a zone effects the zone temperature directly. Hence, ODE corresponding to only zone temperature is effected by conditioned air, rest of the dynamics remain same as defined in (15). The overall dynamics of T_i, the temperature of node i, can be expressed as

$$C_i \frac{dT_i}{dt} = \quad A^a_{i*}T + B^a_{i*}U + m_i^{in} h_i^{in}(T_i^{in}, W_i^{in}) - m_i^{out} h_i^{out}(T_i, W_i) \quad i = 1, 2, ...N \tag{16}$$

where C_i is the thermal capacity of the i^{th} zone, A^a_{i*} and B^a_{i*} represent the i^{th} row of matrix A^a and B^a respectively, $m_i^{out} = m_i^{in} + n_i^p \omega_{H_20}$ is the air rate going out of node i (which is derived from mass balance), n_i^p is the number of people present in i^{th} zone, ω_{H_20} is the amount of water released by a person per second, $h(\cdot)$ is enthalpy. It is assumed that air going out through ventilation has the same temperature and humidity ratio as air present in the zone. In practice, the external load can be computed from measurement of sunlight levels and outside air temperature once the building design parameters such as wall materials, window size, building orientation, etc. are known [4]. Measuring Q_i^p, the internal part of the cooling load, is more challenging and will have to be estimated using sensor measurements as well as mathematical models of occupancy and appliance use. Those models are not considered in this report. Here we assume that the loads Q_i's are inputs to the model that are specified a-priori as functions of time. The enthalpies of incoming and outgoing air in (16) can be computed from psychometric equations [4] as

$$\begin{aligned} h_i^{in} &= C_{pa} T_i^{in} + W_i^{in}(h_{we} + C_{pw} T_i^{in}) \\ h_i^{out} &= C_{pa} T_i + W_i(h_{we} + C_{pw} T_i) \end{aligned} \tag{17}$$

where C_{pa} is specific heat capacity of air at constant pressure, T_i^{in} is dry bulb temperature of incoming air into i^{th} zone, h_{we} is the evaporation heat of water at 0°C, C_{pw} is specific heat capacity of water vapor at constant pressure.

Since humidity plays a significant role in the enthalpy of air, it must be taken into account when examining the air in the zone, leaving the zone, and entering the zone. Therefore, the humidity of each zone is incorporated into the model. Furthermore, the humidity is affected by people and is directly tied to the occupancy of each zone. During normal operating conditions, the occupants of a zone are increasing the moisture content of the zone while the HVAC sytem is removing moisture by introducing drier air (outside air) and removing humid air. This can be represented in differential form as

$$\frac{dW_i}{dt} = \frac{RT_i n_i^p \omega_{H_2O}}{V_i P_i} + \frac{RT_i(W_i^{in} - W_i) m_i^{in}}{V_i P_i (1 + W_i^{in})} \tag{18}$$

where R is ideal gas constant and V_i is the volume of i^{th} zone. Calculation details of above differential equation is shown in Section VIII. It is important to note that (16) does not fully express all the complexities of the dynamics. The mass flow rate m_i^{in} into the i-th zone is related to the total flow rate at the AHU fan, pressure distribution in the ducts, and the open area of the dampers in all the terminal boxes. Changing the position of a damper in one zone may lead to a change in the pressure distribution in the ducts, which will affect the air flows into the nearby zones. Developing accurate models of the entire HVAC system is quite challenging due to these uncertainties [1]. As of now, we leave these complexities aside and assume that the mass flow rates into each zone can be commanded as wished within 0 and pre-designed maximum values.

D. Contaminants

Contaminants, such as CO_2, are usually referred to in quantities of parts per million (ppm). This is essentially a percentage in scientific notation. Multiplying a specified volume of air by the ppm of CO_2 and then dividing by 10^6 provides the volume of CO_2 in the specified volume. In equation form, this is $V_{CO_2} = V_{air} \frac{ppm_{CO_2}}{10^6}$.

To find the concentration of contaminants in a volume of air, the CO_2 production by the occupants must be specified as well as the concentration of the contaminants entering and leaving the zone. An initial CO_2 level of 350 ppm, comparable to the concentration of CO_2 in outdoor air, was used. The CO_2 produced per person was taken to be 0.08 $m^3/hour$ [5]. This is at the lower amount specified for normal work. The outside air was taken as 350 ppm [4]. The following differential equation was used to calculate the concentration of CO_2 in the zone during the time interval. Note, the equation is not specific to CO_2 and can be applied to any contaminant.

$$\begin{aligned} \dot{\psi} =& (CO_2 \text{ produced by people}) + (CO_2 \text{ in}) \\ & - (CO_2 \text{ out}) \\ =& \left[n_i^p \cdot \eta_{CO_2} + \frac{\psi_{chill}(\psi_{RA}, \psi_{OA}) \cdot m_i^{in}(t) \cdot v_{chill}}{10^6} \right. \\ & \left. - \frac{\psi_i \cdot m_i^{out}(t) \cdot v_{avg}}{10^6} \right] \cdot \frac{10^6}{V_i} \end{aligned} \tag{19}$$

Where ψ is the concentration of the contaminant in the zone, η_{CO_2} is the CO_2 produced per person ($\frac{m^3}{s}$), $\psi_{chill}(\psi_{RA}, \psi_{OA})$ is the concentration of CO_2 of the air entering the zone as a function of the return air (RA) and outside air (OA), v_{chill} is the specific volume of the air entering the zone, v_{avg} is the average specific volume of the air in the building. Note, the expression within the brackets is in units of $\frac{m^3}{s}$ and the term outside of the brackets converts the entire expression into units of $\frac{ppm}{s}$.

The differential equations from (16), (18) and (19) can be grouped together and can be written as in state space form.

$$\dot{\mathbf{X}} = \begin{bmatrix} \dot{\mathbf{T}} \\ \dot{\mathbf{W}} \\ \dot{\psi} \end{bmatrix} = f(X, u) \tag{20}$$

where X is a state vector and input u can be mass flow rate, temperature of incoming air or combination of both. It depends on the type of HVAC system. For eg. mass flow rate will be constant in CAV type system, but air temperature can be changed.

E. HVAC System Simplification

The HVAC system for the 4 zone building was simplified to yield results in a shorter amount of time without having to input the dynamics of the entire HVAC system. In addition, the dynamics for the individual VAV boxes were assumed negligible for this early setup and were assumed to be capable of producing the commanded flowrate within a relatively short period of time. Similarly, for the HVAC fan, it was assumed that it would always match the total commanded flow rate of the VAV boxes. Finally, the fan efficiency was assumed to be constant and its power consumption linearly proportional to the commanded flow rate.

The chiller was assumed to always cool the air to a specific temperature and humidity of 12.78°C (55°F) and .0074 humidity ratio. Taking the enthalpy of the incoming air (a known mix of outside air and return air) and the enthalpy of the air leaving the chiller, the change in enthalpy can be calculated. Based on the enthalpy change, the power consumption of the chiller can be found using the assumed efficiency and mass flow rate.

The power consumption of the system can be found by summing the power consumption of the chiller and fan, and if necessary, the reheat coils within the VAV boxes. However, the reheat coils were not used in the preliminary stage of the simulation.

IV. Energy Consumption

A. Chiller Power

Chiller energy is the energy required to cool down the return air or outside air to the desired input air. Diagram 7 shows the brief layout of air flow. It is clear that m^{in} is amount of air (supply air) that goes into the zone. After adding amount of water generated by people, m^{out}

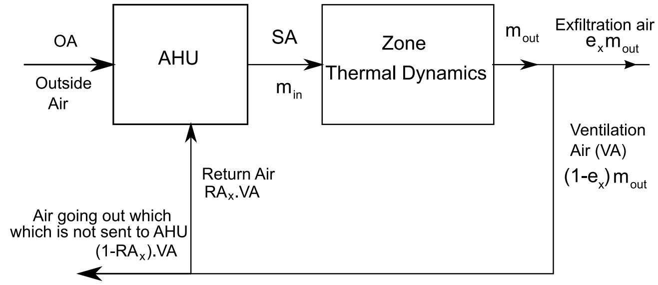

Fig. 7. Layout of air flow

is the total amount of air going out of the zone which can be divided into two parts. One is due to exfiltration (cracks, leakage) and other is air going out via ventilation (ventilation air). Assume that e_x is the ratio of air going out via exfiltration and total amount of air going out. Consider that RA_x is the percentage of air being returned to chiller, which implies $RA_x = RA/VA$. Therefore, outside air rate(m^{OA}) needed can be calculated as

$$m^{OA} = m^{in} - RA_x(m^{in} - e_x m^{out}) \tag{21}$$

Using (17), enthalpy of incoming air, outside air and return air can be calculated as

$$\begin{aligned} h^{OA} &= C_{pa}T^{OA} + W^{OA}(h_{we} + C_{pw}T^{OA}) \\ h^{RA} &= C_{pa}T + W(h_{we} + C_{pw}T) \end{aligned} \tag{22}$$

where T^{OA} and W^{OA} are the dry bulb temperature and humidity ratio respectively of outside air, h^{OA} is the enthalpy of outside air, h^{RA} is the enthalpy of return air. It is assumed that there is air going out through zone is coming back as return air. It means that there is no temperature or humidity ratio drop due to ducts. Therefore, T and W are the zone air temperature and humidity ratio respectively being used in enthalpy calculations of return air. Using (21) and (22), chiller power can be written as

$$P_c = \frac{m^{OA}(h^{OA} - h^{in}) + (m^{in} - m^{OA})(h^{RA} - h^{in})}{\mu_c} \tag{23}$$

where P_c is the chiller power and μ_c is the chiller efficiency

B. Fan Power

Fan power can be written [2] as

$$P_f = 0.1175 p_{in} q_{cfm} / (\mu_f \mu_b \mu_m) \tag{24}$$

where P_f is the fan power in watts, p_{in} is the pressure through the ducts (in. WG), q_{cfm} is the mass flow rate of air (cfm), $\mu_f,\ \mu_b,\ \mu_m$ are the efficiencies of fan, belt and motor respectively. Pressure drop or friction loss is a system specific parameter and depends on the length, diameter, mass flow rate and material of the duct. For example, for a galvanized steel

$$\Delta p = 0.109136 q_{cfm}^{1.9} / d_e^{5.02} \tag{25}$$

where Δp is the friction loss or pressure drop (inches water gauge/100 ft of duct), d_e is equivalent duct diameter (inches). For a circular duct, d_e is the radius of duct, but for a rectangular duct of height a, width b can be calculated as

$$d_e = 1.30(a+b)^{0.625} / (a+b)^{0.25} \tag{26}$$

Friction losses details can be found out in [6]

V. Weather Data

To simulate the temperature dynamics and to calculate the fan and chiller power consumption in (23) and (24), we need to provide outside air temperature and humidity ratio. Hourly weather data is collected from [reference] and interpolated using $interp1$ function in matlab. It is important to note that [reference] gives relative humidity of outside air, but humidity ratio is required here which can be calculated using following series of equations (Equation 6, 22, and 24 from [4]).

$$\begin{aligned}
C_8 &= -5.8002206 \cdot 10^3 \\
C_9 &= 1.3914993 \\
C_{10} &= -4.8640239 \cdot 10^{-2} \\
C_{11} &= 4.1764768 \cdot 10^{-5} \\
C_{12} &= -1.4452093 \cdot 10^{-8} \\
C_{13} &= 6.5459673 \\
C_{14} &= 0
\end{aligned} \tag{27}$$

If temperature $0^\circ < T < 200^\circ$, constants in (28) should be used. If temperature $T < 0^\circ$, constants in following equation (29) should be used.

$$\begin{aligned}
C_8 &= -5.674536 \cdot 10^3 \\
C_9 &= 6.3925247 \\
C_{10} &= -9.677843 \cdot 10^{-3} \\
C_{11} &= 6.221570 \cdot 10^{-7} \\
C_{12} &= -2.074782 \cdot 10^{-9} \\
C_{13} &= 4.163502 \\
C_{14} &= -9.484024 \cdot 10^{-13}
\end{aligned} \tag{28}$$

$$p_{ws} = e^{\left(\frac{C_8}{T} + C_9 + C_{10}T + C_{11}T^2 + C_{12}T^3 + C_{14}T^4 + C_{13}log(T)\right)} \tag{29}$$

where p_{ws} is the saturation pressure.

$$p_w = RH \cdot p_{ws} \tag{30}$$

where p_w is the partial pressure of water vapor.

$$W = 0.62198 \cdot \frac{p_w}{(p_{atm} - p_w)} \tag{31}$$

where W is humidity ratio which we want to calculate and p_{atm} is atmospheric pressure.

A. HVAC Control Laws

In addition, Gouda and others have provided extensive work incorporating adaptive control and prediction [7] [8].

With all of the thermal effects in place in the mathematical building model, the last step is to set up the HVAC control laws. PI controller is designed to track the desired performance. The reason to choose PI instead of PID is that PID takes a long time to simulate instead of PI, but the results are approximately same in both the cases.

ASHRAE requires that there is a minimum of 15-20 CFM (0.007-0.009 m^3/s) of outside air allowed for each person occupying a zone. In this control design, there is no return air which means 100 % of outside air is sent to chiller. Since outside air is used only, dynamics for contaminants are ignored for now. For simplification, number of occupants are assumed to be constant (n_i^p=1, i=1,2...4) in each zone. There is no solar radiation entering into any of the zone. To design a PI controller, desired zone temperature is defined as $T_i^{set} = 19°C$, i=1,2...4. It is important to note that m^{in} is the only input which can be changed. It means that incoming air temperature is fixed as $12.78°C$, but flow rate can be changed using controller. With all these assumptions, PI controller dynamics combined with other states can be written as

$$m_i^{in} = K_p e + K_i \int_0^t e dt, \ \ i = 1, 2...4 \tag{32}$$

where $K_p = -0.00005$, $K_i = 0.0001$ are tuned proportional and integral gain respectively, error e is defined as $e_i = T_i^{set} - T_i, \ i = 1, 2, ..4$. One way to implement this controller is by adding four more states $E = \int_0^t e dt$ into the original states. New states equation can be written as

$$\dot{\mathbf{X}} = \begin{bmatrix} \dot{\mathbf{T}} \\ \dot{\mathbf{W}} \\ \dot{\mathbf{E}} \end{bmatrix} \tag{33}$$

Dynamics of new added state $E = \int_0^t e dt$ can be written as $\dot{E} = e$. One main challenge in integrating these dynamics is how to define initial condition for each state. To simplify the situation, each zone has the same initial temperature as $21°C$ and all other nodes has the initial temperature as $22°C$. Furthermore, each zone has the same initial humidity ratio as 0.008 and E is assumed to be zero initially.

B. Numerical Results

In the simulation results presented here, our goal is to track the desired reference. Total time of simulation is 24 hours and time step is 10 minutes. It means that predefined inputs such as outside temperature,outside humidity ratio no. of people etc. are provided at the interval of every 10 minutes, which implies that these predefined inputs are remained constant for 10 minutes. Similarly, power consumed in chiller and fan are changed only after 10 minutes. However, states and control input are not assumed constant during that time, because ode45 (a matlab function used in this simulation) is intelligent enough to divide the total time of 10 minutes into variable time steps. Figure 8-12 shows the simulation results for a day. It is clear from figure 8 that zone

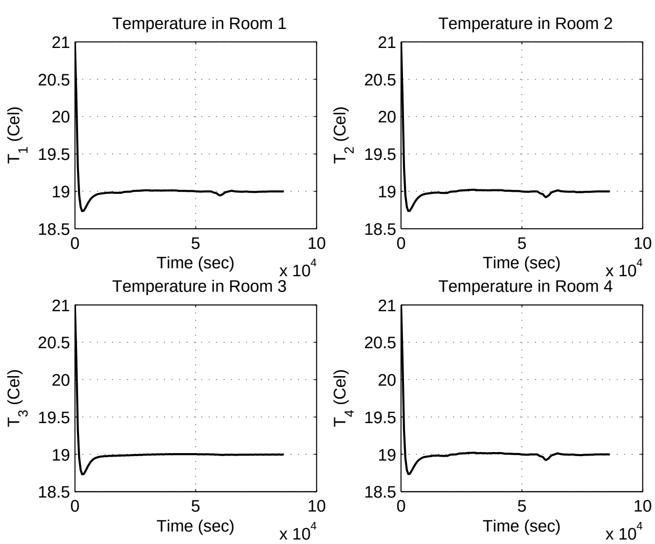

Fig. 8. Temperature in each zone

temperature is able to track the desired temperature very well despite the change of outside temperature change. Temperature profile in zone 3 after 12 hours is different than other three zone temperatures, which is due to zone 3 does not have any window. Therefore, zone 3 is not

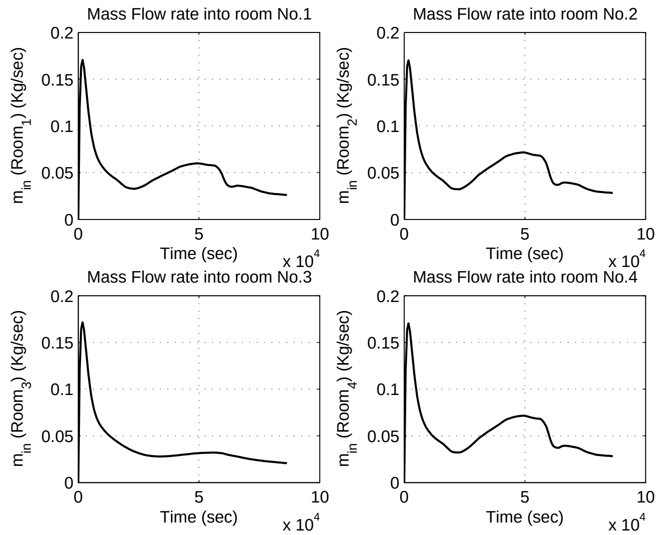

Fig. 9. Air flow rates in each zone

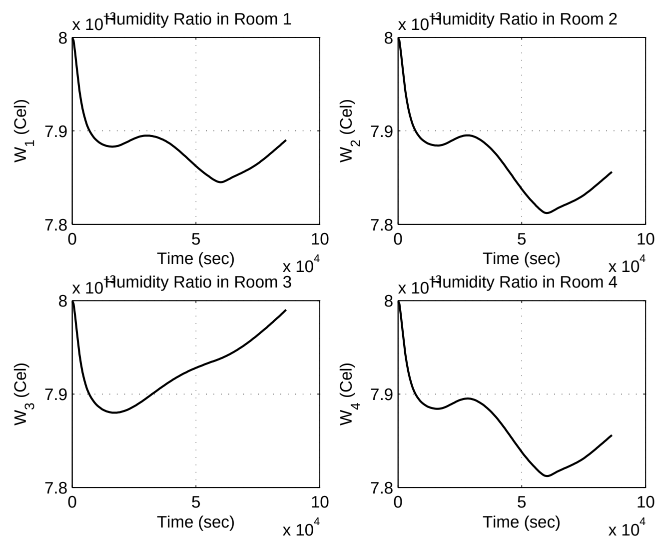

Fig. 10. Humidity ratio of each air

as effected by change in outside temperature than other three zones. Rise time for control input in each zone is less than 25 minutes and it shows smooth behavior. Note that temperature in each zone has an overshoot of less than $0.3°$. Figure 9 shows the mass flow rate in each zone. As temperature difference between desired and actual is large initially, more air flow is required initially which is shown in figure 9. It is obvious that a minimum amount of air flow rate is required to maintain the desired temperature. However there is one more peak after 10 hours, which is due to the increase in outside temperature as shown in figure 12. The main advantages of this controller is that mass flow rate does not hit the

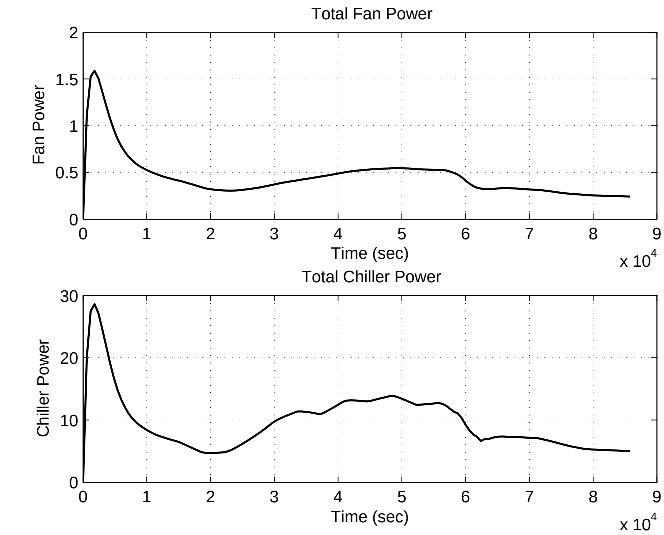

Fig. 11. Total fan and chiller power of all zones

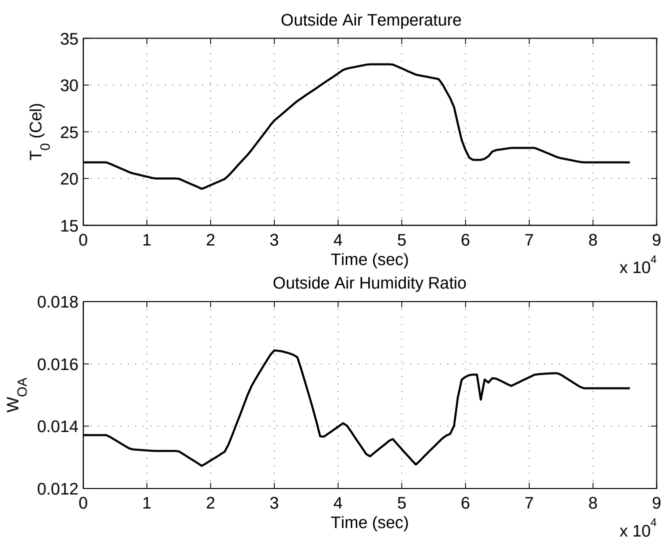

Fig. 12. Temperature and humidity ratio of outside air respectively

upper limit and control input is relatively smoother. zone humidity ratio of each zone is shown in figure 10. As incoming air has the less humidity ratio than the initial humidity ratio of zone, increasing mass flow rate will decrease the humidity ratio which is shown in figure 10. Figure 11 shows the total fan power and chiller power, which is obvious because fan power depends on the mass flow rate.

VI. Conclusion

In this report, a building thermal model has been developed which can be used to test different control strategies performance. A PI controller, which is state of art, is also designed to achieve the desired temperature in each zone with less oscillations which can be used to compare with other control strategies performance to minimize energy consumption.The temperature in each zone is able to reach steady state in fairly good amount of time keeping all the inputs (mass flow rates) within the limits.

VII. Areas for improvement

The simulation created and presented in this paper provides decent results. However, the simulation can be improved. Areas for improvement are listed and then described in the following section.

- Air flow between zones
- Dynamics of the components of the HVAC system including the VAV boxes, fans, and pressure changes
- Improved control algorithms to optimize power consumption and occupant comfort

For simplicity, air transfer between zones was ignored for the 4 zone model. However, in reality air can enter a zone and then move into a hallway area where it is recirculated into the HVAC system. The air entering the hallway area can be modeled similar to air entering a zone, but will require additional programming.

The dynamics of the components of the HVAC system were simplified and in some cases ignored in order to establish a working simulation in a realistic time. VAV boxes, the HVAC fan, and the pressure changes within the air ducts can all be modeled in the future to provide a more accurate model.

Last and foremost, the control algorithms can be drastically improved to save additional power and to maintain a more stable temperature within each zone. Areas for improvement include: reducing the amount of outside air used while reheating air for a particular zone; predictively cooling a zone according to known weather and occupant patterns; using the entire temperature comfort range to save additional power during peak times.

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VIII. Appendix

A. Humidity ratio

Assuming that air going out of a zone is through ventilation and exfiltration only, it can be written as

$$m^{out} = w_{da}^{v} + w_{w}^{v} + w_{da}^{exf} + w_{w}^{exf} \tag{34}$$

where w_{da}^{v} and w_{da}^{exf} are the dry air rates going out through ventilation and exfiltration respectively, w_{w}^{v} and $w_{w}^{exf}$are the water vapor rates going out through ventilation and exfiltration respectively. It is assumed that air going out through ventilation is same as air going out through cracks (i.e. exfiltration). Incoming and outgoing air rates can be rewritten in the following form

$$m^{out} = (1+e_x)(1+W)w_{da}^{v} \tag{35}$$

$$m^{in} = (1+W^{in})w_{da}^{in} \tag{36}$$

where w_{da}^{in} is the dry air rate coming into the zone. Differentiating humidity ratio with respect to time gives us

$$\dot{W} = \frac{d}{dt}\left[\frac{W_w}{W_{da}}\right] = \frac{\dot{W}_w W_{da} - W_w \dot{W}_{da}}{W_{da}^2}$$
$$= \frac{1}{W_{da}}(\dot{W}_w - W\dot{W}_{da}) = \frac{1}{V\rho_{da}}(\dot{W}_w - W\dot{W}_{da}) \tag{37}$$

where W_w, W_{da} are the amount of water vapor and dry air present into the zone respectively, V is the volume of air (which is zone volume here) and ρ_{da} is the density of dry air. Using ideal gas law, it can be written as

$$\dot{W} = \frac{RT}{VP}(\dot{W}_w - W\dot{W}_{da}) \tag{38}$$

where T is zone air temperature, P is zone air pressure which is approximately assumed to be equal to atmospheric pressure. Using mass balance, following equations can be written

$$\begin{aligned} \dot{W}_w &= n^p\omega_{H_2O} + w_w^{in} - (1+e_x)w_w^v \\ \dot{W}_{da} &= w_{da}^{in} - (1+e_x)w_{da}^v \end{aligned} \tag{39}$$

where w_w^{in} is the water vapor air rate entering into the zone (35) and (36) can be rearranged as

$$\begin{aligned} w_{da}^v &= m^{out}/((1+e_x)(1+W)) \\ w_w^v &= Wm^{out}/((1+e_x)(1+W)) \\ w_{da}^{in} &= m^{in}/(1+W^{in}) \\ w_w^{in} &= W^{in}m^{in}/(1+W^{in}) \end{aligned} \tag{40}$$

Using (38), (40)and (39) leads to following equations

$$\dot{W} = \frac{RT}{VP}\left[n^p\omega_{H_2O} + m^{in}\frac{W^{in} - W}{1+W^{in}}\right] \tag{41}$$