Regulating with Limited Information

Overall - not possible to show any scheme is optimal with no information, but given some assumptions, schemes can be devised that are close

Non-Bayesian - Regulator doesn't have any information on the firm's cost (not even beliefs which lead to probability distributions)


Summary - looks a lot like rate of return regulation in that prices are matched to realized cost; under some conditions, even though the regulator has no cost information, the firm will end up using Ramsey prices (i.e., produce where average total cost intersects the demand curve)

Assumptions - only the first one is major
(1) Costs and demand are stationary (this is a very strong assumption)
(2) Firm's cost function exhibits decreasing ray average cost (not a big a deal; V&F relax it)

DRAC - multiproduct firm producing \( q = (q_1, \ldots, q_n) \) units; decreasing ray average cost means that for any constant \( r > 1 \), \( C(rq) \leq rC(q) \); this is the multiproduct counterpart to economies of scale (i.e., declining average cost)

Importance - ensures moving to lower prices (i.e., higher quantity), AC declines so firm doesn't necessarily go out of business
(3) Firm is a myopic profit maximizer (not critical, but makes the math easier)
(4) Firm must supply all demand at the regulated prices (this is a standard assumption for regulated firms so it’s not a big deal)

Information Structure -
Common Knowledge - total cost, output, and price are observable ("accounting data"); regulator is also aware that firm has DRAC
Private Information - only the firm knows the functional form of \( C(q) \)

V-F Mechanism - \( p_t \in R \equiv \{ p \mid p \cdot Q(p_{t-1}) - C(Q(p_{t-1})) \leq 0 \} \)

Single Product Translation - price this year has to be less than last year’s average cost:
\[
R = \left\{ p \mid p \leq \frac{C(Q(p_{t-1}))}{Q(p_{t-1})} = AC_{t-1} \right\}
\]

Sort of like rate of return regulation; firm is allowed to charge any price as long as it doesn’t earn more profit than it did the year before

Theorem - under V-F Mechanism and assumptions (1)-(4), the firm sets prices that converge to Ramsey prices

Proof: look at single product case
As picture shows, firm can set whatever price it wants initially
Future prices will be limited by the firms average cost in the period before
At the limit price will be determined by the intersection of the firm’s AC curve and the demand curve
This is the optimal point (maximizes total surplus while not bankrupting the firm)

Special Case - constant AC = MC
Price converges to AC in second period
**Multiproduct Setting** - V-F Mechanism always forces prices in direction that increases consumer surplus

**Iso-Consumer Surplus** - combinations of prices that lead to same amount of consumer surplus; points closer to origin are better (i.e., consumers prefer lower prices), but can’t get there because of non-negative profit constraint

**V-F Constraint** - tangent to the last period’s iso-CS curve

**Next Period Pricing** - must be at the tangency point (i.e., same as last period’s prices) or must improve CS

**Proof:**

Slope of Iso-CS: \( S(p_{t-1}^1, \ldots, p_{t-1}^n) = \bar{S} \) (constant)

Differentiate wrt any 2 prices: \( S_{p_i}(\cdot)dp^i + S_{p_j}(\cdot)dp^j = 0 \)

Solve: \( \frac{dp^i}{dp^j} = -\frac{S_{p_j}(\cdot)}{S_{p_i}(\cdot)} \frac{Q^j}{Q^i} \) (recall derivative of CS wrt price is quantity)

Slope of V-F constraint: \( p_i \cdot Q(p_{t-1}) - C(Q(p_{t-1})) = 0 \) (= for the boundary)

Differentiate wrt any 2 period \( t \) prices: \( dp^i Q^i(p_{t-1}) + dp^j Q^j(p_{t-1}) = 0 \)

Solve: \( \frac{dp^i}{dp^j} = -\frac{Q^j(p_{t-1})}{Q^i(p_{t-1})} \)

**Problem** - this is a powerful result (get optimal solution even when regulator doesn’t have cost information), but it relies on an implicit assumption: firm is not inflating cost

**Identification Problem** - regulator can’t tell the difference between actual cost and minimum possible cost; firm may have an incentive to drive costs up

**Pure Waste** - to see if this is a problem, drop assumption (3) and check if firm can benefit from “pure waste” in a two period model; (pure waste means the firm is literally just throwing money away to inflate costs, as opposed to driving up costs through R&D, executive perks, or some other potentially useful expenditure... this should be the worst case scenario)

**Simple Case** - neither assumption here is essential, but they make the math easier

- Constant AC = MC
- Linear demand

**No Waste** - in first period, firm will set monopoly price and earn

\[ \pi_1 = (p_1 - c)Q_1 > 0 \]; in second period firm will be forced to set \( p_2 = c \)

so \( \pi_2 = 0 \)

**Pure Waste** - this example will not show the optimal level of waste, but will show the firm can do better by using waste; in first period, firm generates waste \( w_1 = p_1 - c \) (i.e., difference between monopoly price and marginal cost); given this cost, the firm will set \( p_1^w \) at monopoly price for higher cost earning \( \pi_1^w = (p_1^w - w_1 - c)Q_1^w \) (small square); now in second period firm can set original monopoly price so \( p_2^w = p_1 \)

and \( \pi_2^w = \pi_1 \)

**Discount Rate** - \( \beta \leq 1 \)

**Firm’s Choice** - if \( \pi_1^w + \beta \pi_2^w > \pi_1 \) the firm will prefer waste (since \( \pi_2^w = \pi_1 \), if \( \beta \) is near 1, this will be the case)
Basic Problem - firm has no incentive to reduce cost

Blackmail - "This stuff is fun, figuring out ways to outsmart the regulator."


Summary - looks at how much better a regulator can do (relative to V-F Mechanism) if he knows demand (but still doesn’t know cost)

Purpose - proposed new mechanism to avoid the waste problem from V&F

Assumptions
(1) Demand is common knowledge (this requires a lot more information than V&F)
(2) Only firm knows its cost function
Firm’s expenditures ($E(t)$) are observable in each period (but the regulator can’t tell the different between minimum cost and waste)
(4) Firm’s discount factor is $\beta < 1$ (strictly less than 1 is important)
(5) Costs are stationary (don’t need V&F’s assumption (2))

Focus on single product case to make this quick...

Incremental Surplus Subsidy (ISS) Mechanism -

- Firm can set any price it likes in period $t$
- Firm receives revenue $p_tQ(p_t)$ and a subsidy

\[ S_t = \int_{p_t}^{p_{t-1}} Q(p)dp - [p_{t-1}Q(p_{t-1}) - E_{t-1}] \]

(i.e., $\Delta CS$ – last period's profit)

Note: $\Delta CS$ will be < 0 if firm raises price

Bribe - the subsidy is paid by consumers (through taxes or two-part tariff); it’s basically a bribe to get the firm to maximize total surplus

Result -
(i) Firm sets $p = MC$ in each period
(ii) Firm receives positive profit only in first period
(iii) Firm will not engage in any pure waste

Proof: "is really not all that interesting"

Firm's objective:

\[ \max \sum_{t=1}^{\infty} \beta^{t-1} [p_tQ(p_t) - E(Q(p_t)) - w_t + S_t] \]

Note: $p_tQ(p_t) - E(Q(p_t)) = \pi_t$, and can rewrite $S_t = \int_{p_t}^{p_{t-1}} Q(p)dp - \pi_{t-1}$

Mathy Way - plug these in and differentiate wrt $p_t$ and $w_t$

Semi-Math - "prove intuitively why it works with semi-mathematics"

Trick -
\[ \int_{p_t}^{p_{t-1}} Q(p)dp = \int_{p_t}^{\infty} Q(p)dp - \int_{p_t}^{p_{t-1}} Q(p)dp = CS_t - CS_{t-1} \]

effectively $\pm \int_{p_{t-1}}^{\infty} Q(p)dp$ to get integral to equal difference in consumer surplus
Rewrite objective: 
\[
\max_{p_t, w_t} \sum_{t=1}^{\infty} \beta^{t-1} \left[ \pi_t - w_t + (CS_t - CS_{t-1} - \pi_{t-1}) \right]
\]

Rearrange: 
\[
\max_{p_t, w_t} \sum_{t=1}^{\infty} \beta^{t-1} \left[ (CS_t + \pi_t) - (CS_{t-1} + \pi_{t-1}) - w_t \right]
\]

Sub welfare = sum of CS and profit: 
\[
\max_{p_t, w_t} \sum_{t=1}^{\infty} \beta^{t-1} \left[ W_t - W_{t-1} - w_t \right]
\]

∴ firm is maximizing the increments in total welfare; as long as firm discounts the future, it wants the maximum increment in the first period

Not Enough? - Lyon, "Evaluating the Performance of Non-Bayesian Mechanisms," *Journal of Regulatory Economics*, January 1996 - reviews different mechanisms people have developed and discusses which is good to use when