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Applying Robust Optimization to MISO Look-Ahead Commitment

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***Abstract* – Managing uncertainty has been a challenging task for market operations. This paper first reviews the current practice of managing uncertainties at MISO. A framework of using robust optimization based approach on MISO Look-Ahead commitment (LAC) is then introduced. The numerical results show that this type of approaches are promising and yet with challenges to overcome in order to be practical for real world application.**

***Index Terms* – Uncertainty, Look-Ahead Unit Commitment, Mixed-Integer Programming, Two-Stage Robust Optimization.**

I. INTRODUCTION

It has always been important to manage uncertainties for the reliable operations of the power system. Over the years, the industry has developed many effective ways to handle uncertainty. For example, the common practice of carrying operating reserves. Under the market structure, most RTOs commit resources at multiple stages to account for the different level of uncertainties at various stages. At MISO, the commitment process ranges from 7-day Forward Reliability Assessment Commitment (FRAC) to Look-Ahead Commitment (LAC) which runs every 15 minutes. Moreover, various operational procedures have also been developed at MISO to manage the uncertainty.

Managing uncertainty has become more challenging with the integration of renewables. At MISO, the installed wind generation capacity has grown from 500MW in 2005 to about 12,000MW in 2013. The introduction of Dispatchable Intermittent Resources (DIR) helped to manage the wind impact on congestions and over generation situations. However, the commitment and dispatch processes still heavily count on the accuracy of wind forecast.

The source of uncertainties can come from inaccurate input data, unexpected events and market behaviors, and simplified mathematical models. For example, the trading activities in real time may cause the Net Scheduled Interchange (NSI) to deviate significantly from the values used in the earlier scheduling processes. Furthermore, the study interval length in the planning horizon may not be granular enough to reflect the rate of changes. For example, LAC study intervals range from 15 minutes to 30 minutes. The commitment from LAC can meet the 15-min or 30-min forecasted load change; however, it may not have enough ramping capability to meet 5-min real-time dispatch need if any very large load changes happen within 5 minutes.

Uncertainties can result in the deviation of market clearing models from the actual system conditions. If uncertainties are not managed well, the system may require carrying excessive reserves to bridge the gap between the solution of market clearing processes and the actual system needs. It may also cause transmission violations and insufficient generation capacity or ramping capability to meet power balance requirement in real-time, which sometimes leads to expensive remedy actions, such as committing quick start resources, to correct potential near term imbalance or violations.

The expected difference, or level of uncertainties, between the actual system condition and the market clearing models changes overtime. Fig. 1 illustrates the expected level of uncertainties throughout the market clearing processes. The further away from the actual operating point, the larger uncertainty is expected.

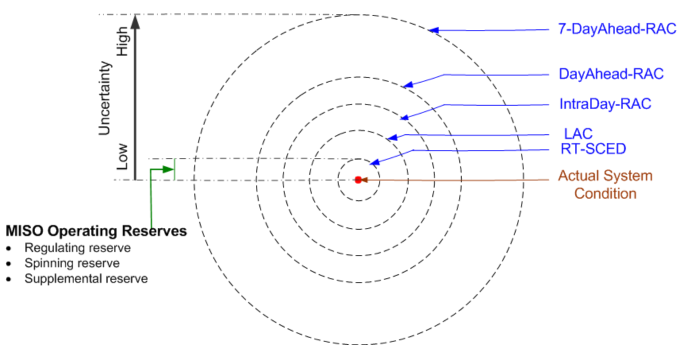

Fig.1 Expected Level of Uncertainties throughout the Market Clearing Processes

At MISO, the operating reserves include regulating reserve, spinning reserve and supplemental reserve. They are cleared to bridge the gap between 5-min Real-Time Security Constrained Economic Dispatch (RT-SCED) and the actual system condition. For example, MISO only carries about 300MW~500MW of regulating reserves with a peak load of about 99GW.

The same operating reserve requirements are applied in all market clearing processes. Apparently, the cleared operating reserves are not sufficient to accommodate larger uncertainties in RAC and LAC. It may also not be able to account for the uncertainty caused by study interval differences. Hence, additional "headroom" has to be incorporated in the planning horizon. For example, for 7-day FRAC and day-ahead FRAC studies, MISO operations require the commitment results to cover certain percentage of capacity headroom beyond the energy balance requirement from demand, NSI, and operating reserves. The capacity headroom is used to cover the uncertainties from the input data and it is determined based on

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the analysis of historical load forecast, wind forecast, NSI, and generation responses. The purpose is to commit slow start resources so that future actions (fast-start resource commitment and economic dispatch) can satisfy additional changes beyond the deterministic input data.

At the LAC time frame, the uncertainty is much less. However, the uncertainty from wind, NSI and load still requires additional capacity headroom. Real-time operations also developed a strategy to solve multiple scenarios at the same time. For example, MISO solves three LAC scenarios with different load, wind and NSI settings. The three scenarios are usually set as follows:

Scenario 1: Coincidental peak forecast level.

Load forecast, wind forecast and NSI are with 5 minutes to 15 minutes granularity. LAC study interval length varies from 15 minutes to 30 minutes. LAC scenario 1 is set with the three inputs at the coincidental peak of each study interval.

Scenario 2: Additional 500MW is proportionally distributed and added to each local balancing area (LBA)'s load forecast value under scenario 1. (Load forecast is performed for each of the 28 LBAs).

Scenario 3: Additional 1000MW is proportionally distributed and added to each LBA's load forecast value under scenario 1.

Similarly, MISO solves six RT-SCED scenarios.

Provided with the solution from multiple scenarios, operators can respond to the latest system condition changes by selecting the proper LAC and RT-SCED scenario. It can result in more targeted commitment and dispatch solution. For example, if a 600MW non-conforming load switched on and it is not included in previous load forecast, the operators can start to look at the LAC scenario 2 before the load is reflected in the load forecast of future LAC cases. The ability to choose from discrete scenarios can help reduce the amount of required regulating reserve, which make it possible, in part, for only carrying 300MW~500MW of regulating reserves with about 99 GW peak load.

MISO LAC is primarily used to commit fast-start resources in real-time. It runs every 15 minutes with a study window of 3 hours. The interval length is 15 minutes for the first two hours and 30 minutes for the third hour. Compared to other commitment processes, LAC has relatively small problem size and narrow range of uncertainty.

The strategies developed by the operations help MISO manage the uncertainty well. With more and more renewable integration, there is a real world need of more systematic ways to manage the uncertainty. Recently, there are many interesting research work in the areas of stochastic and robust optimization. Some of the works look very promising for solving the real world problems [1]-[5]. In 2012, MISO started to investigate these techniques to explore the possibility of using these advanced optimization approaches to incorporate uncertainties in the market clearing processes and to evaluate the benefit from applying those approaches. A prototype of robust optimization based LAC has been developed to consider a range of variations on load forecast from each of the 28 LBAs. It can be extended to model uncertainty in other input data such as scheduled interchanges and wind forecast.

The first stage of the project is to use robust optimization to model the uncertainties currently addressed by the headroom requirement in LAC. In addition to the three scenarios mentioned above, MISO LAC also includes 350MW of capacity headroom requirement for each scenario. It is used to bridge the gap between LAC and RT-SCED, i.e. unexpected generation capacity or ramping requirement that is not covered due to larger study interval length or inaccuracy of input data in LAC. Current deterministic LAC only considers one input profile for each scenario with both the operating reserve requirements and the capacity headroom requirement. The input profile is based on the coincidental peak within each interval and it may not be able to cover the range (e.g. sharp ramp events) between the coincidental valley and the coincidental peak.

There are two options of setting up robust optimization LAC. The first option is to configure uncertainty range to cover all LAC scenarios as well as the headroom requirement and coincidental valley as shown in Fig. 2. The commitment result from this option can cover all the uncertainties currently considered by the operators during the LAC time frame. However, the commitment result can be very expensive.

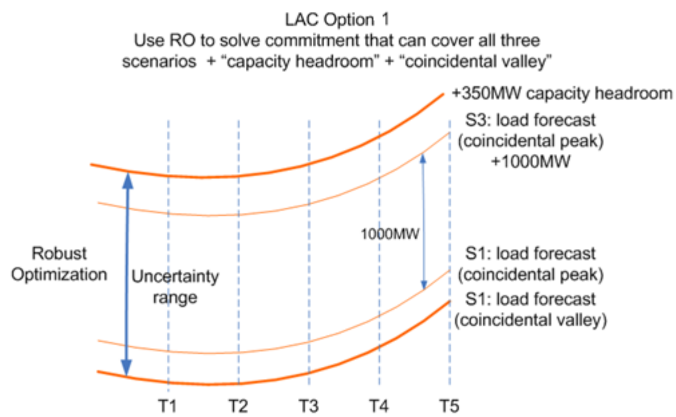

Fig.2 LAC Robust Optimization Setup Option 1

The second option is to configure uncertainty range to cover headroom requirement and coincidental valley within each scenario as shown in Fig. 3. The operators can continue picking the proper scenario based on the latest information. This option matches current operational practice and is chosen for the prototype study.

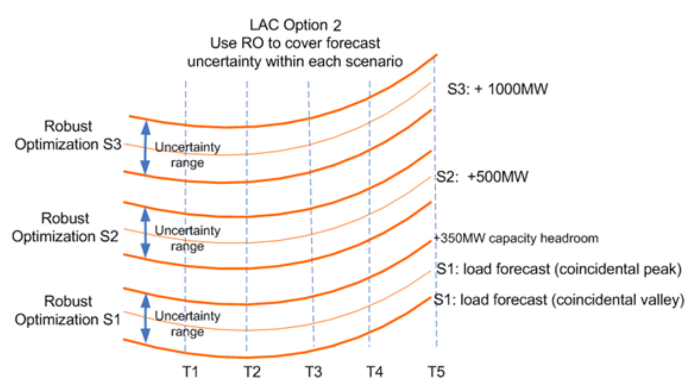

Fig.3 LAC Robust Optimization Setup Option 2

II. Mathematic Formulation

The prototype started from the work described in [4]. It is well known that the robust optimization model proposed in

[1]-[5] considered the worst dispatch cost from the second stage. It may result in conservative commitment results. In real world, the operations may be more interested in the feasibility of the worst-case scenario. Hence, a version of a unified stochastic and robust unit commitment [8] is also implemented.

A. Two-stage robust optimization LAC

In the two-stage robust optimization (TSRO) formulation in [4], the unit commitment decisions are the first stage decision variable; the economic dispatch decisions are the second stage variable so that it is a function of the uncertain load. In the first stage, the unit physical constraints (e.g., start-up/shut-down, min-up/-down time constraints) are imposed. In the second stage, dispatch constraints (e.g., load balance, reserve requirements, transmission line flow limits) and coupling constraints for commitment and dispatch decisions (e.g., generation upper/lower limits, ramping-up/down limits) are enforced. To simplify the discussions, the following compact formulation is used to describe the problem.

$$Min_{\boldsymbol{x}}\{\boldsymbol{c}^{\boldsymbol{T}}\boldsymbol{x} + Max_{d\in\boldsymbol{\mathcal{D}}} Min_{(y,s)\in\Omega(\boldsymbol{x},\boldsymbol{d})}(\boldsymbol{b}^{\boldsymbol{T}}\boldsymbol{y} + \boldsymbol{v}^{\boldsymbol{T}}\boldsymbol{s})\}$$

$s.t.\ \ \boldsymbol{Fx} \le \boldsymbol{f}, \boldsymbol{x}\boldsymbol{:}$ binary

where

$$\Omega(\boldsymbol{x},\boldsymbol{d}) = \{(\boldsymbol{y},\boldsymbol{s}): \boldsymbol{Hy} - \boldsymbol{H_s s} \le \boldsymbol{h},\ \boldsymbol{Ax} + \boldsymbol{By} - \boldsymbol{G_s s} \le \boldsymbol{g},\ \boldsymbol{I_u y} + \boldsymbol{D_s s} = \boldsymbol{d}\}$$

$\boldsymbol{\mathcal{D}}$ represents the uncertainty set for load, $\boldsymbol{x}$ represents the unit commitment decisions (binary variables), $\boldsymbol{y}$ represents the economic dispatch decisions (continuous variables) and $\boldsymbol{s}$ represents the violation slack variables.

Uncertain loads are considered to be within a certain range. Since load forecast is provided on each LBA, the basic uncertainty set is set as follows:

$$\boldsymbol{\mathcal{D}} = \{\boldsymbol{d_{kt}}: \boldsymbol{D_{kt}^{l}} \le \boldsymbol{d_{kt}} \le \boldsymbol{D_{kt}^{u}}, \forall \boldsymbol{t}, \forall \boldsymbol{k} \in \boldsymbol{K}\} \tag{1}$$

where $\boldsymbol{d_{kt}}$ are the uncertain loads, $\boldsymbol{D_{kt}^{l}}$ and $\boldsymbol{D_{kt}^{u}}$ are the corresponding lower/upper bounds for each LBA $\boldsymbol{k}$**,** and $\boldsymbol{K}$ represents the set of the 28 LBAs.

A two-stage decomposition approach is used to solve the original TSRO iteratively as described in [4]. The primal decomposition approach (or scenario-based approach) [5]-[7] is used to generate a group of scenarios based on the extreme load scenario detected by solving the sub-problem.

1) Master problem: unit commitment is considered to be the master problem in the decomposition framework. In the first iteration, the load is set at the nominal level and a deterministic unit commitment is set to obtain the starting point for the whole algorithm. The solution from the master problem is used for the sub-problem in the second stage. At the beginning of each iteration, the master problem is solved again with an additional scenario added.

2) Sub-problem: the sub-problem aims to solve the economic dispatch problem under the worst-case load scenario with the fixed unit commitment decisions. The solution of the sub-problem is used to discover the worst-case scenarios to be added into the master problem for the next iteration. The sub problem is equivalent to solving a bilinear programming problem. To solve it effectively for large sized problems, an alternative bilinear heuristic algorithm is used.

The details of the algorithm framework can be found in [4].

B. Two-stage unified stochastic and robust LAC

A unified stochastic and robust unit commitment approach was proposed in [8]. The model includes stochastic scenarios in the master problem with assigned probability. The second-stage robust optimization is used to generate feasibility cuts. The following version of the unified approach is implemented in the prototype for MISO LAC.

In this unified LAC model, the nominal scenario dispatch and violation cost are included in the master problem with a probability of 1. The second stage sub problem identifies the worst-case scenario violations. Hence, it only includes the worst-case violation cost. The commitment minimizes the nominal scenario total cost and the worst case violation cost.

$$Min_{\boldsymbol{x}}\{\boldsymbol{c}^{\boldsymbol{T}}\boldsymbol{x} + \textstyle\sum_{\boldsymbol{i}\in\boldsymbol{I}} p_i \boldsymbol{Min}_{(y,s)\in\boldsymbol{\Omega}(\boldsymbol{x},\boldsymbol{d_i})}(\boldsymbol{b}^{\boldsymbol{T}}\boldsymbol{y} + \boldsymbol{v}^{\boldsymbol{T}}\boldsymbol{s}) + Max_{d\in\boldsymbol{\mathcal{D}}} Min_{(y,s)\in\Omega(\boldsymbol{x},\boldsymbol{d})}(\boldsymbol{v}^{\boldsymbol{T}}\boldsymbol{s})\}$$

$s.t.\ \ \boldsymbol{Fx} \le \boldsymbol{f}, \boldsymbol{x}\boldsymbol{:}$ binary

where

$$\Omega(\boldsymbol{x},\boldsymbol{d}) = \{(\boldsymbol{y},\boldsymbol{s}): \boldsymbol{Hy} - \boldsymbol{H_s s} \le \boldsymbol{h},\ \boldsymbol{Ax} + \boldsymbol{By} - \boldsymbol{G_s s} \le \boldsymbol{g},\ \boldsymbol{I_u y} + \boldsymbol{D_s s} = \boldsymbol{d}\}$$

$$\Omega(\boldsymbol{x},\boldsymbol{d_i}) = \{(\boldsymbol{y},\boldsymbol{s}): \boldsymbol{Hy} - \boldsymbol{H_s s} \le \boldsymbol{h},\ \boldsymbol{Ax} + \boldsymbol{By} - \boldsymbol{G_s s} \le \boldsymbol{g},\ \boldsymbol{I_u y} + \boldsymbol{D_s s} = \boldsymbol{d_i}\}$$

III. CASE STUDY

A. Master problem solution time under multiple cuts

As shown in [10], under the robust and unified approach, Master 1 solves LAC under nominal load profile without headroom requirement. The average solution time is similar to the time under deterministic model. After that, Sub 1 is called to identified the load profile within uncertainty set (1) that corresponds to the worst-case dispatch cost (for robust) or the worst-case violation (for unified). Then the worst-case load profile is passed to Master 2 as a new scenario to solve for updated commitment. The solution time for Master 2 under robust approach is much more than the one under the unified approach. In this study, the maximum number of scenarios is set to 2 since it takes too long to solve robust master problem with three scenarios.

A case with long Master 2 solution time is investigated to identify the reasons. The Master 2 MIP for the case is solved at ~570 sec. In the CPLEX log, a large portion of efforts are spent in the root node relaxation. For the master problem with two scenarios, it costs around 250 seconds to solve the relaxed root-node problem. The cutting-planes and heuristics in the MIP solving process are actually not very time consuming. The master MIP with three scenarios cannot be solved with timeout happening during the root node relaxation solve. In the dual-simplex log, it can be observed that the LP objective is poorly improved over the 600 sec time limit. CPLEX did not have the chance to enter the cutting-plane/heuristic in this situation.

For LAC, most of the binary commitment variables are fixed based on DA and RAC commitment. There are about 200 binary variables for each of the 10 intervals. The commitment results from Master 1 and Master 2 are very close, with only about 20 total differences. An experiment is done by solving two single-scenario MIP problems, fixing the common binaries and then adding both scenarios to the master problem to solve for the ~20 binaries. This approach only improved the Master 2 solution time slightly.

This can be explained since most of the efforts are spent in the root node relaxation. For the same case, the unified approach with three scenarios (Master 3) took 126 seconds to solve. The log file for the root node relaxation under the robust optimization approach indicates some numerical issue and degeneracy of the relaxation. First of all, the main bottleneck for solving LP is usually the coefficient matrix, which is manipulated by simplex at each iteration. Since the robust model has the same coefficient matrix as the unified model, the slow performance in LP is not due to the complicated matrix structure caused by "adding cuts". In the CPLEX log, there are a lot of "perturbations" in the dual-simplex LP solver and the objective is stuck for similar values for a long time. This is a very typical phenomenon for degeneracy. According to CPLEX manual [11], it is highly recommended to try different LP solvers for this issue.

By changing the LP solver option from Automatic to Barrier during the root node relaxation for the robust model, it only takes 135.24 seconds to solve the Master 2. Moreover, it only takes 168.81 seconds for CPLEX to solve the master problem with three cuts.

However, for some cases, CPLEX still takes very long time to find a solution or even fails to find a solution. After further investigation, it was identified that there were large violation penalty cost in the deterministic solution for those cases primarily driven by the inconsistent input data. As explained in [9], LAC has input data from state estimation, existing commitment plan, override from operators, etc. Sometimes inconsistent input data can cause constraint violation. The violation penalties on resource level constraints, such as limit, ramp rate, maximum daily startup, min-run time, min-down time and max-run time, etc., are usually set very high to avoid violating resource physical limitation. If inconsistent data cause those constraints to be violated, LAC commitment and dispatch usually cannot resolve the violation. However, the huge violation penalty can cause solution time to be very unstable. Fixing slack variables for those penalty terms can significantly improve MIP performance, especially for large sized models under multiple scenarios.

For one of the cases, the input data from the commitment plan already violated maximum daily startup constraints. It introduces penalty cost of $\$10^9$ in the objective. It took 2939 sec for the robust model Master 2 to solve at an objective value of $1,004,248,719. After cleaning the model by freezing the slack variables for those resource level constraints at the end of Master 1, Master 2 can solve at 105 sec with Automatic solver setting and 70 sec with Barrier solver setting during the root node relaxation for the robust optimization model. The solved objective of the clean model is $4,248,594 under both settings.

With the above mentioned two improvements, the solution time for the robust LAC has been reduced significantly.

B. Comparison over an entire day

In this study, a full day of LAC cases are used to compare the commitment differences from the deterministic approach (with headroom requirement), the robust approach (without headroom requirement) and the unified approach (without headroom requirement).

The original LAC cases started from state estimation snapshot [9]. It incorporated the dispatch targets from RT-SCED 5-min dispatch. Also resources may not follow energy target perfectly and the deviations are reflected in state estimation snapshots. To make a consistent and fair comparison, the following steps are performed for running all three approaches:

1) Remove all production LAC commitment. This step creates the same base cases that include commitments from processes other than LAC, e.g. DA, RAC, manual call-on, etc.
2) Run the 96 LAC cases sequentially by feeding the commitment from one case to the next. It also feeds the energy dispatch target from the first interval to the next case assuming resources follow the energy target perfectly. To simplify the testing, RT-SCED is not included in the study.
3) After each commitment run, an economic dispatch is run by freezing the commitment variables and removing the headroom requirements. The same LAC code is used to perform the multi-interval co-optimized economic dispatch. The commitment and dispatch cost from the first interval is summed up from all cases for comparison.

The same set of transmission constraints from production cases are used for the study. The purpose of the study is to get an estimation of the impact of robust/unified approaches on production cost and violation cost.

The study was run on a 64-bit desktop with Intel Core i7-3770CPU @ 3.4GHz and 16GB RAM. LAC is built on AIMMS 3.12.11 with CPLEX 12.5.

Table 1 compares the solution time. For this study, the maximum number of iterations is set to 2. For the robust approach, 54 out of the 96 cases converged within two iterations. For the unified approach, 71 out of the 96 cases converged within two iterations.

Table 1 Comparison of Solution Time

	Average Optimization Solution Time (sec.)				
	Total	Master 1	Sub 1	Master 2	Sub 2
Deterministic	26	-	-	-	-
Robust	225	33	25	141	26
Unified	121	33	28	41	26

Table 2 compares the total commitment and dispatch cost from the first intervals of the economic dispatches. Both the robust approach and the unified approach result in higher commitment cost than the deterministic model. However, the dispatch costs are reduced with the additional commitment. Since robust optimization considers the dispatch cost for the worst case scenarios, its dispatch cost reduction is higher. Unified approach minimizes the worst-case violation cost. But the violation from the sub problem under this approach is corresponding to zero incremental energy and reserve costs. The feasibility cut will ensure minimum violation when energy and reserves are free to dispatch. However, when the worst case scenario happens, the dispatch cost based on earlier unified commitment may not be low. Moreover, the actual violation could be more than what was anticipated in the worst case scenarios in the earlier unified model. Sometimes it might require additional commitment. For this particular day, the commitment and dispatch cost from unified approach is higher than the robust approach and the deterministic approach. The robust approach results in the lowest total cost.

Table 2 Comparison of Commitment and Dispatch Cost

	Sum of 1st Interval Costs ($)		
	Total	Commitment	Dispatch
(Robust increase from Deterministic)%	-0.09%	1.13%	-0.33%
(Unified increase from Deterministic)%	0.05%	1.26%	-0.19%

Table 3 compares the violations. There are some spinning reserve scarcity and transmission violations from the deterministic run. Robust approach results in the least amount of violations. Under unified approach, even though the objective is to minimize the worst-case violation, the worst-case violations are purely feasibility cuts. They are equivalent to the violations under zero incremental energy and reserve offers. Hence, even if the unified approach may result in no violation within the uncertainty set, it may still have violation under the economic dispatch run when the incremental energy and reserve offers are not zero. It just indicates that if the demand curves for reserves and transmission constraints have high enough prices, there is room for clearing more reserves or reliving more transmission violations. It is similar to using headroom under the deterministic model. The headroom capacity may be cleared on resources with expensive incremental energy or reserve offers. When the system has scarcity, the headroom may be too expensive for relieving the violations. Furthermore, the headroom from the deterministic model may be clear on the wrong side of the congestion.

Table 3 Comparison of Violations

	Sum of Violations from the 1st Intervals of 96 Cases	
	Spin Violations (MWh)	Transmission Violations (MWh)
Deterministic	87.56	171.04
Robust	1.69	130.00
Unified	34.95	130.38

C. Discussion

From MISO case study, it indicates that the solution time increases significantly with the addition of new scenarios under the robust optimization approach even though there are only a very small number of unit commitment changes. For MISO sized system, it is currently not practical to solve the master problem with multiple scenarios within the given time limits by calling CPLEX directly. For LAC, each additional scenario results in adding a new set of continuous variables $\boldsymbol{x}$ and constraints $\Omega(\boldsymbol{x}, \boldsymbol{d_j})$ for the new scenario under the profile $\boldsymbol{d_j}$. More research is needed to identify the reasons behind slow performance cases and to develop better strategies to solve the master MIP problem with multiple scenarios.

It is more interesting to apply robust or unified approach in RAC where there are more uncertainties and more future actions at the second stage. The future actions include commitment of fast start resources that introduces binary variables in the second stage. Hence, it is a much more challenging problem to solve.

IV. Conclusion

This paper reviews current practice of managing uncertainties at MISO. A framework of applying robust optimization based approach on MISO Look-Ahead commitment (LAC) is then introduced. Studies on MISO cases indicate that the proposed robust and unified approaches are promising and can potentially improve system reliability. However, there are still computational challenges to overcome in order to implement the framework in MISO production.

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