A Two-Stage Robust Optimization for PJM
Look-Ahead Unit Commitment

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Abstract—Robust optimization recently becomes a state-of-the-art approach to solve decision-making under uncertainty problems in the power system operations. To better quantify and highlight the significance of the robust optimization for reliable unit commitment runs, PJM and Alstom Grid have collaborated to develop a two-stage robust optimization (TSRO) prototype since 2012. In this paper, we present a computational tractable TSRO framework for the PJM Look-Ahead Unit Commitment (LAUC) with the consideration of load uncertainty. Instead of only covering limited number of scenarios in the uncertainty set, TSRO provides a robust solution that immunizes all possible scenario realizations. Linear decision rule (LDR) and two-stage decomposition approaches are considered respectively to solve TSRO in this research. We test the scalability and sensitivity of the proposed models and algorithms with the PJM market data. Finally, the computational results indicate that the proposed TSRO framework provides sufficient ramping capability and improves the security of the large-scale power grid system.

Index Terms—Decomposition Algorithm, Look-Ahead Unit Commitment, Mixed-Integer Programming, Load Uncertainty, Two-Stage Robust Optimization.

I. INTRODUCTION

Look-Ahead Unit Commitment (LAUC) has been carried out widely in several large wholesale electricity markets in the United States recently. On June 9th 2010, PJM Look-Ahead Security Constrained Economic Dispatch (LA-SCED) went live. In PJM LA-SCED, the LA commitment employs a two hours look-ahead fast-start unit commitment; the LA dispatch couples multi-interval SCED. Similar proposals and promotions of LAUC can be found in Midwest ISO [1], ISO-New England [2], and Southwest Power Pool [3] market systems. One of the most primary operational challenges for LAUC is the uncertainty management. The current industry practice is to solve LAUC with the consideration of several uncertain scenarios specifically. The operator picks commitment recommendations from one or more scenarios. The major drawback of this process is that the solution quality of LAUC largely depends on the operator’s preference of the scenarios. Thus, it is necessary and favorable to consider robust optimization for LAUC to achieve cost effectiveness while ensuring system reliability.

Robust optimization has been recently studied to tackle the uncertainty in power system operations. For example, Street et al. [4] propose a robust optimization framework for the contingency-constrained unit commitment. Baringo et al. [5] study a bidding strategy for a price-taking producer via the robust mixed-integer linear programming approach. In [6], a robust optimization approach is presented to handle parameter uncertainties such as price, growth rate of light-duty vehicle, etc., for the sustainable integration of PHEVs into an electric grid. A close related work of this paper is the two-stage robust optimization (TSRO) framework proposed in [7], [8] and [9] to obtain an “immunized against uncertainty” unit commitment solution to accommodate uncertain load or renewable power generation. More recently, TSRO is proposed in [10] to study the contingency-constrained unit commitment incorporating transmission network constraints. The computational results demstrate the efficiency of the two-stage decomposition algorithm for the TSRO framework. To better quantify and highlight the significance of robust optimization in uncertainty management, PJM and Alstom Grid have collaborated to develop a TSRO prototype to study LAUC for PJM since 2012. Linear decision rule (LDR) [11] and two-stage decomposition approaches [7], [8], [9], [10] are studied respectively to solve TSRO.

The contributions of this paper are highlighted as follows:

1) We propose a two-stage robust optimization prototype for LAUC for PJM. The significance of robust optimization is investigated for a real-world large-scale power grid system with computational experiments.

2) Compared with the literature in robust unit commitment, both LDR and two-stage decomposition are studied in this paper to solve TSRO. We also provide a comparison of these two approaches.

3) Compared with the decomposition framework in [7] and [9], we consider an improved bilinear heuristic algorithm as shown in [8] to solve the sub-problem effectively. And we use a primal decomposition instead of the traditional Bender’s dual decomposition to achieve better performance.

The remainder of this paper is organized as follows: Section II describes a TSRO framework; Section III proposes two approaches to solve TSRO; Section IV provides and analyzes case studies for the PJM LAUC problem; finally, Section V makes concluding remarks on this research.
II. MATHEMATICAL FORMULATION

In our LAUC TSRO formulation, the unit commitment decisions are the first stage decision variable; the economic dispatch decisions are the second stage variable so that it is a function of the uncertain load. In the first stage, we impose the unit physical constraints (e.g., start-up/shut-down, min up/down-time constraints). In the second stage, dispatch constraints (e.g., load balance, reserve limits, transmission line flow limits) and coupling constraints for commitment and dispatch decisions (e.g., generation upper/lower limits, ramping-up/down limits) are enforced.

There are different techniques to model the robust unit commitment problem. To simplify our discussions, we use the following compact formulation from [7] throughout this paper.

\[
\min_{x,y} (c^T x + \max_{d \in D} b^T y(d)) \\
\text{s.t. } Fx \leq f, \\
Hy(d) \leq h(d), \forall d \in D, \\
Ax + By(d) \leq g, \forall d \in D, \\
Iy(d) = d, \forall d \in D,
\]

where \(D\) represents the deterministic uncertainty set, \(x\) and \(y\) are unit commitment decisions and economic dispatch decisions, respectively.

In this paper, uncertain loads are considered to be within certain ranges. Accordingly, a basic uncertainty set can be described as follows:

\[D_0 = \{d_{it} : D_{il}^l \leq d_{it} \leq D_{iu}^u, \forall t, \forall i \in N\},\]

where \(d_{it}\) are the uncertain loads and \(D_{il}^l\) and \(D_{iu}^u\) are the corresponding lower/upper bounds. Besides constructing the regular deterministic uncertainty set, it is a common practice to add uncertainty budget constraint into the robust optimization problem to control conservativeness of the model. Polyhedra and cardinality budget constraints are two typical constraints widely applied in the literature [8]. In order to control the conservativeness of the problem, we propose a cardinality budget constraint as shown below in this research.

\[
\sum_{i=1}^{N} \sum_{t=1}^{T} \left| \frac{d_{it} - \bar{d}_{it}}{\bar{d}_{it}} \right| \leq \Gamma,
\]

where \(\bar{d}_{it}\) are the forecasted mean loads and \(\bar{d}_{it}\) are the load deviations.

After introducing a set of binary variables \(z\), we can reformulate (7) with the following mixed-integer programming:

\[
\sum_{i=1}^{N} \sum_{t=1}^{T} z_{it} \leq \Gamma, \\
z_{it} \geq \frac{d_{it} - \bar{d}_{it}}{\bar{d}_{it}}, \\
z_{it} \geq \frac{\bar{d}_{it} - d_{it}}{\bar{d}_{it}}, \\
z_{it} \in \{0,1\}.
\]

III. SOLUTION METHODOLOGY FOR TSRO

In this section, we provide two solution methods to solve the TSRO formulation: LDR and two-stage decomposition algorithms. Note here that LDR is embedded in the current AIMMS software. Therefore, it is very easy to test its performance. That is also the reason we report its performance in this paper.

A. LDR Approach

In LDR, the adjustable variables (i.e., dispatch decision variables) are assumed to be an affine function of the uncertain loads [11]. Therefore, TSRO can be reformulated into a single stage optimization problem. In the remaining part of this subsection, we describe the LDR approach as shown in the Theorem 3.2 in [11]. To apply LDR, we first recast TSRO as follows:

\[
\min_{x,y} (c^T x + Q) \\
\text{s.t. } Q - b^T y(d) \geq 0, \forall d \in D, \\
(2) - (5).
\]

Note that \(Iy = d\) can be substituted by the pair \(Iy \geq d\) and \(-Iy \geq -d\). And \(Q\) can be considered as a first-stage decision variable into the collection \(x\). Therefore, a more general formulation can be achieved as follows:

\[
\min_{x,y} c^T x \\
\text{s.t. } Ax + Uy(d) \geq u(d), \forall d \in D.
\]

After applying LDR, \(y(d)\) can be expressed as an affine function of the uncertain loads \(d\). Thus, we reach the following equivalent reformulation.

\[
\min_{x} c^T x \\
\text{s.t. } Ax + Vd \geq v, \forall d \in D.
\]

Here, we assume \(D\) is a box (i.e. \(D = D_0 = \{D^l \leq d \leq D^u\}\)) to simplify the discussion. To guarantee the feasibility of (17), we want to assure that \(\text{Opt}_i \geq 0, \forall i\), where \(\text{Opt}_i\) is defined as:

\[
\text{Opt}_i = \min_{d} (Ax - v)^i + V^id
\]

\text{s.t. } D^l \leq d \leq D^u.

Let \(\text{Dopt}_i\) denote the optimal objective function in the dual problem of the above optimization. By the strong duality theorem, \(\text{Opt}_i \geq 0\) if and only if \(\text{Dopt}_i \geq 0\), where \(\text{Dopt}_i\) is defined as:

\[
\text{Dopt}_i = \max_{\lambda, \lambda^*} (Ax - v)^i + \lambda^l_i D^l - \lambda^u_i D^u
\]

\text{s.t. } \lambda^l_i - \lambda^u_i = V^i.

Note that \(\text{Dopt}_i \geq 0\) if and only if

\[
\exists (\lambda^l_i, \lambda^u_i) : \lambda^l_i - \lambda^u_i = V^i, (Ax - v)^i + \lambda^l_i D^l - \lambda^u_i D^u \geq 0.
\]

Now we conclude that the adjustable robust optimization counterpart can be reformulated as follows:

\[
\min_{x} c^T x \\
\lambda^l_i - \lambda^u_i = V^i, \forall i \\
(Ax - v)^i + \lambda^l_i D^l - \lambda^u_i D^u \geq 0, \forall i.
\]
Finally, it is worth noting that when the polyhedra budget constraints are imposed, the above steps hold without loss of generality by changing the dual problem formulation.

**B. Two-Stage Decomposition Approach**

Alternatively, we propose a two-stage decomposition approach which solves the original TSRO iteratively.

1) Master problem: unit commitment is considered to be the master problem in our decomposition framework. In the first iteration, we set the load at the nominal level and solve a deterministic unit commitment to obtain the starting point for the whole algorithm. The solution from the master problem is used for the sub-problem in the second stage. At the beginning of each iteration, the master problem is solved again with an additional set of cuts.

2) Sub-problem: the sub-problem aims to solve the economic dispatch problem under the worst-case load scenario with the fixed unit commitment decisions. The solution of the sub-problem discovers the worst-case scenarios which are used to generate the cuts.

According to linear programming and duality theory, we can transform the sub-problem from a max-min programming into a single maximization problem. Readers are referred to (7) – (9) in [7] for this reformulation process. This maximization problem is a nonlinear programming problem composed of a bilinear objective function and linear constraints. To solve such a bilinear programming problem effectively for large-scale problems, we propose a bilinear heuristic algorithm as described in [8]. By fixing different sets of decision variables in the bilinear programming approach, we obtain the following two sub-problems based on equation (9) in [7].

\[
\text{SUB}^1 : \max_{\lambda, \eta} \lambda^T (Ax - g) - \varphi^T h + \eta^T d^* \\
\text{s.t.} \ -\lambda^T B - \varphi^T H + \eta^T I = b^T, \quad \varphi \geq 0, \lambda \geq 0, \eta \text{ free.} \\
\]

\[
\text{SUB}^2 : \max_d \lambda^T (Ax - g) - \varphi^T h + \eta^T d \\
\text{s.t.} \ -\lambda^T B - \varphi^T H + \eta^T I = b^T, \quad d \in D. \\
\]

The heuristic algorithm for the bilinear programming approach is described as follows.

1) Pick an extreme point \(d^* \in D\).
2) Solve \(\text{SUB}^1\) with \(d^*\) and store the objective value as \(\omega_1(y, d)\).
3) Solve \(\text{SUB}^2\) with the dual variable value obtained from step 2, store the objective value as \(\omega_2(\varphi, \lambda, \eta)\).
4) If \(\omega_2(\varphi, \lambda, \eta) > \omega_1(y, d)\), go to step 2, otherwise stop.

When applying this algorithm to the real-world large-scale market software system, a significant bottleneck is to build the dual model \(\text{SUB}^2\). And such a dual model is very vulnerable with any changes of the primal model with thousands of constraints and variables. To improve this bilinear heuristic algorithm, we design a modified algorithm to avoid formulating the dual model. Instead of solving \(\text{SUB}^1\), we consider the dual of \(\text{SUB}^1\), which turns out to be the original primal sub-problem with fixed load and first-stage decision variables:

\[
\text{DSUB}^1 : \min_{y \in \Omega(x, d^*)} b^T y 
\]

where \(\Omega(x, d^*) = \{y : Hy \leq h, Ax + By \leq g, Iy = d^*\}\). Now, we can solve the \(\text{DSUB}^1\) instead of \(\text{SUB}^1\) in the second step of the algorithm. According to strong duality theorem, optimal objective values of \(\text{DSUB}^1\) and \(\text{SUB}^1\) should be equal. And it is easy to acquire the dual solutions (i.e., shadow prices of the constraints) from the optimization solver as the input for the third step. With this method, we avoid formulating the dual model which is risky in a large-scale software system.

Finally, there is no guarantee that such a heuristic algorithm can obtain the optimal solution of the original bilinear programming. However, this algorithm avoids solving the nonlinear programming and provides a near-optimal solution for large-scale problems.

3) Primal decomposition: We use the primal decomposition approach similar to the ones described in [12] and [10] to generate a group of cuts based on the solution obtained from the sub-problem. Compared with the traditional bender’s dual decomposition approach, the primal decomposition approach generates more constraints in each iteration and might achieve better performance for some problems.

4) Algorithm framework: The complete two-stage decomposition algorithm is summarized in this subsection. After solving the master problem, we fix the solutions of the first-stage decision variables in the coupling constraint. The sub-problem is then solved to discover the worst-case load scenario. After generating the cuts according to the sub-problem solution, we add them back into the master problem for the next iteration. The stopping criterion is met when the objective values obtained from the master and sub-problems are equal.

Fig. 1 provides the flowchart of the algorithm for the two-stage decomposition approach.

**IV. Case Studies with PJM Market Data**

In this section, we run the computational experiments for the proposed robust LAUC. All the models and algorithms are implemented in AIMMS 3.11. The mixed-integer programs are solved by CPLEX 12.2. We first compare TSRO approach with the deterministic strategy to address load uncertainty. We investigate the performances of the AIMMS’ embedded LDR and our developed two-stage decomposition approaches through testing different deviations and uncertainty parameters. Finally, we conduct experiments to study the cost of robustness via the cardinality budget constraint within the two-stage decomposition framework.

There are totally around 2000 generation resources available in our case studies for PJM market. 200 of them are fast-start units which can be committed/decommitted by LAUC. 4 look-ahead intervals and 8 zones are considered.

**A. LDR Approach**

1) Uncertainty Set Description: We first consider a simple \(D_0\) for the case study with LDR. The load uncertainty of
2) **Deterministic vs. TSRO:** One deterministic strategy for the system operator to handle load uncertainty is to run deterministic LAUC with different scenarios and pick the commitment recommendations. Based on the uncertain load profile, the system operator usually prefers to look into “High” and “Low” cases with the reliability concern. Because the “High” and “Low” scenarios are probably the worst-case or at least near-worst-case scenarios. Then an investigation of Norm case is necessary to consider system economics. We follow this deterministic process to run three scenarios (i.e., “Norm”, “Low”, and “High”) and compare them with TSRO in Table I.

### TABLE I

**Optimal Objective Values of Three Deterministic Scenarios and TSRO**

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Optimal Obj. ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic Model (Norm)</td>
<td>7,110,998.91</td>
</tr>
<tr>
<td>Deterministic Model (High)</td>
<td>8,115,738.09</td>
</tr>
<tr>
<td>Deterministic Model (Low)</td>
<td>6,363,717.40</td>
</tr>
<tr>
<td>Robust Model</td>
<td>8,311,788.86</td>
</tr>
</tbody>
</table>

It can be observed that neither “High” nor “Low” is the worst-case scenario with this load profile. The robust optimization approach identifies the worst-case and provides a more reliable solution to immunize uncertainties. To better illustrate it, we compare the unit commitment solution of the robust optimization approach with those of “High” and “Low” in Fig. 3. In this figure, “RO” means the robust optimization; “HL” and “LL” mean high load and low load scenarios respectively. We find that most units are shut down in the third period and plenty of units are started up in the fourth period for the robust optimization approach. The reason behind this observation is that the worst-case scenario of this load profile happens when the load drops to valley in the third period and rise to peak in the fourth period. Such a dramatic load increment calls for more start-up/shut-down operations and thus incurs the highest cost among others.

![Unit commitment solution comparisons](image-url)
commitment solution. It can be observed that LDR takes more time as the uncertainty deviation increases. Two-stage decomposition has a better performance and the computational time is not significantly affected by uncertainty deviations. We provide another comparison from the perspective of uncertainty numbers (or budget). In Table III, we observe a similar phenomenon. Increasing the conservativeness of the uncertainty set does not lead to an obvious performance difficulty. There are two reasons for the promising results of the two-stage decomposition approach performance. First, both deviations and budget numbers only affect the sub-problem computational complexity in the decomposition framework. More precisely, they only affect solving SUB, which is a simple linear program, in our heuristic algorithm. Different deviations make slight difference on the feasible regions of SUB. And different uncertainty numbers impose different variable numbers of SUB. But none of them makes SUB significantly more complicated to solve. Second, the performance of the two-stage decomposition approach largely relies on the number of iterations (i.e., convergence rate). In these experiments, all decomposition algorithms converge in three iterations.

<table>
<thead>
<tr>
<th>Uncertainty Dev.</th>
<th>Time (LDR)</th>
<th>Time (Decomposition)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%, 2%, 3%, 5%</td>
<td>160.35s</td>
<td>2.14s</td>
</tr>
<tr>
<td>1%, 5%, 8%, 10%</td>
<td>386.01s</td>
<td>2.34s</td>
</tr>
<tr>
<td>1%, 5%, 10%, 15%</td>
<td>323.80s</td>
<td>2.17s</td>
</tr>
</tbody>
</table>

C. Cost of Robustness

We report the cost of robustness by controlling the conservativeness in the cardinality budget constraint for the proposed TSRO model. The worst-case scenario objective values are reported in Table IV. Two-stage decomposition approach is applied to solve all the cases here and terminates within four iterations. We observe that the operations cost increases as the number of possible load uncertainties increases. The system operators can adjust the number of uncertainty with their preferences to find tradeoff between system economics and robustness.

<table>
<thead>
<tr>
<th>Uncertainty No.</th>
<th>Obj.</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7110998</td>
<td>0.31s</td>
</tr>
<tr>
<td>4</td>
<td>7408009</td>
<td>2.19s</td>
</tr>
<tr>
<td>8</td>
<td>7657384</td>
<td>3.28s</td>
</tr>
<tr>
<td>12</td>
<td>8000696</td>
<td>3.46s</td>
</tr>
<tr>
<td>20</td>
<td>9310763</td>
<td>4.49s</td>
</tr>
</tbody>
</table>

V. Conclusion and Discussions

In this paper, a TSRO model is proposed to study LAUC for the PJM market. The LDR and two-stage decomposition approaches are studied respectively to analyze the computational complexity of TSRO. By substituting the second-stage (adjustable) variables into affine function of uncertain parameters, LDR reformulates TSRO into a deterministic equivalent optimization problem. Two-stage decomposition approach is developed to solve the original TSRO directly. The master problem is the unit commitment. The sub-problem solves economic dispatch under the worst-case scenario. We use the primal decomposition approach after the sub-problem screens the worst-case scenario. However, the bilinear sub-problem is difficult to solve to optimum for large-scale problems. The bilinear heuristic we applied can solve the problem efficiently and near to optimal. Our case study results demonstrate the impact and significance of the robust optimization approach for reliability concerns in real-world power system operations. The efficiency of the proposed algorithms is verified by our case study results.

Note here that in our experiment, we used the default AIMMS setting for the LDR approach (without any decomposition algorithm involved), as a benchmark for our TSRO approach. It is worth mentioning here that the LDR reformulation has a similar problem structure as the deterministic unit commitment problem. There is great potential to develop and implement efficient decomposition and/or heuristic approaches, so as to significantly improve the LDR performance to achieve a similar or even better computational performance, as compared to the TSRO approach does. In future research, we will explore along this direction.

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REFERENCES


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