Two-Stage Robust Optimization for N-k Contingency-Constrained Unit Commitment

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***Abstract*—This paper proposes a two-stage robust optimization approach to solve the N-k contingency-constrained unit commitment (CCUC) problem. In our approach, both generator and transmission line contingencies are considered. Compared to the traditional approach using a given set of components as candidates for possible failures, our approach considers all possible component failure scenarios. We consider the objectives of minimizing the total generation cost under the worst-case contingency scenario and/or the total pre-contingency cost. We formulate CCUC as a two-stage robust optimization problem and develop a decomposition framework to enable tractable computation. In our framework, the master problem makes unit commitment decisions and the subproblem discovers the worst-case contingency scenarios. By using linearization techniques and duality theory, we transform the subproblem into a mixed-integer linear program (MILP). The most violated inequalities generated from the subproblem are fed back into the master problem during each iteration. Our approach guarantees a globally optimal solution in a finite number of iterations. In reported computational experiments, we test both primal and dual decomposition approaches. Our computational results verify the effectiveness of our proposed approach.**

***Index Terms*—Contingency Analysis, N-k Security Criterion, Unit Commitment, Robust Optimization**

NOMENCLATURE

Sets and Parameters

I	Index set of buses
E	Index set of transmission lines
T	Index set of time (e.g., 24 hours)
Λ	Set of all generators
Λ_i	Set of generators at bus i
$\hat{Z}$	Set of all possible contingencies
N	Number of components (e.g., generators and transmission lines) in the power system
R	Number of points selected in a power generation cost curve for piecewise linear approximation
M	Big M: a very large number
P	Unit power balance penalty cost
F''_g	Start-up cost of generator g
F'_g	Shut-down cost of generator g
H''_g	Minimum up time for generator g
H'_g	Minimum down time for generator g
R''_g	Ramp-up rate limit for generator g
R'_g	Ramp-down rate limit for generator g
L_g	Lower limit of generator g's power output
U_g	Upper limit of generator g's power output
θ''_i	Maximum value of the phase angle at bus i
θ'_i	Minimum value of the phase angle at bus i
f''_{ij}	Maximum power flow on transmission line (i,j)
f'_{ij}	Minimum power flow on transmission line (i,j)
x_{ij}	Reactance of transmission line (i,j)
$G_g(q)$	Fuel cost for generator g when its power output is q
D_{it}	Load at bus i at time t
q^r_g	The r^{th} point in the piecewise linear approximation of power output by generator g
O_t	Spinning reserve requirement for the power system at time t

Decision Variables

q_{gt}	Power output by generator g at time t
y_{gt}	Binary decision variable: "1" if generator g is on at time t; "0" otherwise
u_{gt}	Binary decision variable: "1" if generator g is started up at time t; "0" otherwise
v_{gt}	Binary decision variable: "1" if generator g is shut down at time t; "0" otherwise
z_g	Binary decision variable: "0" if generator g is under contingency; "1" otherwise
z_{ij}	Binary decision variable: "0" if transmission line (i,j) is under contingency; "1" otherwise
θ_{it}	Phase angle at bus i at time t
f^t_{ij}	Power flow on transmission line (i,j) at time t
λ^r_{gt}	Weight associated with the r^{th} point q^r_g in the piecewise linear approximation of power output by generator g at time t
d_{it}	Power imbalance amount for bus i at time t
$\hat{\mathcal{Q}}$	Auxiliary variable to represent the optimal objective value for the subproblem

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I. INTRODUCTION

Reliability is a primary concern in power grid operations. Unexpected outages of power grid elements, such as transmission lines and generators, can result in dramatic electricity shortages or even large scale blackouts (cf. [1], [2], [3], and [4]). The well-known N-1 and N-2 security criteria are implemented in industry practice (cf. [5] and [6]). These criteria have also been generalized to consider multiple contingency cases (e.g., N-k criterion [7]). With the N-k rule, a power grid with N components will continue to meet load whenever any k or fewer components suffer a contingency. If unit commitment decisions are involved, the corresponding problem is defined as the contingency-constrained unit commitment (CCUC) problem in the literature (see, e.g., [8], [9], and [10]). For CCUC, some papers consider only generation unit contingencies, such as [8] and [9], while others consider generation unit and transmission line contingencies, such as [10]. The latter case further enhances the reliability of the power grid system as a whole. In principle, CCUC ensures post-contingency energy balance under different contingency scenarios while providing economic UC scheduling and generation dispatch. Due to the computational challenges involved in considering all possible contingency scenarios, the traditional approach relies on identification of a credible contingency set of generators and transmission lines, selected according to power engineering expertise and industry practices. Under a given credible contingency set scheme, different solution approaches and objectives have been proposed. For the solution approach described in [9], all possible contingency scenarios for the components in the credible contingency set are considered. For the security-constrained unit commitment model with joint energy and ancillary services auction described in [10], instead of checking all possible contingency scenarios, a Benders' decomposition approach is proposed. Contingencies are simulated and checked in the Benders' decomposition framework. The proposed algorithm generates Benders' cuts when the network security constraints are violated due to a contingency. Different optimization objectives have also been studied. In [9], the optimization objective is to minimize the total expected cost with each contingency scenario assigned with a predefined probability. In [8] and [10], the optimization objectives are to minimize the sum of pre-contingency dispatch cost and the cost of spinning and non-spinning operating reserve (note here that pre-contingency dispatch cost indicates the cost under the zero-contingency scenario).

In this paper, we consider the N-k contingency-constrained unit commitment problem in which both generator and transmission line contingencies are considered. Our main contribution is to apply robust optimization to detect a set of k critical components that will generate the worst-case contingency scenario, as opposed to using enumeration or a given credible contingency set.

One related work is described in [11]. In this paper, contingencies for both generators and transmission lines are considered. Although the study is focused on the co-optimization of multi-period unit commitment and transmission switching problem, the paper makes a great effort to describe how to incorporate the N-1 security criterion into their formulation. In the approach proposed in [11], a full credible contingency set for the N-1 security criterion is provided for contingency analysis. Meanwhile, due to incorporating the transmission switching options, additional binary switching decisions are introduced. In our approach, we extend this study (without considering transmission switching) to consider the N-k security criterion, and apply decomposition approaches with a separation algorithm embedded to detect the most critical k components, within a two-stage robust optimization framework. This allows for smaller subproblems, allowing us to obtain optimal solutions to larger problems.

The other related work is described in [12], in which the first robust optimization approach is introduced to solve CCUC with the N-k security criterion. Bilevel programming is applied to address the problems of unit commitment and robust contingency analysis. This approach allows a system operator to consider all possible contingency combinations of k out of N generators (transmission line contingencies are not considered); the reported simulation results are very promising. In this paper, we generalize the work in [12] in the following two directions:

1) We extend the study in [12] to include transmission capacity constraints and to consider transmission line contingencies. That is, we include transmission capacity physical constraints in the model, and consider both generator and transmission contingencies.
2) In [12], spinning and nonspinning reserves are adjusted to ensure system reliability under contingency for a single-bus case. In this paper, due to inclusion of transmission capacity constraints for a multi-bus system, we consider economic redispatch to satisfy post-contingency physical constraints.

We provide a two-stage robust optimization formulation for the problem. The mathematical formulation is a min-max-min optimization through two stages. Under the N-k security criterion, for N elements, there are $\sum_{i=1}^{k}\binom{N}{i}$ possible contingencies. Our method can provide a robust optimal unit commitment schedule for system operators for a multi-bus power grid under the N-k security criterion. Note that similar robust optimization concepts have been applied successfully to solve other power system operation problems. For instance, in [13], [14], [15], and [16], robust unit commitment models are studied to ensure system robustness under load and wind power output uncertainties, among which [13], [15], and [16] provide Benders' dual cut approaches, and [14] introduces a primal cut approach. In [17], a price-taking producer offering strategy in a pool-based market is studied by introducing a robust mixed-integer linear programming approach, and confidence intervals are used to describe the uncertain price. In [18], robust optimization is applied to the problem integrating PHEVs into the electric grid to accommodate the most relevant planning uncertainties.

To solve our proposed robust optimization problem, we first reformulate the proposed two-stage robust optimization problem into a deterministic equivalent formulation. This reformulation yields a large scale MILP. To achieve computational tractability, we develop a decomposition framework to solve the MILP iteratively. In our decomposition framework, to solve each subproblem (which is a max-min programming problem), we apply linearization techniques and use duality theory to reformulate the subproblem as a single MILP. In addition, we apply both primal and dual decomposition approaches and compare performances. For the dual approach, we solve the subproblem and obtain a valid Benders' cut that can be added to the master problem. For the primal approach (or scenario-based robust optimization approach), we follow an approach similar to the ones described in [14] and [19].

After solving the subproblem, we identify the most violated contingency scenario and add the corresponding constraints to the master problem. Accordingly, the master CCUC problems proceed in each iteration with the violated Benders' cut (or dual inequality) added for the dual approach or a group of inequalities corresponding to a specific contingency scenario (or primal inequalities) added for the primal approach until the stopping criterion is satisfied. Our decomposition framework provides solutions that converge to the optimal one of the original two-stage robust optimization formulation.

The remainder of this paper is organized as follows. Section II describes our two-stage robust optimization formulation for the N-k CCUC problem, with different objectives: 1) to minimize the total cost under the worst-case contingency scenario, 2) to minimize the total pre-contingency cost, and 3) to minimize the total weighted cost by putting weights for the pre-contingency cost and the worst-case cost. In Section III, we introduce an extended formulation that captures all possible contingencies. We also develop a decomposition framework to solve the problem. In the decomposition framework, we apply primal and dual approaches to generate inequalities for the master problem. Section IV provides and analyzes the computational experiments through several case studies. Finally, Section V summarizes the research contribution.

II. Mathematical Formulation

In our formulation, unit commitment decisions are made in the first stage and economic dispatch is considered in the second stage under the condition of the worst-case contingency scenario. A mathematical formulation of this problem is given as follows:

$$\min_{\{u,v,y\}} \sum_{t\in T}\sum_{g\in\Lambda}(F''_g u_{gt} + F'_g v_{gt}) + \max_{z\in\hat{Z}} Q(z,u,v,y) \tag{1}$$

$$s.t.$$

$$-y_{g(t-1)} + y_{gt} - y_{gk} \le 0, \quad \forall g\in\Lambda, \forall t\in T, \forall k: 1\le k-(t-1)\le H''_g \tag{2}$$

$$y_{g(t-1)} - y_{gt} + y_{gk} \le 1, \quad \forall g\in\Lambda, \forall t\in T, \forall k: 1\le k-(t-1)\le H'_g \tag{3}$$

$$-y_{g(t-1)} + y_{gt} - u_{gt} \le 0, \quad \forall g\in\Lambda, \forall t\in T \tag{4}$$

$$y_{g(t-1)} - y_{gt} - v_{gt} \le 0, \quad \forall g\in\Lambda, \forall t\in T \tag{5}$$

$$\sum_{g\in\Lambda} U_g y_{gt} \ge O_t + \sum_{i\in I} D_{it}, \quad \forall t\in T \tag{6}$$

$$y_{gt}, u_{gt}, v_{gt} \in \{0,1\}, \quad \forall g\in\Lambda, \forall t\in T, \tag{7}$$

where

$$\hat{Z} = \left\{ (z_g, z_{ij}) \in \{0,1\} : \sum_{g\in\Lambda} z_g + \sum_{(i,j)\in E} z_{ij} \ge N-k \right\}, \tag{8}$$

and $Q(z,u,v,y)$ in the objective function is defined to be the minimum redispatch cost during the second stage after the worst-case contingency happens (i.e., (z_g, z_{ij}) are given). It can be observed that $Q(z,u,v,y)$ is equal to

$$\min \sum_{t\in T}\sum_{g\in\Lambda}\sum_{r=1}^{R} \lambda^r_{gt} G_g(q^r_g) + \sum_{t\in T}\sum_{i\in I} P d_{it} \tag{9}$$

$$s.t.\ z_g y_{gt} L_g \le q_{gt} \le z_g y_{gt} U_g, \quad \forall g\in\Lambda, \forall t\in T \tag{10}$$

$$z_{ij} f'_{ij} \le f^t_{ij} \le z_{ij} f''_{ij}, \quad \forall (i,j)\in E, \forall t\in T \tag{11}$$

$$\theta'_i \le \theta_{it} \le \theta''_i, \quad \forall i\in I, \forall t\in T \tag{12}$$

$$(\theta_{it} - \theta_{jt})/x_{ij} - f^t_{ij} + (1 - z_{ij})M \ge 0, \quad \forall (i,j)\in E, \forall t\in T \tag{13}$$

$$(\theta_{it} - \theta_{jt})/x_{ij} - f^t_{ij} - (1 - z_{ij})M \le 0, \quad \forall (i,j)\in E, \forall t\in T \tag{14}$$

$$q_{gt} - q_{g(t-1)} \le (2 - y_{g(t-1)} - y_{gt})L_g + (1 + y_{g(t-1)} - y_{gt})R''_g, \quad \forall g\in\Lambda, \forall t\in T \tag{15}$$

$$q_{g(t-1)} - q_{gt} \le (2 - y_{g(t-1)} - y_{gt})L_g + (1 - y_{g(t-1)} + y_{gt})R'_g, \quad \forall g\in\Lambda, \forall t\in T \tag{16}$$

$$-d_{it} \le \sum_{\forall j\in E_{(\cdot,i)}} f^t_{ji} - \sum_{\forall j\in E_{(i,\cdot)}} f^t_{ij} + \sum_{g\in\Lambda_i} q_{gt} - D_{it} \le d_{it}, \quad \forall i\in I, \forall t\in T \tag{17}$$

$$\sum_{r=1}^{R} \lambda^r_{gt} = y_{gt}, \quad \forall g\in\Lambda, \forall t\in T \tag{18}$$

$$\sum_{r=1}^{R} \lambda^r_{gt} q^r_g = q_{gt}, \quad \forall g\in\Lambda, \forall t\in T \tag{19}$$

$$q_{gt}, \lambda^r_{gt}, d_{it} \ge 0, z_g, z_{ij} \in \{0,1\}, \quad \forall i\in I, \forall (i,j)\in E, \forall g\in\Lambda, \forall t\in T. \tag{20}$$

In the above formulation, the objective function (1) is to minimize the total cost including unit commitment, economic dispatch, and power balance penalty costs under the worst-case contingency scenario. The unit commitment constraints include minimum up/down time and start-up/shut-down constraints through (2–5). In addition, we consider system reserve requirement constraints (6) (note here that we do not consider other operating reserve constraints, following the approach shown in [11], because we explicitly enforce the N-k security criterion and the primary purpose of reserve is to ensure enough generation capacity online to survive contingencies), unit generation upper and lower limit constraints (10), transmission capacity constraints (11), phase angle upper and lower limit constraints (12), power flow constraints (13) and (14) (e.g., if a contingency occurs, these big-M constraints are redundant; otherwise both are activated and identical to the line flow constraint), ramping constraints (15) and (16), and power balance constraints (17). Finally, constraints (18) and (19) implement piecewise linear representations of the generation cost curve using the interpolation method described in [20]. Accordingly, in (9), the second-stage objective function can be written as the sum of the redispatch cost (in the form of a piecewise linear function) and the power balance penalty cost.

Our robust optimization framework can also represent the alternative case of minimizing the total pre-contingency cost. We first describe the pre-contingency feasibility set X^0 as follows:

$$
\begin{aligned}
X^0 = \{(q^0, f^0, \theta^0, \lambda^0):\\
& y_{gt}L_g \le q^0_{gt} \le y_{gt}U_g, \quad \forall g \in \Lambda, \forall t \in T && (21)\\
& f'_{ij} \le f^{t0}_{ij} \le f''_{ij}, \quad \forall (i,j) \in E, \forall t \in T && (22)\\
& \theta'_i \le \theta^0_{it} \le \theta''_i, \quad \forall i \in I, \forall t \in T && (23)\\
& (\theta^0_{it} - \theta^0_{jt})/x_{ij} = f^{t0}_{ij}, \ \forall (i,j) \in E, \forall t \in T && (24)\\
& q^0_{gt} - q^0_{g(t-1)} \le (2 - y_{g(t-1)} - y_{gt})L_g +\\
& \quad (1 + y_{g(t-1)} - y_{gt})R''_g, \ \forall g \in \Lambda, \forall t \in T && (25)\\
& q^0_{g(t-1)} - q^0_{gt} \le (2 - y_{g(t-1)} - y_{gt})L_g +\\
& \quad (1 - y_{g(t-1)} + y_{gt})R'_g, \ \forall g \in \Lambda, \forall t \in T && (26)\\
& \sum_{\forall j \in E_{(\cdot,i)}} f^{t0}_{ji} - \sum_{\forall j \in E_{(i,\cdot)}} f^{t0}_{ij}\\
& \quad + \sum_{g \in \Lambda_i} q^0_{gt} = D_{it}, \quad \forall i \in I, \forall t \in T && (27)\\
& \sum_{r=1}^{R} \lambda^{r0}_{gt} = y_{gt}, \quad \forall g \in \Lambda, \forall t \in T && (28)\\
& \sum_{r=1}^{R} \lambda^{r0}_{gt} q^r_g = q^0_{gt}, \quad \forall g \in \Lambda, \forall t \in T && (29)\\
& q^0_{gt}, \lambda^{r0}_{gt} \ge 0, \ \forall i \in I, \forall g \in \Lambda,\\
& \quad \forall (i,j) \in E, \forall t \in T && (30)\\
\},
\end{aligned}
$$

where $(q^0_{gt}, f^{t0}_{ij}, \theta^0_{it}, \lambda^{r0}_{gt})$ represent the values of $(q_{gt}, f^t_{ij}, \theta_{it}, \lambda^r_{gt})$ for the case without contingency.

With the objective of minimizing the total pre-contingency cost, the corresponding model can be updated as follows:

$$
\begin{aligned}
\min_{\{u,v,y\}} \ & \sum_{t \in T}\sum_{g \in \Lambda}\left(F''_g u_{gt} + F'_g v_{gt} + \sum_{r=1}^{R} \lambda^{r0}_{gt} G_g(q^r_g)\right)\\
& + \max_{z \in \hat{Z}} Q(z,u,v,y)\\
s.t. \ & \text{constraints } (2) - (8)\\
& (q^0_{gt}, f^{t0}_{ij}, \theta^0_{it}, \lambda^{r0}_{gt}) \in X^0\\
& Q(z,u,v,y) = \min \sum_{t \in T}\sum_{i \in I} Pd_{it}\\
& \text{constraints } (10) - (20).
\end{aligned}
$$

Furthermore, the above two formulations can be unified by introducing a weight parameter $\alpha, 0 \le \alpha \le 1$, to indicate the portion for the pre-contingency dispatch cost in the objective function, as follows:

$$
\begin{aligned}
\min_{\{u,v,y\}} \ & \sum_{t \in T}\sum_{g \in \Lambda}\left(F''_g u_{gt} + F'_g v_{gt} + \alpha \sum_{r=1}^{R} \lambda^{r0}_{gt} G_g(q^r_g)\right)\\
& + \max_{z \in \hat{Z}} Q(z,u,v,y)\\
s.t. \ & \text{constraints } (2) - (8)\\
& (q^0_{gt}, f^{t0}_{ij}, \theta^0_{it}, \lambda^{r0}_{gt}) \in X^0\\
& Q(z,u,v,y) = \min \ (1-\alpha) \sum_{t \in T}\sum_{g \in \Lambda}\sum_{r=1}^{R} \lambda^r_{gt} G_g(q^r_g)\\
& \qquad + \sum_{t \in T}\sum_{i \in I} Pd_{it}\\
& \text{constraints } (10) - (20).
\end{aligned}
$$

In the later case study section (i.e., Section IV), we will compare the performances of different objectives: minimizing the worst-case cost (i.e., $\alpha = 0$), minimizing the pre-contingency cost (i.e., $\alpha = 1$), and minimizing the total weighted cost, including the sensitivity analysis for different α values.

III. Primal and Dual Decomposition Approaches

In this section, we describe primal and dual decomposition approaches to solve the general unified model. We first present an extended formulation for the problem.

A. An Extended Formulation

As described in (8), there are $\sum_{i=1}^{k}\binom{N}{i}$ contingency scenarios in total with the consideration of the N-k security criterion. However, since we consider the worst-case contingency scenario, any worst k outage case brings the loss no smaller than any worst ℓ, $\ell < k$, outage case. Thus, considering exact k outages is sufficient for N-k in our case. Mathematically, we add $K \equiv \binom{N}{k}$ groups of constraints into the original formulation. For each contingency scenario m, $1 \le m \le K$, we let $z_g(m)$ and $z_{ij}(m)$, in which $\sum_{g \in \Lambda} z_g(m) + \sum_{(i,j) \in E} z_{ij}(m) = N - k$, represent the contingency statuses for generator g and transmission line (i,j), respectively. The corresponding extended formulation can be described as follows:

$$
\begin{aligned}
\min_{\{u,v,y\}} \ & \sum_{t \in T}\sum_{g \in \Lambda}(F''_g u_{gt} + F'_g v_{gt} + \alpha \sum_{r=1}^{R} \lambda^{r0}_{gt} G_g(q^r_g)) + \hat{\mathcal{Q}} \quad (31)\\
& s.t.\\
& \text{constraints } (2) - (7)\\
& (q^0_{gt}, f^{t0}_{ij}, \theta^0_{it}, \lambda^{r0}_{gt}) \in X^0\\
& \hat{\mathcal{Q}} \ge \sum_{t \in T}\sum_{i \in I}\left((1-\alpha)\sum_{g \in \Lambda_i}\sum_{r=1}^{R} \lambda^{rm}_{gt} G_g(q^r_g) + Pd^m_{it}\right),\\
& \qquad 1 \le m \le K\\
& (q^m, f^m, \theta^m, \lambda^m, d^m) \in X^m, \ 1 \le m \le K,
\end{aligned}
$$

where

$$
\begin{aligned}
X^m = \{&(q^m, f^m, \theta^m, \lambda^m, d^m):\\
& z_g(m) y_{gt} L_g \le q^m_{gt} \le z_g(m) y_{gt} U_g,\\
& \quad \forall g \in \Lambda, \forall t \in T && (32)\\
& z_{ij}(m) f'_{ij} \le f^{tm}_{ij} \le z_{ij}(m) f''_{ij},\\
& \quad \forall (i,j) \in E, \forall t \in T\\
& \theta'_i \le \theta^m_{it} \le \theta''_i, \forall i \in I, \forall t \in T && (33)\\
& (\theta^m_{it} - \theta^m_{jt}) z_{ij}(m)/x_{ij} = f^{tm}_{ij},\\
& \quad \forall (i,j) \in E, \forall t \in T && (34)\\
& q^m_{gt} - q^m_{g(t-1)} \le (2 - y_{g(t-1)} - y_{gt})L_g +\\
& \quad (1 + y_{g(t-1)} - y_{gt})R''_g, \ \forall g \in \Lambda, \forall t \in T && (35)\\
& q^m_{g(t-1)} - q^m_{gt} \le (2 - y_{g(t-1)} - y_{gt})L_g +
\end{aligned}
$$

$$(1 - y_{g(t-1)} + y_{gt})R'_g,\ \forall g \in \Lambda, \forall t \in T \quad (36)$$

$$-d^m_{it} \leq \sum_{\forall j \in E_{(\cdot,i)}} f^{tm}_{ji} - \sum_{\forall j \in E_{(i,\cdot)}} f^{tm}_{ij} + \sum_{g \in \Lambda_i} q^m_{gt} - D_{it} \leq d^m_{it},\ \forall i \in I, \forall t \in T \quad (37)$$

$$\sum_{r=1}^{R} \lambda^{rm}_{gt} = y_{gt},\ \forall g \in \Lambda, \forall t \in T \quad (38)$$

$$\sum_{r=1}^{R} \lambda^{rm}_{gt} q^r_g = q^m_{gt},\ \forall g \in \Lambda, \forall t \in T \quad (39)$$

$$q^m_{gt}, \lambda^{rm}_{gt}, d^m_{it} \geq 0, \quad \forall i \in I, \forall g \in \Lambda, \forall (i,j) \in E, \forall t \in T \quad (40)$$

$$\}.$$

This above developed extended formulation shares the same concept as the one described in [21], in which an extended formulation is developed for a more generalized multistage robust lot-sizing problem with disruptions.

B. Decomposition Framework

The extended formulation above is a large scale MILP, with size a function of N and k. Even the state-of-the-art optimization solvers cannot locate optimal solutions in tractable run-times. In this section, we develop both primal and dual decomposition approaches to solve this MILP efficiently.

1) Master problem: For the master problem, we consider the pre-contingency case as the start point. For instance, the initial master problem can be described as follows:

$$\begin{aligned}\min_{\{u,v,y\}} & \sum_{t \in T}\sum_{i \in I}\sum_{g \in \Lambda_i} (F''_g u_{gt} + F'_g v_{gt} + \sum_{r=1}^{R} \lambda^{r0}_{gt} G_g(q^r_g)) \\ s.t.\ & \text{constraints } (2) - (7) \\ & (q^0, f^0, \theta^0, \lambda^0) \in X^0.\end{aligned}$$

2) Subproblem: After solving each master problem, we can obtain a solution $(\bar{u}, \bar{v}, \bar{y})$. The subproblem is $\max_z Q(z, u, v, y)$, as described in Section II. We expect to solve this problem with $(u, v, y) = (\bar{u}, \bar{v}, \bar{y})$ to identify the worst-case contingency scenario. The detailed subproblem can be described as follows:

$$\begin{aligned}& \max_z Q(z, u, v, y) = \\ & \max_z \min \sum_{t \in T}\sum_{i \in I} \left(\sum_{g \in \Lambda_i}\sum_{r=1}^{R} (1-\alpha)\lambda^r_{gt} G_g(q^r_g) + P d_{it} \right) \\ & s.t.\ \text{constraints } (8), (10) - (20).\end{aligned}$$

Note here that once the unit commitment and contingency scenario are determined, the remaining economic dispatch problem is a linear program. Therefore, we can transfer this max-min problem into a MILP by constructing the dual formulation of the inner minimization problem as follows:

$$\max \mathcal{G}(\gamma, \delta, \eta, \kappa, \varsigma, \pi, \varphi, \tau, z)$$

$$s.t.\ \text{constraint } (8)$$

$$\gamma^+_{gt} - \gamma^-_{gt} - \varsigma^+_{gt} + \varsigma^+_{g(t+1)} + \varsigma^-_{gt} - \varsigma^-_{g(t+1)} + \pi^+_{it} - \pi^-_{it} - \tau_{gt} \leq 0, \quad \forall i \in I, \forall g \in \Lambda_i, \forall t \in T \quad (41)$$

$$\delta^+_{ij,t} - \delta^-_{ij,t} - \kappa^+_{ij,t} + \kappa^-_{ij,t} - \pi^+_{it} + \pi^+_{jt} + \pi^-_{it} - \pi^-_{jt} \leq 0, \quad \forall (i,j) \in E, \forall t \in T \quad (42)$$

$$\eta^+_{it} - \eta^-_{it} + \sum_{j \in E_{(i,\cdot)}} \frac{1}{x_{ij}}\kappa^+_{ij,t} - \sum_{j \in E_{(\cdot,i)}} \frac{1}{x_{ji}}\kappa^+_{ji,t} - \sum_{j \in E_{(i,\cdot)}} \frac{1}{x_{ij}}\kappa^-_{ij,t} + \sum_{j \in E_{(\cdot,i)}} \frac{1}{x_{ji}}\kappa^-_{ji,t} = 0, \quad \forall i \in I, \forall t \in T \quad (43)$$

$$-(1-\alpha)G_g(q^r_g) + \varphi_{gt} + q^r_g \tau_{gt} \leq 0, \quad \forall g \in \Lambda, \forall t \in T, \forall r: 1 \leq r \leq R \quad (44)$$

$$\pi^+_{it} + \pi^-_{it} - P \leq 0, \quad \forall i \in I, \forall t \in T \quad (45)$$

$$\gamma, \delta, \eta, \kappa, \varsigma, \pi \geq 0, \varphi, \tau, \text{unrestricted}, \quad (46)$$

where $\gamma, \delta, \eta, \kappa, \varsigma, \pi, \varphi$, and τ are dual variables for constraints (10), (11), (12), (13)–(14), (15)–(16), (17), (18), and (19), respectively, and

$$\begin{aligned}& \mathcal{G}(\gamma, \delta, \eta, \kappa, \varsigma, \pi, \varphi, \tau, z) \\ &= \sum_{t \in T}\sum_{g \in \Lambda}(\gamma^+_{gt} z_g y_{gt} L_g) - \sum_{t \in T}\sum_{g \in \Lambda}(\gamma^-_{gt} z_g y_{gt} U_g) \\ &+ \sum_{t \in T}\sum_{(i,j) \in E}(\delta^+_{ij,t} z_{ij} f'_{ij}) - \sum_{t \in T}\sum_{(i,j) \in E}(\delta^-_{ij,t} z_{ij} f''_{ij}) \\ &+ \sum_{t \in T}\sum_{i \in I}(\eta^+_{it}\theta'_i) - \sum_{t \in T}\sum_{i \in I}(\eta^-_{it}\theta''_i) \\ &- \sum_{t \in T}\sum_{(i,j) \in E} \kappa^+_{ij,t}(1 - z_{ij})M \\ &- \sum_{t \in T}\sum_{(i,j) \in E} \kappa^-_{ij,t}(1 - z_{ij})M \\ &- \sum_{t \in T}\sum_{g \in \Lambda} \varsigma^+_{gt}((2 - y_{g(t-1)} - y_{gt})L_g \\ &+ (1 + y_{g(t-1)} - y_{gt})R''_g) \\ &- \sum_{t \in T}\sum_{g \in \Lambda} \varsigma^-_{it}((2 - y_{g(t-1)} - y_{gt})L_g \\ &+ (1 - y_{g(t-1)} + y_{gt})R'_g) \\ &+ \sum_{t \in T}\sum_{i \in I}(\pi^+_{it} D_{it}) - \sum_{t \in T}\sum_{i \in I}(\pi^-_{it} D_{it}) \\ &+ \sum_{t \in T}\sum_{g \in \Lambda} \varphi_{gt} y_{gt}.\end{aligned}$$

It is worth noting that we have nonlinear terms in the above objective function with the following format:

$$\max z\mu, \text{ subject to } z \in \{0,1\} \text{ and } \mu \in R^+,$$

which increases the complexity to solve the subproblem. Fortunately, such a bilinear term can be reformulated as

$$\begin{aligned} &\max \mu_z \\ s.t.\ \ &\mu_z \leq (1-z)M + \mu,\ \ \mu_z \leq zM \\ &\mu, \mu_z \in R^+ \end{aligned}$$

to finally make the subproblem a MILP. The overall approach to solve the subproblem is similar to the one described in [15]. This approach is also called "separation" procedure in optimization, which shows the main difference between our approach and the traditional credible contingency set approach.

3) Primal approach: After solving the subproblem (or separation problem) $\max \mathcal{G}$, we obtain the optimal solution $z(m^*)$, which indicates the worst-case contingency scenario. If $\hat{\mathcal{Q}} < \max \mathcal{G}$, the corresponding most violated primal inequalities

$$\hat{\mathcal{Q}} \geq \sum_{t\in T}\sum_{i\in I}\left(\sum_{g\in\Lambda_i}\sum_{r=1}^{R}(1-\alpha)\lambda_{gt}^{rm^*}G_g(q_g^r) + Pd_{it}^{m^*}\right)$$
$$(q^{m^*}, f^{m^*}, \theta^{m^*}, \lambda^{m^*}, d^{m^*}) \in X^{m^*}$$

will be added into the master problem. This (scenario based robust optimization) approach is similar to the ones described in [14] and [19]. In this approach, we try to solve the problem optimally with fewer contingency scenarios considered, as compared to the extended formulation. Not all scenarios must be considered in order to obtain an optimal solution, only some extreme cases. It is easy to observe that this primal approach converges and obtains an optimal solution of the original problem.

4) Dual approach: In the traditional Benders' decomposition approach, both feasibility and optimality cuts are considered. In this research, feasibility is guaranteed after we consider the power balance penalty cost. After solving the subproblem $\max \mathcal{G}$, if $\hat{\mathcal{Q}} < \max \mathcal{G}$, the following dual inequality

$$\hat{\mathcal{Q}} \geq \mathcal{G}(\gamma^*, \delta^*, \eta^*, \kappa^*, \varsigma^*, \pi^*, \varphi^*, \tau^*, z^*), \tag{47}$$

where $(\gamma^*, \delta^*, \eta^*, \kappa^*, \varsigma^*, \pi^*, \varphi^*, \tau^*, z^*)$ is the optimal solution for the subproblem, will be added to the master problem.

5) Algorithm framework: For the entire algorithm, after obtaining a solution of the master problem, we solve the subproblem to detect the worst-case contingency scenario and add the most violated primal or dual inequalities into the master problem to conduct the next iteration. The algorithm terminates when the time limit is reached or an optimal solution is found (e.g., $\hat{\mathcal{Q}} = \max \mathcal{G}$). The proposed decomposition framework is summarized in Fig. 1.

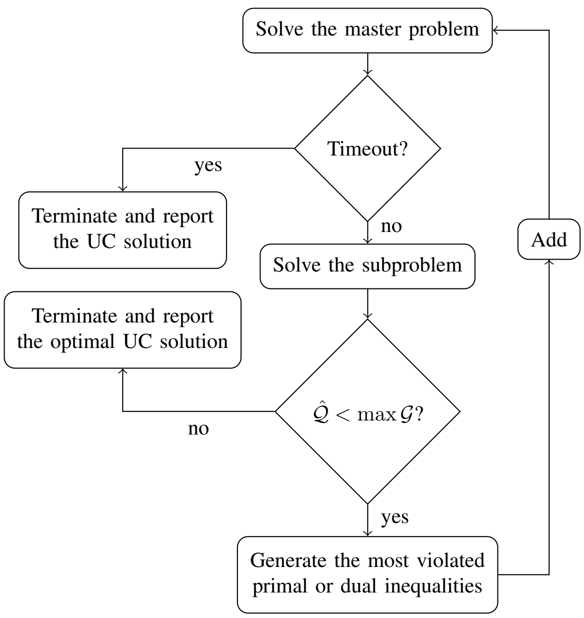

Fig. 1. Proposed Decomposition Framework

IV. Computational Results

In this section, we first report the performance of our proposed approach for a six-bus system for illustrative purposes. Then we investigate performance on modified IEEE 118-bus systems, including 33 thermal generators and 186 transmission lines (available from http://motor.ece.iit.edu/data), to show the computational advantage of our robust optimization approach. We compare the computational times between the primal and dual decomposition approaches and describe the differences obtained by using different objective functions. Note here that it is not hard to imagine that our approach performs much better than the current practice using the enumeration approach. If we use the extended formulation, the size of the problem will be huge. For instance, for the $k = 2$ case, the size of contingency scenarios is up to $\mathcal{O}(10^4)$ (i.e., in the size of $\binom{219}{2}$, where 219 is the sum of the number of generators and the number of transmission lines), and the corresponding sizes of decision variables and constraints are up to $\mathcal{O}(10^8)$ (i.e., in the size of $\binom{219}{2} \cdot 186 \cdot 24$). For the $k = 4$ case, the size of contingency scenarios is up to $\mathcal{O}(10^8)$ (i.e., in the size of $\binom{219}{4}$), and the corresponding sizes of decision variables and constraints are up to $\mathcal{O}(10^{11})$ (i.e., in the size of $\binom{219}{4} \cdot 186 \cdot 24$). In our experiment, a piecewise linear function with three linear pieces, based on the interpolation method [20], is utilized to approximate the fuel cost function. The power balance penalty cost is set to be \$1500 per MWh. All algorithms are implemented in C++ using the CPLEX 12.1 callable library, and all experiments are performed on a quad-core Intel workstation with 8GB RAM.

A. Six-Bus System

We now report results for a six-bus system which is composed of three generators, six loads, and eight transmission lines, with the N-2 security criterion. The system layout is shown in Fig. 2. Note here that this system meets the N-2 security criterion: for each bus with no more than two transmission lines adjacent to it, there is a thermal generator with sufficient generation capacity to cover the load at the bus. In Tables I–IV, we summarize the characteristics of all the buses, thermal units, and transmission lines. The objective is to minimize the total cost under the worst-case contingency scenario (i.e., $\alpha = 0$).

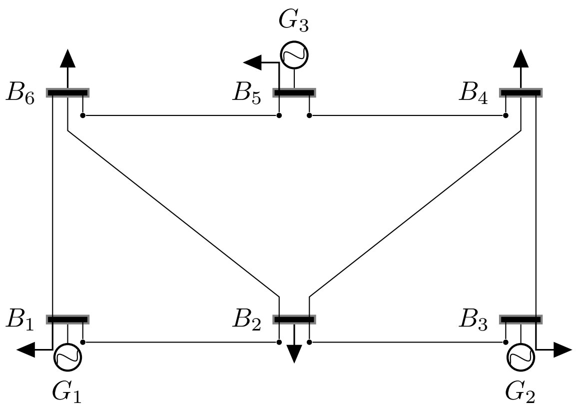

Fig. 2. Six-Bus System

TABLE I
BUS PROFILES

Bus ID	Type	Unit	Hourly Load (MW)
B_1	Thermal/Load	G_1	20
B_2	Load	-	20
B_3	Thermal/Load	G_2	20
B_4	Load	-	20
B_5	Thermal/Load	G_3	20
B_6	Load	-	20

TABLE II
GENERATOR PARAMETERS

Unit	Lower (MW)	Upper (MW)	Min-Down (h)	Min-Up (h)	Ramp (MW/h)
G_1	20	150	2	4	50
G_2	50	200	3	3	40
G_3	100	300	3	2	15

TABLE III
FUEL COST DATA

Unit	**a** (MBtu)	**b** (MBtu/ MWh)	**c** (MBtu/ MW^{2}h)	Start-Up Fuel (MBtu)*	Fuel Price ($/MBtu)
G_1	50	6	0.0004	100	1.2300
G_2	40	5.5	0.0001	300	1.2461
G_3	60	4.5	0.005	0	1.2462

* : The shut-down fuel is assumed to be the same as the start-up fuel

TABLE IV
TRANSMISSION LINE PARAMETERS

Line ID	From	To	X	Flow Limit (MW)
L_1	B_1	B_2	0.170	100
L_2	B_1	B_4	0.150	100
L_3	B_2	B_3	0.258	100
L_4	B_2	B_4	0.197	100
L_5	B_2	B_6	0.197	100
L_6	B_3	B_6	0.140	100
L_7	B_4	B_5	0.150	100
L_8	B_5	B_6	0.160	100

The primal approach is performed in the decomposition framework for the six-bus system. The algorithm terminates in seven iterations. In the first iteration, only G_1 is committed over all periods in the operational time interval when no contingency occurs, because it has the lowest generation cost. The worst-case contingency scenario detected by the separation problem at iteration 1 involves contingencies on G_1 and L_1. In the second iteration, we add the corresponding contingency scenario into the master problem; the resulting solution indicates that G_2 (instead of G_1) is now committed over all periods. Our separation problem then returns the worst-case contingency scenario, which involves contingencies on G_2 and L_3. Similarly, G_3 (instead of G_1 or G_2) is committed over all periods at the third iteration. In the fourth iteration, the master problem yields a solution in which both G_1 and G_2 are committed over all periods. Then, accordingly, the separation problem indicates that the worst-case contingency scenario involves contingencies on G_1 and G_2. In the following two iterations, different combinations of two generators are committed. Finally, in the seventh iteration, the master problem yields a solution with all of G_1, G_2, and G_3 committed over all periods. At this moment, our separation problem returns $\max \mathcal{G} = \hat{\mathcal{Q}}$. Based on the stopping criterion, the algorithm terminates, providing an optimal solution for the original problem.

B. IEEE 118-Bus System

In this section, we study modified IEEE 118-bus systems to show the computational effectiveness of our proposed approach. All of the thermal generators in the original IEEE 118-bus system are retained.

1) Experiments with the N-1 security criterion: We make the system meet the N-1 security criterion by adding five additional transmission lines $\{l_{(10,73)}, l_{(70,73)}, l_{(89,112)}, l_{(111,115)}, l_{(111,116)}\}$, and test our approach using the N-1 security criterion.

Comparison of primal and dual approaches: We test the performances of both the primal and dual approaches. We first consider the objective of minimizing the total cost under the worst-case contingency scenario. Table V summarizes the solution procedure of the primal approach. The algorithm converges in only four iterations. We also notice that the subproblems are significantly more difficult to solve than the master problems.

TABLE V
COMPUTATIONAL RESULTS FOR THE 118-BUS SYSTEM: PRIMAL APPROACH

Iteration	Type	Obj. ($)	CPU Time (sec)
1	master	754,507	2.27
	subproblem	1,309,680	328.5
2	master	764,379	6.45
	subproblem	762,264	724.9
3	master	782,815	19.2
	subproblem	785,137	697.5
4	master	783,606	39.06
	subproblem	762,264	724.6

As described in Section III, the traditional Benders' decomposition approach adds the corresponding dual inequality (after solving the subproblem) into the master problem. In this study, we also report the result of this dual approach in Table VI for comparison. The dual approach terminates due to the predefined one-hour time limit. From the table, we observe that the dual approach does not converge as fast as the primal approach, and that the objective value for the master problem increases very slowly. After five iterations, the objective value for the dual approach ($\$755,134$) is much smaller than the optimal objective value for the original problem ($\$783,606$).

TABLE VI
COMPUTATIONAL RESULTS FOR THE 118-BUS SYSTEM: DUAL APPROACH

Iteration	Type	Obj. ($)	CPU Time (sec)
1	master	754,507	3.12
	subproblem	1,309,680	464.4
2	master	754,830	7.01
	subproblem	1,218,430	597.2
3	master	754,948	10.16
	subproblem	1,218,370	708.07
4	master	755,047	15.6
	subproblem	1,218,450	687.2
5	master	755,134	12.3
	subproblem	1,218,430	700.5

Comparison of robust optimization and credible contingency set approaches: For the credible contingency set approach, one common practice is to use the spinning reserve requirements to cover the generator outage and the credible contingency set with a simultaneous feasibility test (SFT) approach to cover the transmission network contingencies [22]. This approach can be summarized as follows:

1. Run the security-constrained unit commitment (SCUC) problem and identify the credible contingency set, which usually corresponds to the binding transmission capacity constraints in SCUC.
2. Run SFT for contingency transmission security testing. SFT is a contingency analysis tool that checks the feasibility of the transmission network before and after contingencies by calculating the power flow for each. For any branch or interface that violates its limit, SFT will compute bus sensitivities to those violation elements and return the corresponding linear constraints to SCUC.
3. Go to step one and continue the iteration between SCUC and SFT until the process converges (no new constraints found) or reaches the maximum number of iterations (e.g., three to four iterations in practice due to time restrictions).

This is a preventive action and there is no post-contingency corrective action involved. The performance of this approach is highly dependent on the selection of the credible contingency set. In our experimental testing of the SFT approach, the same steps described above are taken and a credible contingency set with elements in which the flows reach their capacities is selected. The experimental results indicate that the original unit commitment solution survives SFT. However, the worst-case cost for this approach reaches $1,309,680, which is much larger than the value (i.e., $783,606) obtained from the robust optimization approach.

Comparison of different objectives and sensitivity analysis on α: We now apply our two-stage robust optimization approach to test the case with the objective of minimizing the total pre-contingency cost. The ultimate objective value is $\$764,379$, which is smaller than the minimum worst-case objective value of $\$783,606$. We also observe that, for this instance, both approaches provide the same robust unit commitment decision, even though the conclusion is not in general true. Finally, we apply the two-stage robust optimization approach to conduct sensitivity analysis on different α values and report the results in Table VII. From the table, we observe that the cost decreases as the α value increases, because the dispatch cost under the pre-contingency scenario is smaller than that under the worst-cast contingency scenario.

TABLE VII
COMPUTATIONAL RESULTS ON DIFFERENT α VALUES

α	Obj. ($)	CPU Time (sec)
0	783,606	2541.1
50%	773,923	2089.4
80%	768,180	2022.5
100%	764,379	2018.3

2) Experiments with the N-2 security criterion: To test the case with the N-2 security criterion, we add 31 transmission lines and two thermal generators into the original system. This approach can provide a solution with zero power imbalance for the system under the N-2 security criterion (parameters for each transmission line and generator are described in Appendix A). The objective is to minimize the total cost under the worst-case contingency scenario. Under this setting, the robust optimization approach can obtain an optimal solution (with the objective value $\$774,488$) in three iterations with the N-1 security criterion.

As indicated in the previous subsection, the subproblem takes a much longer time as compared to the master problem for the $k = 1$ case. As k increases, the subproblem takes an even longer time to obtain an optimal solution. In this subsection, we propose a heuristic approach to ensure that the proposed decomposition approach obtains a reasonable good solution (and accordingly obtains a lower bound of the optimal objective value for the original problem). To solve the separation problem, we set the time limit to be one hour, obtain the two worst contingencies within this time limit, and report the objective value for the subproblem $\mathcal{G}'$. If $\hat{\mathcal{Q}} < \mathcal{G}'$, then the constraints for the corresponding scenario can be added into the mater problem and the algorithm can continue. That is, we can use a feasible solution of the subproblem to continue the primal approach. The algorithm terminates in five iterations since we set the total time limit to be five hours (see, e.g., Table VIII). This heuristic approach can detect severe contingencies even though they might not be the worst-case contingency scenarios. In addition, this heuristic approach can detect more critical components as the time limit increases.

As compared to the solution for the N-1 security criterion case, one more expensive generator is turned on in the final unit commitment solution. With the pre-contingency cost as

TABLE VIII
COMPUTATIONAL RESULTS FOR THE 118-BUS SYSTEM: THE N-2 CASE

Iteration	Type	Obj. ($)	CPU Time (sec)
1	master	749,727	3.74
	subproblem	1,700,200	2378.29
2	master	766,615	13.02
	subproblem	1,018,590	3600.09
3	master	775,288	33.05
	subproblem	1,214,330	3600.06
4	master	784,406	273.18
	subproblem	801,187	3600.02
5	master	805,186	265.30
	subproblem	-	Timeout

the objective, the cost for the N-2 case ($\$760,179$) is $\$10,452$ larger than that for the N-1 case ($\$749,727$).

3) Cost of robustness: Finally, we test the scalability of the proposed approach and evaluate the objectives (e.g., minimizing the total cost under the worst-case contingency scenario, without loss of generality) of the CCUC problem at different k values. That is, we compare solutions with different N-k rules (e.g., $k = 1, 2, 3,$ and 4). The same heuristic method and power grid setting as those for the N-2 case are applied to study the N-$k, k \geq 3$ security criterion cases. All cases (including $k = 3$ and $k = 4$) are terminated in four to five iterations. The results are summarized in Table IX and shown in Fig. 3. It can be observed that the objective value obtained from our proposed heuristic approach goes higher as the k value increases. On the other hand, more units are turned on as the k value increases. We can consider this as the cost estimation of robustness, which is due to obtaining more conservative but reliable solutions for the CCUC problem.

TABLE IX
COMPUTATIONAL RESULTS FOR THE 118-BUS SYSTEM: THE N-k CASE

k	Obj. ($)	Number of On-line Units	Cost Increment
0	749,727	17	0
1	774,488	17	3.3%
2	805,186	18	7.4%
3	830,196	20	10.7%
4	844,489	20	12.6%

V. CONCLUSION AND DISCUSSION

In this paper, we propose a two-stage robust optimization model to solve the CCUC problem with the N-k security criterion. Our approach can help identify the most critical components in a power system, as compared to the traditional credible contingency set approach. A decomposition framework with both primal and dual approaches is studied to solve the problem. Our case study indicates that the primal approach significantly outperforms the dual approach. We also observed that our approach can accommodate other objective functions. Finally, we observe that for the N-k security criterion, as k increases, the problem is hard to solve to optimality. As indicated in the computational experiment results, the empirical complexity for the master problem is not very bad, so the

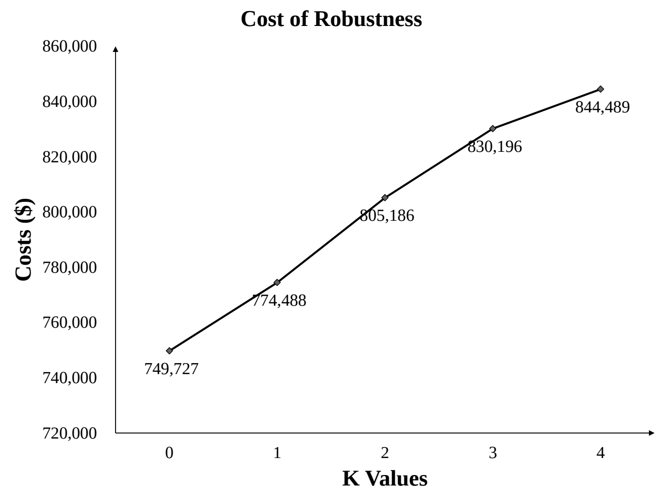

Fig. 3. CCUC Objective with Different k Values

bottleneck is the subproblem. In particular, it is mainly due to the big-M formulation in the subproblem (e.g., the big-M formulation usually leads to a big optimality gap). In addition, too much redundancy/symmetry (in terms of which component is to be selected as a contingency) also makes it challenging for the solver to identify the optimal solution. A similar issue arose in [11], in which an optimal solution could not be found in a short time for the IEEE 73-bus network (RTS 96) without considering transmission switching (i.e., the similar setting as ours). Meanwhile, the heuristic approach proposed in this paper can detect severe contingency scenarios (i.e., provide good feasible solutions for the subproblems) easily and in a short time, according to our experimental results, although it is very difficult to prove the optimality of the obtained solution. In future research, we will explore alternative formulations and other methods to solve large scale problems to optimality.

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APPENDIX A
ADDITIONAL TRANSMISSION LINES AND GENERATORS

We add 31 transmission lines and two thermal generators to construct a more reliable network structure for our experiments. The additional transmission lines are reported in Table

X. The flow limit is 100MW and the reactance is 0.072 for each additional transmission line. Two thermal generators are added to B_{117} and B_{118} respectively, with the same generator parameter setting as G_2 in Table II.

TABLE X
TRANSMISSION LINE PARAMETERS

Line ID	From	To	Line ID	From	To
L_1	B_1	B_4	L_{17}	B_{73}	B_{74}
L_2	B_2	B_6	L_{18}	B_{73}	B_{76}
L_3	B_7	B_9	L_{19}	B_{78}	B_{84}
L_4	B_{10}	B_{13}	L_{20}	B_{79}	B_{86}
L_5	B_{10}	B_{14}	L_{21}	B_{87}	B_{88}
L_6	B_{16}	B_{18}	L_{22}	B_{87}	B_{90}
L_7	B_{20}	B_{22}	L_{23}	B_{91}	B_{95}
L_8	B_{26}	B_{28}	L_{24}	B_{97}	B_{98}
L_9	B_{29}	B_{33}	L_{25}	B_{101}	B_{107}
L_{10}	B_{35}	B_{39}	L_{26}	B_{102}	B_{108}
L_{11}	B_{36}	B_{43}	L_{27}	B_{109}	B_{111}
L_{12}	B_{44}	B_{48}	L_{28}	B_{111}	B_{112}
L_{13}	B_{50}	B_{52}	L_{29}	B_{113}	B_{114}
L_{14}	B_{53}	B_{57}	L_{30}	B_{115}	B_{116}
L_{15}	B_{58}	B_{63}	L_{31}	B_{116}	B_{117}
L_{16}	B_{67}	B_{72}			

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