Price-Based Unit Commitment with Wind Power Utilization Constraints

Qianfan Wang, *Student Member, IEEE,* Jianhui Wang, *Member, IEEE,* Yongpei Guan, *Member, IEEE*

***Abstract*—This paper proposes an optimal bidding strategy for independent power producers (IPPs) in the deregulated electricity market. The IPPs are assumed to be price takers, whose objectives are to maximize their profits considering price and wind power output uncertainties, while ensuring high wind power utilization. The problem is formulated as a two-stage stochastic price-based unit commitment problem with chance constraints to ensure wind power utilization. In our model, the first stage decision includes unit commitment and quantity of electricity submitted to the day-ahead market. The second stage decision includes generation dispatch, actual usage of wind power, and amount of energy imbalance between the day-ahead and real-time markets. The chance constraint is applied to ensure a certain percentage of wind power utilization so as to comply with renewable energy utilization regulations. Finally, a Sample Average Approximation (SAA) approach is applied to solve the problem, and the computational results are reported for the proposed SAA algorithm showing the sensitivity of the total profit as the requirement of wind power utilization changes.**

***Index Terms*—Price based unit commitment, wind power, stochastic programming, chance constrains, mixed integer programming, sample average approximation**

NOMENCLATURE

A. Sets and Indices

N	Number of scenarios.
T	Time horizon (24 hours).
B	Number of buses.
Λ_b, Υ_b	Sets of thermal generators and hydro units in bus b, respectively.

B. Parameters

SU_i^b	Start-up cost of thermal generator i in bus b.
SD_i^b	Shut-down cost of thermal generator i in bus b.
$F_c(p_{it}^b)$	Fuel cost of thermal generator i in time t at bus b when its generation is p_{it}^b.
MU_i^b	Minimum-up time for thermal generator i in bus b.
MD_i^b	Minimum-down time for thermal generator i in bus b.
UR_i^b	Ramp-up rate limit for thermal generator i in bus b.
DR_i^b	Ramp-down rate limit for thermal generator i in bus b.
L_i^b	Lower limit of electricity generated by thermal generator i in bus b.
U_i^b	Upper limit of electricity generated by thermal generator i in bus b.
S_{ib}^{begin}	Water reserve level of pumped-storage unit i in bus b in the first time period.
S_{ib}^{end}	Water reserve level of pumped-storage unit i in bus b in the last time period.
L_{ib}^H	Lower limit of power pumped in/out by pumped-storage unit i in bus b in one time period.
U_{ib}^H	Upper limit of power pumped in/out by pumped-storage unit i in bus b in one time period.
γ_t^b	Penalty cost per MW for energy imbalance in time t in bus b.
η_1	Efficiency of generating power by the pumped-storage unit.
η_2	Efficiency of absorbing power by the pumped-storage unit.

C. Decision Variables

p_{it}^b	Electricity generation amount by thermal generator i in time t in bus b.
y_{it}^b	Binary decision variable: "1" if thermal generator i is on in time t in bus b; "0" otherwise.
o_{it}^b	Binary decision variable: "1" if thermal generator i is started up in time t in bus b; "0" otherwise.
v_{it}^b	Binary decision variable: "1" if thermal generator i is shut down in time t in bus b; "0" otherwise.
h_{it}^b	Binary decision variable to indicate whether pumped-storage unit i generates (e.g., "1") or consumes (e.g., "0") power in time t in bus b.
s_{it}^b	Water reserve level of pumped-storage unit i in time t in bus b.
q_{tb}^B	Electricity bid into the day-ahead market in time t in bus b.
q_{tb}^G	Total amount of thermal unit generation in time t in bus b.
q_{tb}^W	Wind power sold in the real-time market in time t in bus b.
q_{itb}^{H+}	Power generated from pumped-storage unit

Qianfan Wang and Yongpei Guan are with the Industrial and Systems Engineering Department, University of Florida, Gainesville, FL, 32611, USA. E-mail: qfwang@ufl.edu; guan@ise.ufl.edu.
Jianhui Wang is with the Decision and Information Science Division, Argonne National Laboratory, Lemont, IL 60439, USA. E-mail: jianhui.wang@anl.gov.

i in time t in bus b.

q^{H-}_{itb} Power consumed by pumped-storage unit i in time t in bus b.

q^{imb}_{tb} Power imbalance in time t in bus b.

Some of these decision variables are second stage variables when they are followed by (ξ), where ξ represents a random vector following a certain probabilistic distribution.

D. Random Numbers

$R^{DA}_{tb}(\xi)$ Day-ahead market price of electricity in time t in bus b.

$R^{RT}_{tb}(\xi)$ Real-time market price of electricity in time t in bus b.

$W_{tb}(\xi)$ Wind power output in time t in bus b.

I. Introduction

A Number of initiatives have been launched to increase the utilization of wind power in different countries and regions (e.g., [1] and [2]). One common approach is to introduce green certificates to ensure utilization of renewable energy as effectively as possible [3]. This approach is basically imposing the national target for renewable energy utilization on either the demand side including consumers or distribution companies (e.g., Denmark and Germany) [4], or the generation side (e.g., Italy) [4]. If the regulation is applied to the demand side, consumers or distributors will be required to prove that they consume at least the specified amount of renewable energy by submitting certificates to the authorities at a given time. If the regulation is applied to the generation side, every supplier, except renewable energy producers or importers, is required to ensure that a certain percentage of the energy produced by them, is renewable energy.

This regulation has exerted a large impact on electricity market economics and operations, in particular market participants such as independent power producers (IPPs) that own thermal units as well as renewable generation resources like wind power. Under this regulation, each producer has to utilize as much renewable energy as possible for possible extra profit obtained from the green certificate market. On the other side, an IPP owning traditional thermal units and wind power turbines has to face two-fold uncertainties - price and wind power output uncertainties when submitting bids to the market. If there is a mismatch between the amount submitted in day-ahead and the real-time outputs [5], a penalty will be imposed (e.g., [6] and [7]). Due to the intermittent nature of wind power, significant penalties can be generated. To avoid such significant penalties, an efficient approach to handle the uncertainties is based on the mixed utilization of wind power and pumped-storage units [5]. In addition, it is easy to observe that thermal units can also help accommodate wind power output uncertainty. For instance, among others, the WILMAR model is studied in [8] and [9], a stochastic security-constrained unit commitment model is analyzed to integrate uncertain wind power output into a thermal power system in [10], and the impact of wind power on power generation in the Dutch system is investigated in [11].

In this paper, we propose to study the optimal bidding strategy for an IPP whose generation portfolio may consist of thermal, hydro and wind power units. The objective of an IPP's self-scheduling problem is to maximize its profit while ensuring high utilization of wind power output to comply with related regulations. In our approach, the IPP is assumed to be a price taker in the market, which means the IPP does not have control over the market prices by bidding strategically. One of the reasons for the IPP to be a price taker is its relatively small share of generation in the total generation capacity in the market. Since the IPP is a price-taker, the market prices are purely input to the IPP's own profit maximization problem. Thus, the objective of the IPP is to optimize its own generation portfolio based on the forecasted day-ahead and real-time price information. With this price-based decision making, the IPP tries to come up with its best generation schedule that can be bid into the market subject to physical constraints of the generators such as min on/off, capacity limits, etc. Along with the generation quantity obtained from the optimization, the IPP can bid a low price to ensure the acceptance of its bids in the market. This problem is typically defined as the Price-based Unit Commitment (PBUC) problem as shown in the literature (see, e.g., [12], [13], [14], [15]). In this paper, the IPP is considered to operate and schedule a few number of thermal generators, several wind farms and pumped-storage units. We propose a novel bidding strategy that can maximize the expected profit with the consideration of wind power forecasting errors and ensure high utilization of wind power output for IPPs.

The chance constraint requires a certain probability at which a given portion of wind power must be utilized. For example, we can define a probability of 95% at which the utilization of wind power is no smaller than a certain number (e.g., 85%). Applications of chance-constrained optimization in power system have been studied recently. In [16], the chance constraints are applied to address the hourly load uncertainty. The chance constraint is used to formulate a stochastic unit commitment problem which guarantees the load satisfied with a pre-defined chance. In [17], the not-overload-probability is addressed using chance-constrained optimization for the transmission planning problem. In [18], the chance constraint is presented to solve the unit commitment problem with uncertain wind power output from the system operator's point of view. In [19], the chance constraint is utilized to ensure the load balance at each time period to be satisfied by a certain percentage while the demand response uncertainty is considered. However, none of the previous studies has investigated wind power bidding strategies with chance constraints, which is the main focus of this paper.

In this paper, we study a sample average approximation (SAA) method to solve the chance constrained power producer bidding problem. As compared to the SAA algorithms recently developed for the two-stage stochastic program described in [18], [20] and [21], and for the chance-constrained single-stage stochastic program described in [22] and [23], we develop a different SAA algorithm due to the binary decision variables for the hydro units in the second stage. The validation process and convergence proof of the proposed approach are also investigated. The case studies show the proposed algorithm in this paper provides tight lower and upper bounds, which illustrate the effectiveness of our approach.

The contributions of this paper are summarized as follows:

(1) Much of the previous research studied the stochastic unit commitment problem from an ISO's perspective. In this paper, our contributions focus on the price-based unit commitment (PBUC) from a price-taking IPP's perspective. A two-stage stochastic programming model is studied to solve the problem.
(2) Compared with other works in PBUC [12], [13], this is the first paper to apply the chance-constrained stochastic programming to address bidding strategies on a thermal-wind-hydro generation portfolio.
(3) The SAA framework is adjusted specifically for our chance-constrained stochastic programming model. We also propose a heuristic-based SAA algorithm for this specific problem structure.

The rest of the paper is organized as follows: Section II describes the problem and presents the mathematical formulation of the problem. Section III introduces the proposed SAA algorithm and discusses the convergence property of the algorithm and the related solution validation process. Section IV reports the numerical examples and computational results for both single-bus and multi-bus systems. Finally, Section V summarizes the contributions of this research.

II. Mathematical Formulation

A. Market Framework

In a deregulated electricity market, the IPPs submit bids each day, and the market operator provides the market clearing prices of electricity. The IPPs with both thermal and wind power units in the generation portfolio face at least two main sources of uncertainties: market prices and variable wind power output. IPPs have to consider the uncertainty of wind power in their bidding as their wind power forecasts will not be perfectly accurate and errors always exist. In addition, some of the electricity markets (such as PJM [24]) are enforcing penalties on the mismatch between IPPs' day-ahead bids and their actual generation in the real-time market. One can easily observe that uncertain wind power output contributes significantly to the mismatch. In the meantime, IPPs can not choose to abandon too much wind power to avoid its fluctuation as they may be subject to certain regulations that require renewable energy like wind power accounting for a certain share of their total generation output [4]. Hence, IPPs need to optimize their bidding strategies that can balance the objective of profit maximization and the risks associated with wind power realizations, while making sure the wind power is utilized to the greatest degree. Based on the above discussion, the proposed method in this paper models such a bidding process for IPPs by formulating the problem as a two-stage stochastic program. To maximize the total expected profit, the IPPs decide the quantity to be submitted to the market in the first stage (day-ahead market), considering the possible realizations of uncertain market prices and wind power output in the second stage (real-time market). Chance constraints are used to model the least percentage of wind power utilization. Our modeling framework also captures the two-settlement (day-ahead and real-time) market procedure as in most of the U.S. markets by modeling the two markets in the objective function (e.g., PJM [24]).

B. Problem Formulation

The PBUC problem is formulated as a two-stage chance constrained stochastic programming problem. The first stage of the model is a unit commitment problem with decisions on commitment and quantity of the electricity offered to the day-ahead market. The second stage of the model is an economic dispatch problem with decisions on thermal and hydro unit dispatch, the actual usage of wind power, and energy imbalance. We consider the chance constraint at the second stage, in which the actual wind power utilized could be different from the wind power output. We describe the final formulation of the problem (denoted as the true problem) as follows.

$$\begin{aligned}
\max -&\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{i\in\Lambda_b}(SU_i^b o_{it}^b + SD_i^b v_{it}^b) \\
&+E[\mathcal{Q}(y,o,v,q^B,\xi)] && (1)\\
s.t.\ &-y_{i(t-1)}^b + y_{it}^b - y_{ik}^b \leq 0, \\
&\quad 1\leq k-(t-1)\leq MU_i^b, \forall i\in\Lambda_b, \forall b, \forall t && (2)\\
&y_{i(t-1)}^b - y_{it}^b + y_{ik}^b \leq 1, \\
&\quad 1\leq k-(t-1)\leq MD_i^b, \forall i\in\Lambda_b, \forall b, \forall t && (3)\\
&-y_{i(t-1)}^b + y_{it}^b - o_{it}^b \leq 0, \\
&\quad \forall i\in\Lambda_b, \forall b, \forall t && (4)\\
&y_{i(t-1)}^b - y_{it}^b - v_{it}^b \leq 0, \\
&\quad \forall i\in\Lambda_b, \forall b, \forall t && (5)\\
&y_{it}^b, o_{it}^b, v_{it}^b \in \{0,1\}, \forall i\in\Lambda_b, \forall b, \forall t, && (6)
\end{aligned}$$

where $\mathcal{Q}(y,o,v,q^B,\xi)$ is equal to

$$\begin{aligned}
\max &\sum_{t=1}^{T}\sum_{b=1}^{B}(R_{tb}^{DA}(\xi)q_{tb}^B + R_{tb}^{RT}(\xi)q_{tb}^{imb}(\xi)) \\
&-\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{i\in\Lambda_b}F_c(p_{it}^b(\xi)) - \sum_{t=1}^{T}\sum_{b=1}^{B}\gamma_t^b|q_{tb}^{imb}(\xi)| && (7)\\
s.t.\ &L_i^b y_{it}^b \leq p_{it}^b(\xi) \leq U_i^b y_{it}^b,\ \forall i\in\Lambda_b, \forall b, \forall t && (8)\\
&p_{it}^b(\xi) - p_{i(t-1)}^b(\xi) \leq (2 - y_{i(t-1)}^b - y_{it}^b)L_i^b + \\
&\quad (1 + y_{i(t-1)}^b - y_{it}^b)UR_i^b,\ \forall i\in\Lambda_b, \forall b, \forall t && (9)\\
&p_{i(t-1)}^b(\xi) - p_{it}^b(\xi) \leq (2 - y_{i(t-1)}^b - y_{it}^b)L_i^b + \\
&\quad (1 - y_{i(t-1)}^b + y_{it}^b)DR_i^b,\ \forall i\in\Lambda_b, \forall b, \forall t && (10)\\
&\sum_{i\in\Lambda_b} p_{it}^b(\xi) = q_{tb}^G(\xi),\ \forall b, \forall t && (11)\\
&q_{tb}^W(\xi) + q_{tb}^G(\xi) + \sum_{i\in\Upsilon_b}(q_{itb}^{H^+}(\xi) - q_{itb}^{H^-}(\xi)) = q_{tb}^B \\
&\quad +q_{tb}^{imb}(\xi),\ \forall b, \forall t && (12)\\
&s_{it}^b(\xi) = s_{i(t-1)}^b(\xi) + \eta_2 q_{itb}^{H^-}(\xi) - \frac{q_{itb}^{H^+}(\xi)}{\eta_1}, \\
&\quad \forall i\in\Upsilon_b, \forall b, \forall t && (13)
\end{aligned}$$

$$h_{it}^b(\xi)L_{ib}^H \le q_{itb}^{H^+}(\xi) \le h_{it}^b(\xi)U_{ib}^H,\ \forall i \in \Upsilon_b, \forall b, \forall t \quad (14)$$

$$(1-h_{it}^b(\xi))L_{ib}^H \le q_{itb}^{H^-}(\xi) \le (1-h_{it}^b(\xi))U_{ib}^H, \quad \forall i \in \Upsilon_b, \forall b, \forall t \quad (15)$$

$$s_{iT}^b(\xi) = S_{ib}^{end}, s_{i0}^b(\xi) = S_{ib}^{begin},\ \forall i \in \Upsilon_b, \forall b \quad (16)$$

$$Pr(\beta \sum_{b=1}^{B} W_{tb}(\xi) \le \sum_{b=1}^{B} q_{tb}^W(\xi), \forall t) \ge 1-\epsilon \quad (17)$$

$$p_{it}^b(\xi), q_{tb}^W(\xi), q_{tb}^G(\xi), q_{itb}^{H^+}(\xi), q_{itb}^{H^-}(\xi), s_{it}^b(\xi) \ge 0, \quad h_{it}^b(\xi) \in \{0,1\}, q_{tb}^B \text{ free}, \forall i, \forall b, \forall t. \quad (18)$$

The objective function (1) is to maximize the expected total profit. It is equal to the expected revenue $E[\sum_{t=1}^T \sum_{b=1}^B (R_{tb}^{DA}(\xi)q_{tb}^B + R_{tb}^{RT}(\xi)q_{tb}^{imb}(\xi))]$ which follows the two-settlement market procedure in most U.S. electricity markets, minus the expected power generation cost $\sum_{t=1}^T \sum_{b=1}^B \sum_{i\in\Lambda_b}(SU_i^b o_{it}^b + SD_i^b v_{it}^b) + E[\sum_{t=1}^T \sum_{b=1}^B \sum_{i\in\Lambda_b} F_c(p_{it}^b(\xi))]$, and the expected penalty cost $E[\sum_{t=1}^T \sum_{b=1}^B \gamma_t^b |q_{tb}^{imb}(\xi)|]$. It should be noted here that $q_{tb}^{imb}(\xi)$ captures the amount of imbalance between the day-ahead bid amount and the real-time generation output. As this imbalance is mainly caused by the variable wind power and inaccurate wind power forecasting, the additional penalty γ_t^b which is imposed by market operators can reduce the uncertainty of the market related to uncertain wind generation. The unit commitment constraints at the first stage listed above include constraints (2) to (5), representing the unit minimum-up time requirement when the unit is turned on (e.g., constraints (2)), the unit minimum-down time requirement when the unit is turned off (e.g., constraints (3)), the unit start-up condition (e.g., constraints (4)), and the unit shut-down condition (e.g., constraints (5)). The hourly economic dispatch constraints at the second stage include unit generation upper and lower limit constraints (8), unit ramping up constraints (9), unit ramping down constraints (10), total thermal generation output (11) (it sums up all the generation by thermal units), power balance constraints (12) (the total system generation should be equal to the amount of energy offered in the day-ahead market plus the imbalance), hydro water inventory balance constraints (13), hydro unit pump in/out limit constraints ((14) and (15)), and first/last period water reservation amount constraints (16). The chance constraint (17) is associated with a risk level ϵ (e.g., $\epsilon = 10\%$), which means the total utilization of wind power has to be larger than or equal to β (e.g., $\beta = 85\%$) for at least $100(1-\epsilon)$ percent of chance. As can be seen, adding this constraint can help IPPs comply with regulations which require a certain percentage of wind power utilization at a high probability. In addition, y_{it}^b, o_{it}^b and v_{it}^b are first-stage decision variables, and others are second-stage decision variables.

In the objective function of our model, there exists an absolute value which indicates the imbalance penalty. It can be reformulated by using linear programming as shown in [25]. For instance, the following minimization problem

$$\min \gamma_t^b |q_{tb}^{imb}|, \text{ subject to } Ax = b \quad (19)$$

can be reformulated as follows:

$$\min\{\gamma_t^b d_{tb} | -d_{tb} \le q_{tb}^{imb} \le d_{tb} \text{ and } Ax = b\} \quad (20)$$

after introducing an auxiliary variable d_{tb}. Thus, we can replace the absolute value part in (19) with a linear function (20).

III. Sample Average Approximation

In this section, we apply a sample average approximation (SAA) method to solve the stochastic program shown above. SAA is composed of three steps: 1) scenario generation to approximate the true distribution, 2) convergence analysis to show the convergence property of the algorithm, and 3) solution validation to verify that the solution converges to the optimal one. The readers are referred to [21] for more details regarding the traditional SAA method description. For notation brevity, the proposed mathematical model can be abstracted as follows:

$$\min_{x^f \in X} f(x^f) + E[\mathcal{Q}(x^f, \xi)]$$

where

$$\begin{aligned} \mathcal{Q}(x^f,\xi) = &\min cx^s(\xi) \\ s.t.\quad & Ax^s(\xi) = g - Dx^f, x^s(\xi) \ge 0 \\ & Pr\{H(x^f, x^s(\xi), \xi) \le 0\} \ge 1-\epsilon. \end{aligned}$$

In the above formulation, x^f and x^s represent the first and second stage decision variables, and $f(x^f)$ and $\mathcal{Q}(x^f,\xi)$ represent the first and second stage objective functions. In addition, X represents the feasible region of x^f, c, g, A, D are vectors/matrices of parameters, and H is the constraint mapping.

A. SAA Problem

In our approach, the SAA problem is generated similarly to the one described in [18] and [21]. For instance, a Monte Carlo simulation method is utilized for scenario generation purposes. After the scenarios are generated (e.g., N scenarios), the objective function $E[\mathcal{Q}(x^f,\xi)]$ can be linearized and replaced by the sample average function $N^{-1}\sum_{j=1}^N \mathcal{Q}(x^f, \xi^j)$ [20]. Meanwhile, an indicator function $\mathbb{1}_{(0,\infty)}(H(x^f, x^s(\xi^j), \xi^j)$ is introduced to estimate the chance constraint as described in [22]. We have $\mathbb{1}_{(0,\infty)}(H(x^f, x^s(\xi^j), \xi^j)) = 1$ if $H(x^f, x^s(\xi^j), \xi^j) \in (0,\infty)$ and $\mathbb{1}_{(0,\infty)}(H(x^f, x^s(\xi^j), \xi^j)) = 0$ when $H(x^f, x^s(\xi^j), \xi^j) \le 0$. By introducing binary decision variables z to indicate if a constraint is satisfied, when a sample size N is given, we can linearize the chance constraint as the following constraints (22)–(24), and the SAA problem can be described as follows:

$$\max -\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{i\in\Lambda_b}(SU_i^b o_{it}^b + SD_i^b v_{it}^b) + N^{-1}\sum_{j=1}^{N} \Bigg[\sum_{t=1}^{T}\sum_{b=1}^{B}(R_{tb}^{DA}(\xi^j)q_{tb}^{B} + R_{tb}^{RT}(\xi^j)q_{tb}^{imb}(\xi^j)) -\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{i\in\Lambda_b}F_c(p_{it}^b(\xi^j)) - \sum_{t=1}^{T}\sum_{b=1}^{B}\gamma_t^b|q_{tb}^{imb}(\xi^j)|\Bigg] \quad (21)$$

$$s.t. \quad (2)-(6), (8)-(16), \text{and } (18)$$

$$\beta\sum_{b=1}^{B}W_{tb}(\xi^j) - \sum_{b=1}^{B}q_{tb}^{W}(\xi^j) \le M\times z_j, \forall t, \forall j \quad (22)$$

$$\sum_{j=1}^{N} z_j \le N\times\epsilon \quad (23)$$

$$z_j \in \{0,1\},\ j=1,2,\ldots,N. \quad (24)$$

B. Convergence Analysis and Solution Validation

We use statistical methods to analyze the solution of the SAA problem and provide convergence analysis and solution validation. As the sample size N goes to infinity, we claim that the objective of the SAA problem converges to that of the true problem. To prove the convergence property, we need to first prove the convergence of the chance constrained part. This result can be achieved using a similar approach as described in [22]. Secondly, after converting the chance constraint into the MILP formulation, we should notice that the first-stage problem in the whole SAA problem of the true problem is a pure integer program, and the second-stage problem is a mixed integer linear program. According to [21], the solution of such an SAA problem will converge to that of the true problem.

In [21] and [22], the procedures for solution validation of SAA problems have been developed for the two-stage problem and for the chance-constrained problem, respectively. Let $\bar{x}$ and $\bar{v}$ be an optimal solution and the corresponding optimal objective value for the SAA problem, respectively. To validate the quality of $\bar{x}$, the validation process obtains upper and lower bounds for $\bar{v}$ of the true problem. Usually, the solution validation needs to consider the feasibility when dealing with chance constraints (e.g., chance constraints contain both first and second stage decision variables), because it is not guaranteed that the solution of the SAA problem always satisfies the chance constraints with a large scenario size. However, in this paper, the chance constraints are only considered in the second stage. The second-stage decision is made after the scenarios are realized. Thus, the chance constraints can always be satisfied by tuning the second stage decision variables. We apply directly the validation process in [21] to construct statistical bounds for the objective value of our SAA problem.

C. SAA Algorithm Framework

To describe the SAA algorithm, we first introduce additional notation. For instance, we let N be the scenario size of the SAA problem, K be the iteration number, N' be the scenario size of the validation process to obtain a lower bound, $\widehat{g}$ be the lower bound of the true problem, $\bar{x}_k$ and $\bar{v}_k$ be the optimal solution and optimal objective value in iteration k, and $\bar{v}$ be the upper bound of the true problem. The SAA algorithm can be summarized as follows with a flowchart in Fig. 1:

1. Set $k = 1, 2, \ldots, K$ and repeat the following steps for each k:
   (1) For a given sample size N, generate a corresponding SAA problem and solve the SAA problem to obtain $\bar{x}_k$ and $\bar{v}_k$;
   (2) For a given sample size N' for the validation process, generate independent scenarios $\xi^1, \xi^2, \ldots, \xi^{N'}$, and estimate the lower bound of the problem using the following formula:

$$\widehat{g}^k = f(\bar{x}_k^f) + \frac{1}{N'}\sum_{n=1}^{N'}\mathcal{Q}(\bar{x}_k^f, \xi^n), \quad (25)$$

   where $\mathcal{Q}(\bar{x}_k^f, \xi^n)$ is the second-stage problem defined in (7)-(18) with x^f fixed as $\bar{x}_k^f$.
2. Take the average of $\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_K$. The upper bound can be obtained as $\bar{v} = \frac{1}{K}\sum_{k=1}^{K}\bar{v}_k$ following Theorem 1 in [26].
3. Take the maximum of $\widehat{g}^1, \widehat{g}^2, \ldots, \widehat{g}^K$. The lower bound can be obtained as $\widehat{g} = \max_{1\le k\le K}\widehat{g}^k$.
4. Estimate the optimality gap: $(\bar{v}-\widehat{g})/\widehat{g}\times 100\%$.

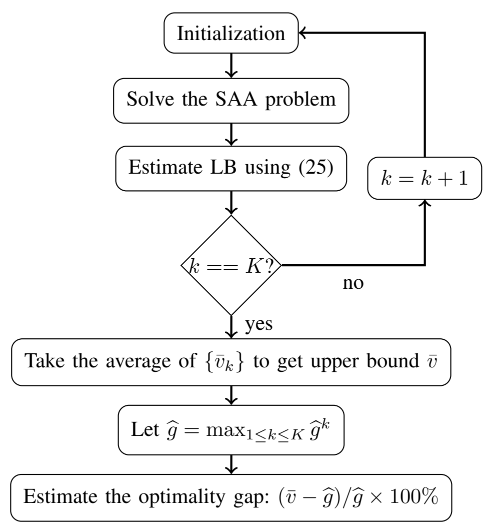

Fig. 1. Proposed SAA Algorithm

D. Heuristics for Solving Each SAA Problem

As shown in Section III-A, each SAA problem contains many integer variables which make the problem hard to solve by commercial solvers like CPLEX [27] under default settings. In addition, there are many binary decision variables in the second stage due to hydro operations. When the system becomes larger, the number of integer variables will increase significantly. To reduce the computational complexity of the problem, we apply heuristic methods to obtain good feasible solutions, and tight inner upper and lower bounds of the optimal objective value for each SAA problem. Based on optimization theory, in our heuristic approach, we solve a relaxation problem by relaxing the second stage binary decision variables to be fractional, which provides an upper bound for our maximization problem. In addition, any feasible solution leads to a corresponding lower bound.

1) Inner upper bound (Inner UB): The basic idea is to use the relaxation to get an inner upper bound for a maximization problem. Since the integer variables lead to the difficulty of solving each SAA problem, we relax the integrality for constraints (14) and (15) while maintaining the integrality of the z variables in (24) to get an inner upper bound for the given SAA problem. Note here that this approach is more effective for small sample sizes.

2) Inner lower bound (Inner LB): We create a feasible solution to get an inner lower bound for the given SAA problem in this part. Note that the first-stage solution from obtaining the inner upper bound in III-D1 should also satisfy the first-stage constraints in the SAA problem. Therefore, we can fix the first-stage solution obtained from the above part in III-D1, and solve the second-stage sub-problem to obtain a feasible solution and a corresponding inner lower bound for the SAA problem.

Moreover, when we use the inner upper bound and corresponding solution derived in III-D1 to replace $\bar{v}_k$ and $\bar{x}_k$ in Step 1.1 in III-C in the calculation of obtaining the upper bound in the validation process, we still obtain the upper bound for the original true problem. Similarly, using the inner lower bound and corresponding solution obtained in III-D2 to replace $\bar{v}_k$ and $\bar{x}_k$ in Step 1.1 in III-C in the calculation of obtaining the lower bound in the validation process provides the lower bound for the original true problem.

IV. Computational Results

In this section, we first study a three-generator system in a single bus to illustrate the proposed algorithm. Second, we consider a more complicated large system in a single bus to evaluate the performance of the heuristic approach. Finally, we evaluate the performance of our algorithm on a generalized multi-bus system (e.g., thermal, wind and hydro units are located in different buses), by comparing it with the case in which each bus is considered separately. It should be noted that our SAA solution framework can be applied to larger systems as well although the sizes of the test systems used in this paper are moderate. The reason for us to use a moderate size instance is that an IPP with an excessively large generation portfolio may most likely be able to influence the market prices, which violates our assumption of a price-taker. For the three-generator system, we run the computational experiments on the SAA problems at different risk levels and different sample sizes for comparison. The SAA algorithm described in Section III is also tested for this system. For the complicated system, we consider the heuristic method described in Section III-D to solve the SAA problem and run the computational experiments to test the heuristic-based SAA algorithm. The codes are written in C++ and the problem is solved with CPLEX 12.1. All the experiments are implemented on a computer workstation with 4 Intel Cores and 8GB RAM.

A. Scenario Generation for Uncertain Wind Power and Price

In the SAA framework, we need to generate scenarios by Monte Carlo simulation. In our approach, MISO wind power and price historical data are utilized for our case studies. The wind power data is available in the National Renewable Energy Laboratory (NREL) 2006 eastern wind data set. The Locational Marginal Price (LMP) historical data is provided by MISO. The state-of-the-art time series models for wind power generation are in two categories: wind speed-based approaches and wind power-based approaches. The wind speed-based approaches (see, e.g., [28], [29]) apply the time series model to generate wind speed scenarios and convert them into wind power output. The wind power-based approaches consider wind power time series directly (see, e.g., [30]). To capture the wind power uncertainty, we now first apply the time series model to analyze the historical wind data available from NREL [31]. The method in [30] is applied to construct the ARIMA-based model. A Monte Carlo simulation is then performed on random noise which is subject to a normal distribution in the ARIMA model to generate scenarios. As LMPs are very difficult to forecast themselves due to a variety of factors such as strategic bidding or transmission congestion, we assume the uncertainties of day-ahead and real-time LMPs to follow a Gaussian distribution and follow the method described in [32] to generate price scenarios. That is, we use the historical data to get the mean and variance for the Gaussian distribution for the day-ahead and real-time LMPs, respectively. Then, the iid samples are generated from the Gaussian distribution with the estimated mean and variance, plus the white noise following the standard normal distribution. Note that the proposed model in this paper can be applied to other scenario generation approaches without loss of generality by changing the scenario generation approach accordingly. For example, as described in [10], the wind power can be assumed to follow a multivariate Gaussian distribution in Monte Carlo simulation.

B. Three-Generator System

In this subsection, we study a simple case in which an IPP owns and operates three thermal generators, one wind farm, and one pumped-storage unit. For this small instance, each SAA problem can be solved by CPLEX with default settings directly. Therefore, the heuristic method in Section III-D is not considered. We report the computational results at different risk levels and scenario sizes. The characteristics of thermal and pumped-storage units are described in Tables I-III.

TABLE I
GENERATOR DATA

Unit	Lower (MW)	Upper (MW)	Min-down (h)	Min-up (h)	Ramp (MW/h)
G_1	50	100	2	4	40
G_2	100	150	3	3	30
G_3	20	50	3	2	15

In order to run the model in CPLEX effectively, we linearize the fuel cost function by using the interpolation method [33].

TABLE II
FUEL DATA

Unit	a (MMBtu)	b (MMBtu /MWh)	c (MMBtu /MW^{2}h)	Start-up (MMBtu)	Fuel Price ($/MMBtu)
G_1	50	6	0.0004	100	1.246
G_2	40	5.5	0.0001	300	1.246
G_3	60	4.5	0.005	0	1.246

TABLE III
PUMPED-STORAGE

Upper (MW)	Lower (MW)	Start Level (MWh)	End Level (MWh)	η_1	η_2
20	0	5	10	0.9	0.9

Accordingly, the fuel cost function in (7) is replaced by a piecewise linear function.

1) Optimal solution with ten scenarios: We report the optimal solution of the SAA algorithm with ten scenarios and risk level $\epsilon = 10\%$ in this subsection. Table IV reports the unit commitment status for each generator. It can be observed that G_1 is committed mostly, The reason is that G_1 has more flexible lower/upper bounds and ramp limits than G_2 and G_3. The flexibility of these characteristics allows G_1 to accommodate the wind power better.

TABLE IV
OPTIMAL UNIT COMMITMENT

T	Hours (1-24)
G_1	0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1
G_2	0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 0 0 0 0
G_3	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

To show the effectiveness of the pumped-storage unit, we compare the average imbalance q_{tb}^{imb} value with and without the pumped-storage unit. The average imbalance decreases from 6.8 to 2.5 when we have the pumped-storage unit in the system.

2) Sensitivity analysis for different risk levels and different scenario sizes: The numerical results on different risk levels and different scenario sizes are reported in the following Figs. 2 and 3. From Fig. 2, it can be observed that the total profit increases as the risk level increases. This is reasonable because the total profit will be lower if the utilization requirement of wind power output is more restrictive. To verify the convergence property of the SAA algorithm shown in Section III, the SAA algorithm is tested numerically by setting different sample sizes. The results shown in Fig. 3 (with the risk level to be 10%) indicate that the objective function oscillates at the beginning when the sample size is small and then converges slowly to the optimal objective value.

C. Computational Results for a Complicated System

In this subsection, we report the case study result of a more complicated system. We assume the IPP owns five generators,

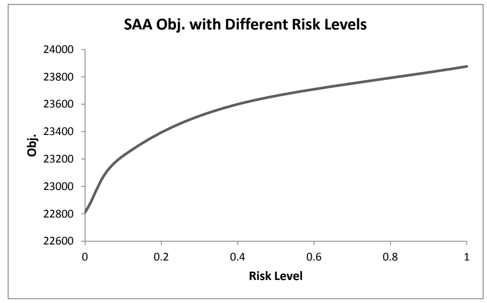

Fig. 2. Obj.($) of the SAA Problem with Different Risk Levels

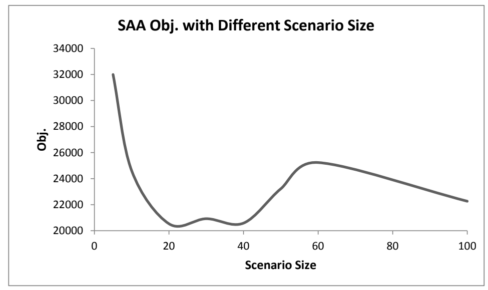

Fig. 3. Obj.($) of the SAA Problem with Different Scenario Sizes

five wind farms, and two hydro units. G_1 and G_2 used in the three-generator system are duplicated in this case study setting.

Under this setting, each SAA problem cannot be solved to optimality within the two-hour time limit when the scenario size reaches 50. The reason is that the wind power scenarios vary extensively such that the computational complexity is dominated by the chance constraint. However, our heuristic approach can still provide tight inner lower and upper bounds. As shown in Table V, the lower bound matches the upper bound when the sample size is no larger than 50. When the sample size increases, we can stop our heuristic algorithm if the time limit is reached. Accordingly, we can still report the corresponding lower and upper bounds for each SAA. The drawback for this approach is that it potentially increases the optimality gap for each SAA problem. However, as we increase the iteration number K and the validation process sample size N', the final estimated optimality gap for the SAA framework can still be reduced to a small number, as shown in Table VI.

TABLE V
COMPUTATIONAL RESULTS FOR A COMPLICATED SYSTEM FOR EACH SAA PROBLEM - HEURISTIC METHOD (RISK LEVEL: 10%)

N	Inner LB	Inner UB	CPU Time(s)
10	128690	128690	3.8
30	123820	123820	605.6
50	124688	124688	2476.9

TABLE VI
RESULTS OF SOLUTION VALIDATION FOR A COMPLICATED SYSTEM (RISK LEVEL: 10%)

(K, N')	LB	UB	Gap	CPU Time(s)
(10, 200)	122410	129672	5.9%	1334.5
(30, 500)	121040	128765	6.3%	3243.8
(50, 800)	122818	126637	3.1%	5453.2

TABLE VIII
COMPUTATIONAL RESULTS FOR DISTRIBUTED SYSTEM

Risk Level	Obj. with Coupling Chance Constraint	Obj. with Separating Chance Constraints
0%	79873	62968
10%	92266	76452
40%	113419	104145
100%	153631	153631

D. Multi-bus System

In the above larger system in Section IV-C, we assume all the IPP's power generation resources are in a single bus, or aggregated. It is common in practice that the IPP's power generation resources are distributed at different buses as described in the model in Section II. In such a case, the electricity prices at different buses might be different. One can separately solve the problem for each bus, which can save the computational time. In this subsection, we use our model to consider different buses simultaneously since the objective of the IPP should be to maximize the profit of its entire generation portfolio located at different buses. While each bus contains its own price information and power bidding balance constraints, one chance constraint is applied on the total wind utilization for the whole multi-bus generation portfolio. The chance constraint is the coupling constraint for all buses. In our experiment, we assume the IPP's power generation resources are distributed in five different buses. G_1 used in the three-generator system is duplicated as the fourth generator in this case study. The detailed settings are summarized in Table VII.

TABLE VII
BUS SETTINGS

Bus No.	Wind	Thermal	Hydro
1	1	1	1
2	1	2	0
3	1	1	1
4	2	1	2
5	1	0	0

The computational results are reported in Table VIII. It can be observed that the proposed method which considers the coupling chance constraint provides a larger total profit. This matches the theoretical result. That is, any solution of the separated chance constrained problem must be a feasible solution of the coupling chance constrained problem, which leads to the fact that the coupling chance constrained problem provides a better solution with a higher total profit. We therefore conclude that sharing resources inside the multi-bus system can tackle the uncertainties better and offer a higher profit in general.

V. CONCLUSION

In this paper, a stochastic programming model is proposed to address the price-based unit commitment problem with wind power utilization constraints. Our model incorporates day-ahead price, real-time price, and wind power output uncertainties. In the first stage, an IPP makes decisions on unit commitment and the amount of energy offered for the day-ahead market. The economic dispatch of generators is made in the second stage. A chance constraint is considered to ensure the utilization of the volatile wind power to a large extent. In other words, there is a great chance the usage of the wind power satisfies a pre-defined percentage. The chance constraint allows the power producer to adjust the utilization of wind power based on different regulations.

Our model maximizes the profit and accommodates the required usage of wind power output. An SAA algorithm is developed to solve the problem, and the objective value of the SAA problem converges to the optimal one as the scenario size increases. For more complicated systems, we propose a heuristic approach to accelerate the SAA algorithm. Our implementation provides the overall upper and lower bounds for the true problem. The reasonable estimated optimality gap and moderate computational time verify that our approach is effective in solving this problem.

ACKNOWLEDGMENTS

The authors would like to thank the editor and reviewers very much for the sincere suggestions which help improve the quality of this paper significantly. This research was supported in part by University of Chicago Argonne, LLC, Operator of Argonne National Laboratory ("Argonne"). Argonne, a U.S. Department of Energy Office of Science laboratory, is operated under Contract No. DE-AC02-06CH11357.

REFERENCES

[1] U. S. Department of Energy, "20% wind energy by 2030: Increasing wind energy's contribution to U.S. electricity supply," *Washington, DC*, July 2008.

[2] Global Wind Energy Council. Global wind 2008 report. [Online]. Available: http://www.gwec.net.

[3] S. Jensen and K. Skytte, "Interactions between the power and green certificate markets," *Energy Policy*, vol. 30, no. 5, pp. 425–435, 2002.

[4] L. Nielsen and T. Jeppesen, "Tradable green certificates in selected European countries–overview and assessment," *Energy policy*, vol. 31, no. 1, pp. 3–14, 2003.

[5] J. García-González, R. de la Muela, L. Santos, and A. González, "Stochastic joint optimization of wind generation and pumped-storage units in an electricity market," *Power Systems, IEEE Transactions on*, vol. 23, no. 2, pp. 460–468, 2008.

[6] C. Monteiro, R. Bessa, V. Miranda, A. Botterud, J. Wang, and G. Conzelmann, "Wind power forecasting: state-of-the-art 2009," *Argonne National Laboratory, ANL/DIS-10-1*, November, 2009.

[7] P. Pinson, H. Madsen, H. Nielsen, G. Papaefthymiou, and B. Klöckl, "From probabilistic forecasts to statistical scenarios of short-term wind power production," *Wind Energy*, vol. 12, no. 1, pp. 51–62, 2009.

[8] Wind power integration in liberalised electricity markets (wilmar) project. [Online]. Available: http://www.wilmar.risoe.dk.

[9] R. Barth, H. Brand, P. Meibom, and C. Weber, "A stochastic unit-commitment model for the evaluation of the impacts of integration of large amounts of intermittent wind power," in *9th International Conference on Probabilistic Methods Applied to Power Systems*, June 2006, pp. 1–8.

[10] J. Wang, M. Shahidehpour, and Z. Li, "Security-constrained unit commitment with volatile wind power generation," *Power Systems, IEEE Transactions on*, vol. 23, no. 3, pp. 1319–1327, 2008.

[11] B. Ummels, M. Gibescu, E. Pelgrum, W. Kling, and A. Brand, "Impacts of wind power on thermal generation unit commitment and dispatch," *IEEE Transactions on Energy Conversion*, vol. 22, no. 1, pp. 44–51, 2007.

[12] T. Li, M. Shahidehpour, and Z. Li, "Price-based unit commitment: A case of lagrangian relaxation versus mixed integer programming," *Power Systems, IEEE Transactions on*, vol. 20, no. 4, pp. 2015–2025, 2005.

[13] ——, "Risk-constrained bidding strategy with stochastic unit commitment," *Power Systems, IEEE Transactions on*, vol. 22, no. 1, pp. 449–458, 2007.

[14] M. Shahidehpour, H. Yamin, and Z. Li, *Market Operations in Electric Power Systems*. John Wiley and Sons, 2002.

[15] E. H. Allen and M. Ilic, *Price-Based Commitment Decisions in the Electricity Market*. Springer-Verlag London Series on Advances in Industrial Control, 1998.

[16] U. Özturk, M. Mazumdar, and B. Norman, "A solution to the stochastic unit commitment problem using chance constrained programming," *Power Systems, IEEE Transactions on*, vol. 19, no. 3, pp. 1589–1598, 2004.

[17] H. Yu, C. Y. Chung, K. P. Wong, and J. H. Zhang, "A chance constrained transmission network expansion planning method with consideration of load and wind farm uncertainties," *Power Systems, IEEE Transactions on*, vol. 24, no. 3, pp. 1568–1576, 2009.

[18] Q. Wang, Y. Guan, and J. Wang, "A chance-constrained two-stage stochastic program for unit commitment with uncertain wind power output," *Power Systems, IEEE Transactions on*, vol. 27, no. 1, pp. 206 –215, 2012.

[19] Q. Wang, J. Wang, and Y. Guan, "Stochastic unit commitment with uncertain demand response," *Power Systems, IEEE Transactions on, to be published*, 2012.

[20] A. Kleywegt, A. Shapiro, and T. Homem-De-Mello, "The sample average approximation method for stochastic discrete optimization," *SIAM Journal on Optimization*, vol. 12, no. 2, pp. 479–502, 2002.

[21] S. Ahmed and A. Shapiro, "The sample average approximation method for stochastic programs with integer recourse," ISyE, Georgia Institute of Technology, GA, Tech. Rep., Feb. 2002.

[22] B. Pagnoncelli, S. Ahmed, and A. Shapiro, "Sample average approximation method for chance constrained programming: theory and applications," *Journal of Optimization Theory and Applications*, vol. 142, no. 2, pp. 399–416, 2009.

[23] J. Luedtke, S. Ahmed, and G. Nemhauser, "An integer programming approach for linear programs with probabilistic constraints," *Mathematical Programming*, vol. 122, no. 2, pp. 247–272, 2010.

[24] PJM course: Two settlements. [Online]. Available: http://www.pjm.com/sitecore/content/Globals/Training/Courses/ol-two-sett.aspx.

[25] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[26] W.-K. Mak, D. P. Morton, and R. K. Wood, "Monte carlo bounding techniques for determining solution quality in stochastic programs," *Operations Research Letters*, vol. 24, pp. 47–56, 1999.

[27] IBM ILOG CPLEX Optimizer. [Online]. Available: http://www.ibm.com/.

[28] A. Papavasiliou, S. Oren, and R. O'Neill, "Reserve requirements for wind power integration: A scenario-based stochastic programming framework," *Power Systems, IEEE Transactions on*, vol. 26, no. 4, pp. 2197–2206, 2011.

[29] B. Brown, R. Katz, and A. Murphy, "Time series models to simulate and forecast wind speed and wind power," *Journal of Climate and Applied Meteorology*, vol. 23, pp. 1184–1195, 1984.

[30] P. Chen, T. Pedersen, B. Bak-Jensen, and Z. Chen, "Arima-based time series model of stochastic wind power generation," *Power Systems, IEEE Transactions on*, vol. 25, no. 2, pp. 667–676, 2010.

[31] NREL eastern wind dataset. [Online]. Available: http://www.nrel.gov/wind/integrationdatasets/eastern/data.html

[32] A. Botterud, Z. Zhou, J. Wang, R. Bessa, H. Keko, J. Sumaili, and V. Miranda, "Wind power trading under uncertainty in LMP markets," *Power Systems, IEEE Transactions on*, vol. 27, no. 2, pp. 894 –903, 2012.

[33] H. Triebel, *Interpolation Theory, Function Spaces, Differential Operators*. North-Holland Amsterdam, 1978.