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# Two-stage Robust Optimization for Power Grid with Uncertain Demand Response

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## Abstract

The Demand Response Program (DRP) has recently attracted much attention from Independent System Operators (ISO) in managing end-use customers' electricity consumption behavior. In this paper, we propose a model that considers the uncertainty in customers' responses to time-varying prices. We develop a two-stage robust optimization model that maximizes the social welfare under unit commitment constraints. We use the Bender's Decomposition method to handle the two-stage robust optimization problem. Finally, we test the performance of the proposed approach through extensive case studies, and verify that the power system obtained by the robust optimization approach is more reliable.

## Keywords
Demand Response Uncertainty, Price-elastic Demand Curve, Bender's Decomposition, Robust Optimization

## 1. Introduction
In most electricity markets, electricity price is constant and independent of the time it was used. But constant price may cause the imbalance between consumption and production due to the inability to meet the high electrical demand during peak hours. The Demand Response Program (DRP) aims at managing end-user's electricity consumption pattern via time-varying price, or offering incentive payments to reduce the consumption of electricity at times of high electricity demand or when system stability is jeopardized [1]. The program can benefit the load-serving entities, consumers, and Independent System Operators (ISO) [1],[2],[3], including:

- Load-serving entities:
  - Demand response can reduce electricity production cost by shifting the demand of electricity from peak hours to off-peak hours.
  - Demand response can lower the capacity requirements for load-serving entities, which leads to the reduction of electricity production cost.
- Consumers:
  - Those who adjust their electricity demand from high price periods to low price periods will reduce their electricity costs.
  - Consumers that have no response to time-varying prices may also save money due to lower electricity production cost.
- ISO:

- ○ Demand response can help balance the electricity consumption and production, and therefore ensure a more stable, reliable and controllable grid system.

U.S. Department of Energy reported that in 2004, demand response potential was 3% of the total U.S. peak demand. In order to "ensure that demand response is treated comparably to other resources", the Federal Energy Regulatory Commission (FERC) requires ISO and regional transmission organizations (RTOs) to "accept bids from demand response resources in their markets for certain ancillary services, comparable to other resources"[4]. On the other hand, several regional grid operators (e.g., NYISO, PJM, ISO-NE, and ERCOT) provide opportunities for customers to participate in the DRP, progressively integrating demand response resources into the wholesale energy market.

To see the effects of customers' participation in the DRP, we model how customers respond to time-varying prices, or spot prices. The spot price is set by ISO every hour, half-hour or 15 minutes, and customers can adjust their electricity consumptions based on the change in spot price [5]. For example, consumers can cook dinner at off-peak hours, or they may switch off the air-conditioning when facing high electricity prices. In general, the lower the retail electricity price, the higher the corresponding electricity consumption. However, electricity producers are usually more willing to supply electricity under higher electricity prices. We can model the demand-price curve and the supply-price curve as Fig. 1 [6]. The electricity supply and demand reach an equilibrium at the intersection point $(P^*, Q^*)$, corresponding to price $P^*$.

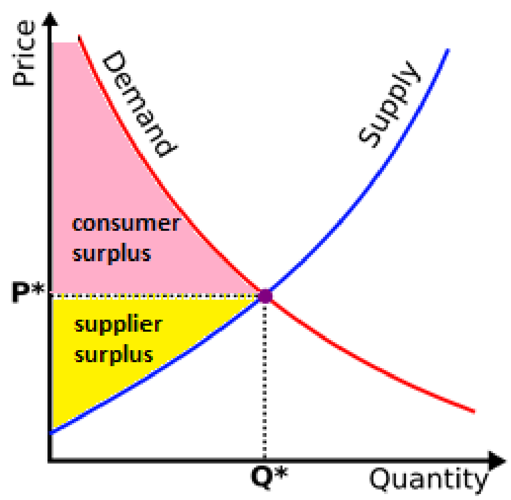


Figure 1: Demand-price curve and supply-price curve

The social welfare is defined as the summation of customer surplus and supplier surplus. The objective of ISO is to maximize the social welfare in real time market, while satisfying a series of operational constraints and maintaining the reliability of the power grid. However, there are several challenges:

- How to properly model customer behavior under varying spot prices. In other words, how to describe the demand-price curve.
- For a given spot price, the corresponding electricity consumption may be uncertain. Possible reasons for the uncertainty include lack of attention, change in consumption behavior and weather conditions. As a result, modeling the uncertain demand-price curve in real time market is challenging.
- In order to balance supply and demand, ISO should schedule the output of each unit to obey the unit commitment constraints while maximizing the social welfare.

In most articles, the demand-price curve is measured by demand price elasticity, which represents the sensitivity of electricity demand to price changes [7], [8]. For a given reference point $(P_0, Q_0)$, the price elasticity is defined as:

$$\alpha = \frac{\Delta P / P_0}{\Delta Q / Q_0} \tag{1}$$

In [3], the elasticity value is simplified as $\alpha = \frac{\Delta P}{\Delta Q}$ (i.e. linearize the price-elastic demand curve). In [9], the author approximates the price-elastic demand curve as a stepwise linear curve. In [10], the author develops the concept of "self-elasticity" and "cross-elasticity" when the change in price of one commodity affects both its own demand and the demand of another commodity. They show how these elasticities can model customers' behaviors and the set of spot price. In our model, we consider an uncertain price-elastic demand curve that fluctuates within a certain range.

The remaining part of the paper is organized as follows: In section 2, we describe the mathematical formulation. In section 3, we discuss the uncertain set for the demand-price curve, and develop the solution approach to solve the problem. In section 4, we provide some case studies and present our computational results. Finally, we conclude our research in section 5.

## 2. The Model and Assumption

For a T-period power grid optimization problem, we denote $M$ as the number of generators. We use $t$ as the index of time periods and $i$ as the index of generators. For each time period t, and each generator $i$, let $S_i$ represent the start-up cost, $W_i$ represent the shut-down cost, $L_i$ represent the minimum-up time, $G_i$ represent the minimum-down time, $U_i$ represent the maximal generating capacity, $Q_i$ represent the minimal generating capacity, $R_i$ represent the ramp-up limit, and $P_i$ represent the ramp-down limit. Let $C_1$ represent the lower bound of the total demand, and $C_2$ represent the upper bound of the total demand.

In the first stage, the unit commitment decisions are binary variables $y_{it}$, $u_{it}$, and $v_{it}$. $y_{it}$ indicates if the generator $i$ is on during time period $t$, $u_{it}$ indicates if the generator $i$ is started up during time period $t$, and $v_{it}$ is to indicate if the generator $i$ is shut down during time period $t$. In the second stage, let $d_t$ represent the demand in time period $t$, and $x_{it}$ represent the amount of electricity generated or received by the generator $i$ during time period $t$.

According to the definition of social welfare, the objective value is the integral of the demand curve minus the integral of the supply curve. Let $r_t(d_t)$ represent the integral of the demand curve, and $f_{it}(x_{it})$ represent the fuel cost for the generator $i$ in time period $t$. The nominal model is:

$$\max \sum_{t=1}^{T} r_t(d_t) - \sum_{t=1}^{T}\sum_{i=1}^{M}(f_{it}(x_{it}) + S_i u_{it} + W_i v_{it}) \tag{2}$$

$$s.t. \quad -y_{i(t-1)} + y_{it} - y_{ik} \le 0, \quad (1 \le k-(t-1) \le L_i, i = 1,2,\cdots,M, t = 1,2,\cdots,T), \tag{3}$$

$$y_{i(t-1)} - y_{it} + y_{ik} \le 1, \quad (1 \le k-(t-1) \le G_i, i = 1,2,\cdots,M, t = 1,2,\cdots,T), \tag{4}$$

$$-y_{i(t-1)} + y_{it} - u_{it} \le 0, \quad (i = 1,2,\cdots,M, t = 1,2,\cdots,T), \tag{5}$$

$$y_{i(t-1)} - y_{it} - v_{it} \le 0, \quad (i = 1,2,\cdots,M, t = 1,2,\cdots,T), \tag{6}$$

$$Q_i y_{it} \le x_{it} \le U_i y_{it}, \quad (i = 1,2,\cdots,M, t = 1,2,\cdots,T), \tag{7}$$

$$x_{it} - x_{i(t-1)} \le y_{i(t-1)} R_i + (1-y_{i(t-1)}) U_i, \quad (i = 1,2,\cdots,M, t = 1,2,\cdots,T), \tag{8}$$

$$x_{i(t-1)} - x_{it} \le y_{i(t-1)} P_i + (1-y_{i(t-1)}) U_i, \quad (i = 1,2,\cdots,M, t = 1,2,\cdots,T), \tag{9}$$

$$\sum_{i=1}^{M} x_{it} = d_t, \quad (t = 1,2,\cdots,T), \tag{10}$$

$$C_1 \le \sum_{t=1}^{T} d_t \le C_2, \tag{11}$$

$$y_{it}, u_{it}, v_{it} \in \{0,1\}, x_{it}, d_t \ge 0, \quad (i = 1,2,\cdots,M, t = 1,2,\cdots,T). \tag{12}$$

In the above formulation, the objective function is to maximize the social welfare. Constraint (3) means once the unit is started up, it should not be turned off within a certain time. Constraint (4) describes that once the unit is turned down, a minimum time required before it can be started up again. The following two constraints indicate the status of the units (i,e, switched on or switched off). Constraint (7) describes the upper and lower bound of the unit's power output. Ramping constraints (8), (9) limit the maximum increase or decrease of generated power from one time period to the next. Constraint (10) ensures the demand is met. Constraint (11) describes the lower and upper bound for the total demand.

## 3. Solution Methodology

### 3.1 The approximation of objective function

In the above formulation, there are two nonlinear terms in the objective function:

- $r_t(d_t)$

- $f_{it}(x_{it})$

Now we will discuss the approximation of these two nonlinear terms.

- The approximation of $r_t(d_t)$:

In our model, the customer demand response is represented as a price-elastic demand curve. If the price elasticity is constant for the demand curve, we can describe the price-elastic demand curve as: $d_t = A_t p_t^{\alpha_t}$, where $\alpha_t$ is the given price elasticity for time period $t$. $A_t$ is a parameter that can be decided by a given reference point $(D_t^{ref}, P_t^{ref})$ [8]. Then, as Fig. 2 shows, a step-wise function is applied to approximate this demand-price function. We approximate $r_t(d_t)$ as:

$$r_t(d_t) = \sum_{k=1}^{K} p_t^k h_t^k, d_t = \sum_{k=1}^{K} h_t^k, 0 \le h_t^k \le l_t^k, \quad (t = 1,2,\cdots,T, k = 1,2,\cdots,K) \tag{13}$$

where $(p_t^k, l_t^k)$ is the point at step k for the step-wise function; $h_t^k$ is the variable introduced for demand at step k; K is the number of steps.

Notice that $p_t^k$ is strictly decreasing with k. Since we are maximizing $r_t(d_t)$, we will have: $h_t^z = \begin{cases} l_t^z, & \text{if } z < z_0; \\ [0, l_t^z], & \text{if } z = z_0; \\ 0, & \text{if } z > z_0. \end{cases}$

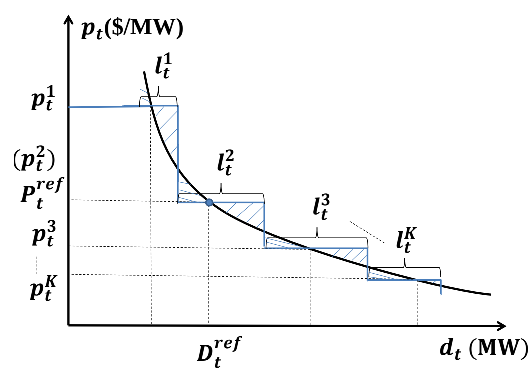


Figure 2: Step-wise approximation of price-elastic demand curve

when $\sum_{k=1}^{z_0-1} l_t^k < d_t \le \sum_{k=1}^{z_0} l_t^k$ for a certain $z_0$. In this case, we can prove that (13) is justified.

- The approximation of $f_{it}(x_{it})$:

In practice, the fuel cost function $f_{it}(x_{it})$ can be expressed as a quadratic function, which we approximate as the following $N$-piece piecewise linear function [11]:

$$\phi_{it} \ge \alpha_{it}^j y_{it} + \beta_{it}^j x_{it}, \quad (i = 1,2,\cdots,M, t = 1,2,\cdots,T, j = 1,2,\cdots,N) \tag{14}$$

where $\alpha_{it}^j$ is the intercept of the $j$th segment line and $\beta_{it}^j$ is the slope of the $j$th segment line.

### 3.2 The uncertain set

In this part, we model the uncertainty of demand-price curve as an uncertain set. As illustrated in Figure 3, for a given certain price $p_0$, the corresponding demand is uncertain. Similarly, for a certain demand $d_0$, the price will fluctuate within a corresponding range. For computational convenience, in our model, we consider for each demand $d_t$ in the step-wise curve obtained by the previous steps, the corresponding $p_t^k$ is allowed to wing in the range $p_t^k \in [p_t^{k*} - \bar{\varepsilon}_t, p_t^{k*} + \underline{\varepsilon}_t]$, where $p_t^{k*}$ is the forecasted value for $p_t^k$, and $\bar{\varepsilon}_t$ and $\underline{\varepsilon}_t$ are the deviations for $p_t^k$. To adjust the degree of conservation, we restrict the number of time periods that allow the price's uncertainty, and call it "uncertain budget" $\upsilon$. It can be observed that the greater the $\upsilon$, the more conservative the system. So we can adjust the robustness through changing the value of $\upsilon$. We describe uncertainty set as follows:

$$P_c = \{p : p_t^{k*} - \underline{\varepsilon}_t z_t \le p_t^k \le p_t^{k*} + \bar{\varepsilon}_t z_t, \sum_{t=1}^{T} z_t \le \upsilon, z_t \in \{0,1\}, \forall k = 1,\cdots,K\} \tag{15}$$

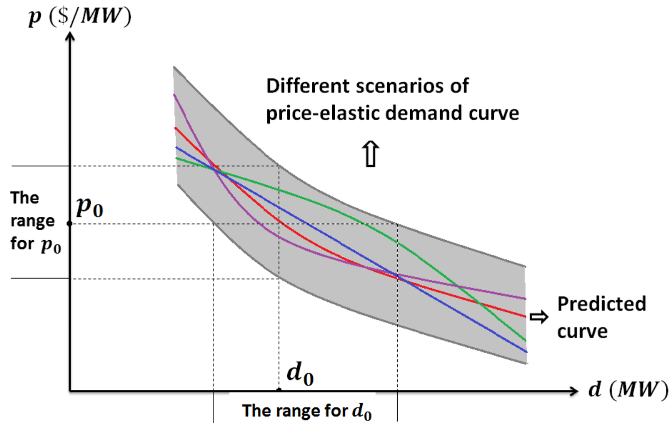


Figure 3: The uncertainty of price-elastic demand curve

**3.3 Two-stage robust optimization problem**

We consider a two-stage robust optimization problem. In the first stage, we determine a turn-on and turn-off schedule of electrical power generating units by satisfying unit commitment constraints. In the second stage, we decide how much electricity each unit should generate to maximize the social welfare under the worst case scenario.

Then we can rewrite the model as:

$$\begin{aligned}
&\max_{y,u,v} -\sum_{t=1}^{T}\sum_{i=1}^{M}(S_i u_{it} + W_i v_{it}) + \min_{p\in P_c}\max_{x,h,\phi\in\chi}\Big(\sum_{t=1}^{T}\sum_{k=1}^{K} p_t^k h_t^k - \sum_{t=1}^{T}\sum_{i=1}^{M}\phi_{it}\Big)\\
s.t.\quad & (3),(4),(5),(6)\\
& y_{it}, u_{it}, v_{it} \in \{0,1\} \quad (i=1,2,\cdots,M, t=1,2,\cdots,T) \qquad (16)
\end{aligned}$$

where $\mathcal{X} = \Big\{$

$$(7),(8),(9),(14)$$

$$\sum_{i=1}^{M} x_{it} = \sum_{k=1}^{K} h_t^k \quad (t=1,2,\cdots,T) \tag{17}$$

$$h_t^k \le l_t^k \quad (t=1,2,\cdots,T, k=1,2,\cdots,K) \tag{18}$$

$$\sum_{t=1}^{T}\sum_{k=1}^{K} h_t^k \le C_1 \tag{19}$$

$$\sum_{t=1}^{T}\sum_{k=1}^{K} h_t^k \ge C_2 \tag{20}$$

$$x_{it} \ge 0, h_t^k \ge 0, \quad (i=1,2,\cdots,M, t=1,2,\cdots,T, k=1,2,\cdots,K)\Big\} \tag{21}$$

By dualizing the constraints in $(\mathcal{X})$ and combining the constraints of price $p$, we can transform the second-stage problem as follows:

$$\omega(y) = \min \quad \sum_{t=1}^{T}\sum_{i=1}^{M}((U_i y_{it}\gamma_{it}^{+} - Q_i y_{it}\gamma_{it}^{-}) + (U_i + (R_i - U_i)y_{i(t-1)})\tau_{it}^{+}$$

$$+(U_i + (P_i - U_i)y_{i(t-1)})\tau_{it}^{-}) - \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{N}\alpha_{it}^{j} y_{it}\zeta_{it}^{j} + \sum_{t=1}^{T}\sum_{k=1}^{K} l_t^k \delta_t^k + C_1\mu^{+} - C_2\mu^{-} \tag{22}$$

$$s.t. \quad \gamma_{it}^{+} - \gamma_{it}^{-} + \tau_{it}^{+} - \tau_{it}^{-} - \tau_{i(t+1)}^{+} + \tau_{i(t+1)}^{-} + \eta_t + \sum_{j=1}^{N}\beta_{it}^{j}\zeta_{it}^{j} \geq 0, \quad (i = 1,2,\cdots,M, t = 1,2,\cdots,T), \tag{23}$$

$$-\eta_t + \delta_t^k + \mu^{+} - \mu^{-} - p_t^k \geq 0, \quad (t = 1,2,\cdots,T, k = 1,2,\cdots,K), \tag{24}$$

$$\sum_{j=1}^{N}\zeta_{it}^{j} = 1, \quad (i = 1,2,\cdots,M, t = 1,2,\cdots,T), \tag{25}$$

$$p_t^{k*} - \underline{\varepsilon}_t z_t \leq p_t^k \leq p_t^{k*} + \bar{\varepsilon}_t z_t, \quad (t = 1,2,\cdots,T, k = 1,2,\cdots,K), \tag{26}$$

$$\sum_{t=1}^{T} z_t \leq \upsilon, \tag{27}$$

$$\gamma,\tau,\eta,\mu,\zeta,\delta \geq 0, z_t \in \{0,1\} \quad (i = 1,2,\cdots,M, t = 1,2,\cdots,T, k = 1,2,\cdots,K) \tag{28}$$

where $\gamma,\tau,\eta,\mu,\zeta,\delta$ are dual variables for constraints (7), (8-9), (10), (11), (14) and (18) respectively.
Now we use the Bender's decomposition algorithm to solve it. Replace $\min\omega(y)$ with $\theta$ and then consider the following master problem. By adding feasibility cuts and optimality cuts, the problem can be solved iteratively:

$$\omega^{\mathcal{M}} = \max_{y,u,v} \quad -\sum_{t=1}^{T}\sum_{i=1}^{M}(S_i u_{it} + W_i v_{it}) + \theta$$

$$(\mathcal{M}) \quad s.t. \quad \sum_{t=1}^{T}\sum_{i=1}^{M}\sigma_{it}^{s} y_{it} \geq \xi_s, \quad (s = 1,\cdots,S) \tag{29}$$

$$\theta - \sum_{t=1}^{T}\sum_{i=1}^{M}\hat{\sigma}_{it}^{l} y_{it} \leq \hat{\xi}_l, \quad (l = 1,\cdots,L) \tag{30}$$

$$(3),(4),(5),(6),(16)$$

where constraints (29) represent the feasibility cuts, while constraints (30) represent the optimality cuts.

### 3.4 Feasibility cuts

We use the L-shaped method to generate feasibility cuts. In this case, we don't need to consider the constraint (14) since it will not affect the feasibility. The corresponding formulation is shown as follows:

$$\omega^{l}(y) = \min\sum_{t=1}^{T}\sum_{i=1}^{M}((U_i y_{it}\hat{\gamma}_{it}^{+} - Q_i y_{it}\hat{\gamma}_{it}^{-}) + (U_i + (R_i - U_i)y_{i(t-1)})\hat{\tau}_{it}^{+}$$

$$+(U_i + (P_i - U_i)y_{i(t-1)})\hat{\tau}_{it}^{-}) + \sum_{t=1}^{T}\sum_{k=1}^{K} l_t^k \hat{\delta}_t^k + C_1\hat{\mu}^{+} - C_2\hat{\mu}^{-} \tag{31}$$

$$s.t. \quad \hat{\gamma}_{it}^{+} - \hat{\gamma}_{it}^{-} + \hat{\tau}_{it}^{+} - \hat{\tau}_{it}^{-} - \hat{\tau}_{i(t+1)}^{+} + \hat{\tau}_{i(t+1)}^{-} + \hat{\eta}_t \geq 0, \quad (i = 1,2,\cdots,M, t = 1,2,\cdots,T), \tag{32}$$

$$-\hat{\eta}_t + \hat{\delta}_t^k + \hat{\mu}_{tk}^{+} - \hat{\mu}_{tk}^{-} \geq 0, \quad (t = 1,2,\cdots,T, k = 1,2,\cdots,K), \tag{33}$$

$$0 \leq \hat{\gamma}_{it}^{+} \leq 1, 0 \leq \hat{\gamma}_{it}^{-} \leq 1, \quad (i = 1,2,\cdots,M, t = 1,2,\cdots,T), \tag{34}$$

$$0 \leq \hat{\tau}_{it}^{+} \leq 1, 0 \leq \hat{\tau}_{it}^{-} \leq 1, \quad (i = 1,2,\cdots,M, t = 1,2,\cdots,T), \tag{35}$$

$$-1 \leq \hat{\eta}_t \leq 1, \quad (t = 1,2,\cdots,T), \tag{36}$$

$$0 \leq \hat{\delta}_t^k \leq 1, \quad (t = 1,2,\cdots,T, k = 1,2,\cdots,K), \tag{37}$$

$$0 \leq \hat{\mu}^{+} \leq 1, 0 \leq \hat{\mu}^{-} \leq 1 \tag{38}$$

And we have the following conclusions:
(1) If $\omega^{l}(y) = 0$, y is feasible;

(2) If $\omega^l(y) < 0$, generate a feasibility cut as follows:

$$\sum_{t=1}^{T}\sum_{i=1}^{M}\sigma_{it}^{s}y_{it} \geq \xi_s \tag{39}$$

where

$$\sigma_{it}^{s} := U_i\hat{\gamma}_{it}^{+} - Q_i\hat{\gamma}_{it}^{-} + (R_i - U_i)\hat{\gamma}_{i(t+1)}^{+} + (P_i - U_i)\hat{\gamma}_{i(t+1)}^{-}, \tag{40}$$

$$\xi_s := -\sum_{i=1}^{M}\sum_{t=1}^{T}(U_i\hat{\gamma}_{it}^{+} + U_i\hat{\gamma}_{it}^{-}) - \sum_{t=1}^{T}\sum_{k=1}^{K} l_t^k\hat{\delta}_t^k - C_1\hat{\mu}^{+} + C_2\hat{\mu}^{-} \tag{41}$$

### 3.5 Optimality cuts

Assume in the $i$th iteration, we solve the master problem and get $\theta^i$ and $y^i$. Since we let $\theta = \min\omega(y)$, so if we substitute $y^i$ into the subproblem and get $\omega(y^i)$, we should have $\omega(y^i) \geq \theta^i$. If $\omega(y^i) < \theta^i$, we can claim that $y^i$ is not optimal and generate a optimality cut:

$$\theta - \sum_{t=1}^{T}\sum_{i=1}^{M}\hat{\sigma}_{it}^{l}y_{it} \leq \hat{\xi}_l \tag{42}$$

where

$$\hat{\sigma}_{it}^{l} := U_i\gamma_{it}^{+} - Q_i\gamma_{it}^{-} + (R_i - U_i)\gamma_{i(t+1)}^{+} + (P_i - U_i)\gamma_{i(t+1)}^{-} - \sum_{j=1}^{N}\alpha_{it}^{j}\zeta_{it}^{j}, \tag{43}$$

$$\hat{\xi}_l := -\sum_{i=1}^{M}\sum_{t=1}^{T}(U_i\gamma_{it}^{+} + U_i\gamma_{it}^{-}) - \sum_{t=1}^{T}\sum_{k=1}^{K} l_t^k\delta_t^k - C_1\mu^{+} + C_2\mu^{-} \tag{44}$$

## 4. Case Studies

In this section, we study the IEEE 118-bus system given online at *motor.ece.iit.edu/data*. In this experiment, we have 33 generators and the time horizon is 24 hours. All the experiments are implemented using CPLEX 12.2, at Intel Quad Core 2.40GHz with 8GB memory.

### 4.1 Robust case: uncertain budget vs. elasticity value

The optimal objective values, number of start-ups and CPU times corresponding to different uncertain budgets and different elasticity values are reported in TABLE 1. From the results we can observe several conclusions: 1) when the uncertain budget increases, the predicted social welfare decreases due to the augment of uncertainty; 2) when the uncertain budget raises, more generators should be started up to guarantee the balance between electricity supply and demand; 3) when the uncertain budget increases, it takes more CPU times to calculate the optimal objective value. It can also be observed that as the demand becomes more elastic (high $\alpha$), the total social welfare decreases.

Table 1: Different $\upsilon$ vs. different $\alpha$

| $\alpha$ | $\upsilon$ | Objective value | Start-ups | CPU time(s) |
|---|---|---|---|---|
| $\alpha = -1$ | 2 | 6300362 | 10 | 40 |
| | 4 | 6273063 | 10 | 256 |
| | 6 | 6246180 | 10 | 1480 |
| $\alpha = -2$ | 2 | 2994692 | 9 | 27 |
| | 4 | 2983314 | 10 | 244 |
| | 6 | 2971791 | 10 | 811 |
| $\alpha = -4$ | 2 | 2354530 | 9 | 37 |
| | 4 | 2350413 | 10 | 173 |
| | 6 | 2344642 | 10 | 1220 |

### 4.2 Deterministic case vs. robust Case

In this part, we discuss why robust demand response model performs better. In the deterministic case, we can set the uncertain budget to 0. By running the same framework, we achieve the first stage decisions based on deterministic demand response. Then considering the uncertain set in the second stage with the uncertain budget 6, we can get the optimal social welfare based on the first stage variables' values. We compare the results with the robust case in TABLE 2. From the results, we can see that the social welfare of the deterministic DR is less than the social welfare of the robust DR.

Table 2: Deterministic DR vs. Robust DR

| | | Deterministic | Robust |
|---|---|---|---|
| $\alpha = -1$<br>$\upsilon = 6$ | Generators that started up | 5,9,10,16,17,<br>18,24,26,27,28 | 2,4,5,9,10,<br>16,17,18,24,27 |
| | Obj. | 6243943 | 6245050 |
| $\alpha = -2$<br>$\upsilon = 6$ | Generators that started up | 4,5,9,10,16,<br>17,24,26,27,28 | 2,4,5,9,10,<br>16,17,18,24,27 |
| | Obj. | 2926002 | 2971791 |
| $\alpha = -4$<br>$\upsilon = 6$ | Generators that started up | 4,5,9,10,16,<br>17,18,24,27,28 | 2,4,5,9,10<br>16,17,18,24,27 |
| | Obj. | 2284540 | 2344642 |

## 5. Conclusion

In this article, we develop a robust optimization approach to maximize the social welfare under the worst case scenario. We use an uncertain price-elastic demand curve to model customer's response to price signals, and the Bender's decomposition framework to solve the problem. Finally, our computational results on an IEEE 118-bus system verify that our robust model gives better solutions than the deterministic model under the worst case scenario.

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