Stochastic Unit Commitment with Uncertain Demand Response

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***Abstract*—Although demand response (DR) encourages customers to voluntarily schedule electricity consumption based on price signals, the response from the consumer side could be uncertain due to a variety of reasons. In this letter, we study the stochastic unit commitment problem with uncertain demand response to enhance the reliability unit commitment process for ISOs. We use a stochastic representation of DR by scenario, and each scenario corresponds to a price-elastic demand curve. Contingency constraints are considered and in addition, a chance constraint is applied to ensure the loss of load probability (LOLP) lower than a pre-defined risk level. Finally, a sample average approximation (SAA) method is applied to solve the problem.**

***Index Terms*—Stochastic Programming, Unit Commitment, Contingency Analysis, Demand Response, Chance Constraint.**

I. Introduction and the Model

The objective of an Independent System Operator (ISO) is to maximize the social welfare for electricity producers and customers. Customers participating in the Demand Response (DR) program can expect savings by reducing their electricity usage during peak periods [1]. In the literature, DR was mostly modeled as a fixed demand curve. However, due to a variety of reasons such as lack of attention, latency in communication, and change in consumption behavior, the actual response from the consumers to a price signal is uncertain in nature. Hence, the customer behavior is explicitly modeled by an uncertain demand elasticity in this letter, which means customers have different responses to the electricity prices under different scenarios. We also consider generator outages and transmission line contingencies which can be addressed by DR programs to avoid or reduce forced load curtailment. Our proposed approach can be applied to enhance the reliability unit commitment process for ISOs.

We consider a two-stage stochastic programming formulation with unit commitment decisions at the first stage and real-time generation and load amount decisions at the second stage. The objective is to maximize the social welfare:

$$\max -(c_{gs} + \textstyle\sum_{t=1}^{T}\sum_i E[f_i(x_{it}(\xi))]) \\ + \textstyle\sum_{t=1}^{T}\sum_b E[F_{t,b,\xi}(d_t^b(\xi))] - \sum_{t=1}^{T} E[\gamma_t w_t(\xi)], \quad (1)$$

where c_{gs} denotes generator start-up and shut-down costs [2], $f_i(x_{it}(\xi))$ represents the fuel cost for generator i at time t when the generation amount is $x_{it}(\xi)$, $F_{t,b,\xi}(\cdot)$ represents the consumer benefit at bus b at time t with $d_t^b(\xi)$ representing the amount of elastic load at bus b at time t (note here each bus load includes both inelastic and elastic loads, and the consumer benefit for the inelastic load is zero, cf. [3]. Therefore, the objective function only includes elastic loads), and $w_t(\xi)$ and γ_t represent the total amount of load curtailment and unit penalty cost at time t, respectively.

Our model includes generation upper/lower bound constraints, min-up/-down time constraints, start-up/shut-down constraints, ramp-up/-down constraints, spinning reserve constraints, and transmission capacity constraints (cf. [2]). Both the generation upper/lower bound and transmission capacity constraints consider contingencies. For instance, the generation bound constraints with uncertain generator contingency consideration are modeled as follows:

$$L_i y_{it}(1 - C_i(\xi)) \leq x_{it}(\xi) \leq U_i y_{it}(1 - C_i(\xi)) \quad \forall i, \forall t, \quad (2)$$

$$Pr(C_i(\xi) = 1) = \tau_i \quad \forall i, \quad (3)$$

where L_i and U_i are lower and upper bounds of generator i, y_{it} is a binary variable to indicate if generator i is on during time period t, $C_i(\xi)$ is a random binary parameter indicating the contingency of generator i, and τ_i is the given probability value that the contingency happens for generator i. Constraints (2) enforce the generation output to be zero during the contingency. Finally, a chance constraint is introduced to formulate the loss of load probability (LOLP) as follows:

$$Pr\left(\sum_b (d_t^b(\xi) + \hat{d}_t^b(\xi)) \leq \sum_i x_{it}(\xi), \ \forall t\right) \geq 1 - \epsilon, \quad (4)$$

where ϵ is defined as risk level and $\hat{d}_t^b(\xi)$ represents the amount of inelastic load at bus b at time t.

II. Solution Methodology and Case Study

A. Solution Methodology

An SAA method is utilized to solve the problem. In our approach, a Monte Carlo method is first applied to generate scenarios (e.g., N scenarios). Then, the expected value function is replaced with the sample average function, and accordingly the chance constraint is replaced with an MILP reformulation as in [4]. The price-elastic demand curve for each ISO could be different with the common part that the demand is a non-increasing function of price (cf. [3] and [5]). This curve can be obtained by simulation and historical data analysis. Without loss of generality, in this paper, the price-elastic demand curve is described as $d_t^b(\xi) = A_t^b p_{t,b}^{\alpha_t^b(\xi)}$ (cf. [6]) with the purpose to illustrate our proposed solution approach. For a given elasticity $\alpha_t^b(\xi)$, A_t^b can be decided by the given reference point $(D_{t,b}^{ref}, P_{t,b}^{ref})$. Then, a step-wise

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function is applied to approximate this demand-price function as described in [6]:

$$F_{t,b,\xi}(d_t^b(\xi)) = \sum_{k=1}^{K} p_{t,b}^k r_{t,b}^k(\xi) \quad (5)$$

$$d_t^b(\xi) = \sum_{k=1}^{K} r_{t,b}^k(\xi), 0 \le r_{t,b}^k(\xi) \le l_{t,b}^k, \quad (6)$$

where K is the number of steps (see Fig. 1), $p_{t,b}^k$ and $l_{t,b}^k$ are given for each k, and $r_{t,b}^k(\xi)$ is an auxiliary decision variable. Based on (5) and (6), and the max objective, we have $r_{t,b}^k(\xi) = l_{t,b}^k$ in the solution if $d_t^b(\xi) \ge \sum_{u=1}^{k} l_{t,b}^u$. Thus, the discrepancy between the approximation and the integral of the curve (consumer benefit) is equal to the difference between the shaded area above the curve and the one below the curve, and this discrepancy converges to zero as $K \to +\infty$. In addition, $\alpha_t^b(\xi)$ is assumed to follow a normal distribution (our methodology can also be applied to other distributions) and a Monte Carlo method is applied to generate $\alpha_t^b(\xi)$ for obtaining $F_{t,b,\xi}$ under different scenarios. For the probability constraints, during the scenario generation process, we randomly set generator i under contingency status for $\tau_i \times N$ scenarios, and transmission line (m, n) under contingency status for $\kappa_{mn} \times N$ scenarios, where κ_{mn} is the given probability value that the contingency happens for transmission line (m, n) and N represents the total number of scenarios.

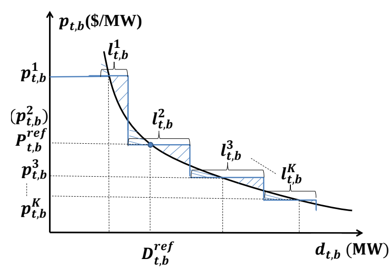

Fig. 1. Step-wise approximation of price-elastic demand curve

B. Case Study

We study the revised IEEE 118-bus system (online at ece.iit.edu/data) with 33 generators to illustrate the results.

1) Deterministic DR vs. Stochastic DR: We first set the risk level to be zero and compare the results based on the deterministic DR and stochastic DR representations, to show how stochastic DR works better. We assume possible contingencies occur on two transmission lines and two generators in this subsection. For the deterministic DR, the price-elastic demand curve is certain in which the mean value of the elasticity is taken to generate the curve. Several indicators are given in Table I for comparison purposes. It can be observed that the

TABLE I
DETERMINISTIC VS. STOCHASTIC

	Deterministic	Stochastic
Number of Start-ups	82	144
Expected Reserve Amount (MW)	1191	2100
Expected Load Loss (MW)	833	120
Solution Time (sec.)	35.2	50.6

stochastic formulation approach puts more generators online to provide additional capacity for unexpected consumption behaviors. This approach provides more reserves (for each scenario, it is measured as the difference between the total generation capacity of online generators and the load) which lead to less load curtailment.

2) Risk Levels and Demand Response Effect: The optimal objective values corresponding to different risk levels are reported in Table II. The risk level is represented by the probability defined in the chance constraint (4), which indicates the possibility of the load being curtailed. It can be observed that the social welfare increases when the risk level increases, because allowing load curtailment provides more flexibility for generation scheduling. To show the effectiveness of DR, we assume α_t^b the same for each b and t and compare the optimal social welfare using a group of elasticities with different mean and standard deviation values (e.g., μ_α and σ_α). It can be observed in Table II that the total social welfare has

TABLE II
COMPUTATIONAL RESULTS FOR DIFFERENT RISKS AND ELASTICITIES

ϵ	Obj. ($)	Time (sec.)	$(\mu_\alpha, \sigma_\alpha)$	Obj. ($)	Time (sec.)
0	1252530	24.5	$(-0.8, 0.2)$	1252530	24.90
10%	1323200	57.8	$(-2, 1)$	1494270	30.28
30%	1464530	131.9	$(-3, 2)$	1738670	14.57

a tendency to increase as the demand elasticity increases. But it is not guaranteed that there is always a positive correlation between elasticity and welfare. It depends on each specific price-elastic demand curve. Also, our conclusion is based on the "reference point" modeling approach we used. It may not be generalized to other modeling methods. However, the general modeling framework we described in this paper can accommodate other demand side modeling approaches. Our proposed solution approach can solve theses models efficiently and numerically.

III. CONCLUSION

In this letter, we provided a general modeling framework that considers the uncertain demand-side response in which price-elastic demand curves vary by scenario. This framework can accommodate different demand side modeling approaches. In addition, the proposed chance constraint controls the LOLP and the sample average approximation method can solve the IEEE 118-bus system efficiently. Final case studies indicate that the stochastic representation of uncertain demand response can lead to more available generation capacity, as compared to its deterministic counterpart.

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