EEL 4514L
COMMUNICATION LABORATORY

Laboratory Manual

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TABLE OF CONTENTS

INTRODUCTION TO THE COMMUNICATION LABORATORY

1 Purpose of the Laboratory Course

The goals of the communication laboratory are:

1. to allow you to perform experiments that demonstrate the theory of signals and communication systems that will be discussed in the lecture course,

2. to introduce you to some of the electronic components that make up communication systems (which are not discussed in the lecture course because of time limitations), and

3. to familiarize you with proper laboratory procedure; this includes precise record-keeping, logical troubleshooting, safety, and learning the capabilities as well as the limitations of your measurement equipment.

2 General Laboratory Procedure

The most important rule to follow in any laboratory is: **think before you do anything**. If you follow this one rule you will avoid injury to yourself, damage to the system you are testing, damage to your measurement equipment, and you will not waste time going down dead-end streets.

Safety In general you will not be using voltage levels high enough to cause injury; nevertheless, you should always pay attention to what you are doing.

Circuit Damage Your voltage levels **can** cause damage to the circuit under test if you are not careful. Make sure that your circuit diagram is correct, and be careful to build the circuit correctly on the protoboard. If you need to make changes to the circuit, disconnect the power supply and the input signal.

Equipment Each lab station has the following permanent equipment that you will use for most labs:

Spectrum Analyzer Agilent E4411B Spectrum Analyzer

Oscilloscope Agilent 54622D Mixed Signal Oscilloscope

Signal Generator Agilent 33120A Arbitrary Function Generator (2 per station)

Multimeter Agilent 34401A Digital Multimeter

Power Supply Agilent E3631A Triple-Output DC Power Supply

Before you use any measurement equipment, know the maximum input signal level it can withstand, and make sure that the signal you are trying to measure does not exceed it. (All good measurement equipment has overload protection, but it is still possible to do damage; do not rely on the equipment to protect you from your own mistakes.) In general, the signals in this laboratory course will not cause damage to the oscilloscope. (You can find the maximum voltage ratings on the front panel, next to the connectors.) The same is **not** true of the spectrum analyzer; you must be very careful what signal you apply to it. (Again, the maximum signal that can be applied is printed on the front panel.)

A big part of this laboratory course is learning how to use measurement equipment; you learn how to make good measurements by actually using the instruments to measure things. The lab experiments in this manual will not be a step-by-step procedural list—you will not be told which button to push, which menu to bring up in order to make the instrument do something. Rather, you will be told things such as "display the output signal on the oscilloscope and determine its frequency components". You will have to learn how to accomplish this. To help you, the complete User's Guide for each instrument is on the PC at each station. On the PC desktop you will find a shortcut to a folder called Equipment Manuals; all of the User's Guide are there in PDF format. Double-click the one you want to open in the Acrobat Reader.

Troubleshooting Things will not always go as expected; that is the nature of the learning process. When you are testing a circuit, especially one that you have built, if the output signal is not what you expect do not go in and randomly replace chips and other components. The key is to be logical and systematic; don't just try things at random hoping to get lucky. First, look for obvious errors that are easy to fix. Is your measuring device correctly set and connected? Is the power supply set for the correct voltage and is it connected correctly? Is the signal generator correctly set and connected? Next, check for obvious misconnections or broken connections, at least in simple circuits. If the problem is not one of these trivial ones, then you need to get to work. As you work through your circuit, use your notebook to record tests that you make and changes that you make as you go along; don't rely on your memory for what you have tried. Identify some test points in the circuit at which you know what the signal should be, and work your way backwards from the output through the test points until you find a good signal. Now you have a section of the circuit to focus your efforts on. (Here is where a little thought about laying out your board before connecting it up will pay off; if your board looks like a bird's nest, it is going to be very hard to troubleshoot, but if it is well organized and if the wires are short, it is going to make your job a lot easier.) Final remark: if you do discover a bad component or wire, do not just throw it back in the box.

Neatness When you have finished for the day, return all components to their proper storage bins, return all test leads and probes to their storage racks or pouches, return all equipment to its correct location, and clean up the lab station.

Computers On occasion you will find that measurements made in lab do not check with your prelab calculations or simulations; the PC's at each station have Mathcad and (Microsim) PSpice on them so that you can check your prelabs. The PC's are not conncected to the campus network. The PC's are also used to give you access to a printer so you can print out oscilloscope and spectrum analyzer displays. **Do not** install other software on the computers, change the system settings (such as the display), change the desktop, install your own wallpaper or screen saver, etc. You may temporarily save your own files on the hard disk; you will find a shortcut to the My Documents directory on the desktop. You may create your own folders under My Documents to store files in. Do not, however, expect those files to be there next time you use the computer; the computers will be cleaned up periodically to provide disk space. **Always copy any files you need to save onto your own floppy before you leave the lab**.

> Final note: when you start the PC, do not logon. When the logon screen comes up, just hit the Esc key.

3 Record-Keeping

You will be working in groups of 2 at the lab stations, but each student will maintain a standard laboratory notebook into which all calculations, measurements, prelabs, answers to questions, etc. are entered. Your notebook will be checked each week for adequate progress during the course. The laboratory notebook is a record of your lab activity, not a series of formal lab reports. You should try to keep the notebook neat and organized, but perfection is not expected. Occasionally you will make an entry that is simply wrong; **do not** erase or tear out the page, but merely cross out the entry. (In industry you will be required to keep a patent notebook in ink—no erasures at all are allowed. We shall be more relaxed—small errors may be erased, but do not waste time erasing a half-page, just cross it out.)

Most of the lab experiments have prelabs, involving PSpice, Mathcad, or Matlab, as well as derivations or calculations to do by hand. All of the prelabs must be entered into your notebook; any printouts they include should be securely pasted or taped into your notebook. The same is true of any printouts you make of the oscilloscope and spectrum analyzer displays. (You may also paste the experiments from this lab manual into your notebook, but that is not required, nor is it recommended.)

Each student is expected to participate in the lab and to maintain a notebook; remember, your notebook will be checked each week, and there will be a final practical exam—if you have not kept up with the labs, you will not do well on the final.

4 Prelabs

Most of the experiments have prelabs. **You will be expected to have the prelab completed before the lab period— you will not be permitted to do the in-lab part of the experiment without a complete prelab**. You are encouraged to use any computer tool that you consider appropriate to help you complete the prelab. The tools available in the ECE computer lab (NEB 288) that you will find most useful are PSpice, Mathcad, and Matlab. The computers at each station in the lab also have Microsim PSpice and Mathcad installed. If you use one of these tools to produce a circuit diagram, a graph, or a table, then you must secure that page in your lab notebook; your graphs must have titles and axis labels, and if you have more than one trace on a graph the traces must be labeled. Circuit diagrams drawn by hand should be entered directly into your notebook, as neatly as possible, with all components clearly labeled. If you choose to draw a graph by hand, then you must do it on appropriate graph paper, using a straightedge to draw axes. **You are an engineer—you are expected to present data and calculations clearly and precisely**.

LABORATORY 1
THE DIGITAL STORAGE OSCILLOSCOPE, THE FUNCTION GENERATOR, AND MEASUREMENTS

OBJECTIVES

1. To become familiar with the features and basic operation of the Agilent 54622D oscilloscope and the Agilent 33120A function generator.

2. To investigate signals in the time and frequency domains.

PRELAB

1. Review Appendix A of this manual; it contains basic information on how a digital storage oscilloscope works in general, with some specific information on the Agilent 54622D DSO.

2. Calculate and plot[1] the exponential Fourier series coefficients for a sinusoidal voltage of amplitude A, frequency f_0, phase angle θ, and dc value (i.e. average value) of K.

3. Calculate and plot the exponential Fourier series coefficients of a square wave of amplitude A, frequency f_0, duty cycle 50%, and dc value K. (Use an *odd* square wave.)

4. Calculate and plot the transfer function of an RC lowpass filter for a given time constant $\tau = RC$. Indicate the 3-dB bandwidth on your plot.

5. For your RC lowpass filter, calculate and plot the output spectrum and the output time signal for a sinusoidal input and for a square input.

[1]Be sure to heed the advice in the Introduction about plots and graphs.

6. Design an RC lowpass filter having time constant $\tau = 10\,\mu s$. What is the 3-dB break frequency?

IN LAB

1. On the desktop of the computer at your station you will find a shortcut to a folder called "Equipment Manuals". This folder contains, in PDF format, the complete User's Guides to the oscilloscope, function generator, multimeter, DC power supply, and spectrum analyzer. (In addition there is a Quick Reference Guide and a Front Panel Guide for the function generator.) Locate these manuals and be ready to open them as needed. (Double-click on the name to open the manual with the Acrobat reader.)

2. Use the function generator to produce a sine wave of frequency 2.5 kHz and peak-to-peak amplitude 200 mV, with zero dc offset. Use a coaxial cable with BNC connectors on the ends to connect the output of the signal generator to one of the analog inputs on the oscilloscope. Display the sine wave on the oscilloscope and measure the frequency and amplitude in two ways: (1) By counting divisions on the screen to determine the amplitude and the period. (Use the cursors to help you make the measurements—see the oscilloscope manual for information on using cursors.) (2) By having the oscilloscope automatically make the measurements. (Manual again.) Always pay attention to the information on the status line (above the waveform display) and on the measurement line (below the waveform display); see p.2-11 in the manual.

   Is there a discrepancy between your measured amplitude and the amplitude you entered into the function generator? Explain. (Hint: check the output impedance of the function generator and the input impedance of the oscilloscope. Take a look at the Function Generator Front Panel guide in the Equipment Manuals folder.)

3. Take a few minutes to become familiar with the front panel controls of the two devices.

   On the function generator, learn how to select waveshapes, amplitudes and frequencies using the keypad and the control knob. What is the maximum frequency and maximum amplitude sine wave that the function generator can produce? What is the minimum frequency and minimum amplitude that it can produce? (Make sure that the maximum amplitude does not exceed the maximum input rating of the oscilloscope.)

On the oscilloscope, learn how to select channels to display, and how to get a good display without using the Autoscale button. (Autoscale does not do anything you cannot do with the controls, and there is no guarantee that it will give the display settings you need.) Spend some minutes investigating the following features (you do not need to record this in your notebook, unless you want to for your own reference):

(a) What does the Delayed Sweep feature do?

(b) What are the three triggering modes that this oscilloscope provides?

(c) What are the trigger coupling modes?

(d) The signal must also be coupled to the input of the oscilloscope—what is the difference bewtween AC and DC input coupling?

(e) What are the different acquisition modes that this oscilloscope has?

(f) What do the RUN/STOP and SINGLE buttons do?

You must learn to become familiar with these features and to pay attention to them. Every time you make a measurement with an oscilloscope, you must know how the input is coupled, how the waveform is acquired, how the oscilloscope is triggered, and the sampling rate being used. If you do not pay attention, you could end up displaying on the screen a waveform that in no way represents the signal you are trying to measure.

4. Reset the function generator to produce the 2.5 kHz sine wave from Step 2.

   (a) Find out how to save the trace and the oscilloscope settings to one of the three internal memories, and do so. Disconnect the signal generator. Recall the saved trace from the internal memory location and display it. (This is useful when you want to compare a measurement to a known good measurement that has been stored.)

   (b) Clear the recalled trace from the screen. Reconnect the signal generator and redisplay the "live" sine wave. Now save the trace and oscilloscope settings to a floppy disk, and recall the saved trace from the floppy. Saving the trace and settings on a disk allows you to transfer them to another oscilloscope (the same or compatible model, of course). Note that you can also save the screen in other formats, such as Windows bitmap (`*.bmp`).

5. Display the amplitude spectrum of the sine wave on the oscilloscope. Remember that the oscilloscope does this by calculating the FFT of the samples of the signal it has acquired. You will need to adjust the sampling rate (through the horizontal sweep control), the center frequency, and the frequency span to get a good display. Compare with your prelab calculations. Why is the spectrum as shown by the oscilloscope not a pure line spectrum as in your prelab plot? In particular, address these two points:

   (a) Why is there more than one line? (Hint: measure the amplitude level, in dB, of the higher order lines relative to the fundamental line. How much power is contained in the higher order lines? Is the signal generator producing a perfect sine wave?)

   (b) Why are the lines not truly lines? That is, they have non-zero width. (Hint: In order to calculate the FFT, the oscilloscope can only use a *finite* number of samples; i.e., the signal is *windowed* to have a finite time duration. What is the Fourier transform of a sinusoidal pulse?)

6. Save the display of the spectrum on a floppy as a bitmap, print it out and include it in your notebook.

7. Use the HP function generator to produce a 10 kHz square wave with peak-to-peak value 200 mV, 50% duty cycle, and zero dc offset. Display it on the oscilloscope, and display its FFT. Include a printout of the square wave and its FFT in your lab notebook. Compare its amplitude spectrum, out to the first five peaks, with your prelab calculations.

8. Build the RC lowpass filter having time constant $\tau = 10\,\mu$s from your prelab. Use the square wave from Step 7 as the input to the RC filter. Display the output signal and its FFT; insert a printout in your notebook. Compare the output to your prelab calculations.

9. **Measure** the time constant τ of the RC circuit and compare with the designed value. Hint: use a square wave test input, and measure the rise time of the output. Calculate τ from the measured rise time. The Delayed Sweep feature of the oscilloscope will be helpful here—you

can use it to “zoom-in” on the rising edge of the output waveform and get a more accurate measurement of the rise time.

LABORATORY 2
THE SPECTRUM ANALYZER AND MEASUREMENTS

OBJECTIVES

1. To become familiar with the features and basic operation of the Agilent E4411B spectrum analyzer.

2. To investigate signals in the frequency domain.

PRELAB

1. Review Appendix B on the basic operation of the spectrum analyzer.

2. You will need your Prelab calculations from Laboratory 1: Fourier series for sine and square waves, transfer function for an RC lowpass filter, and the outputs of an RC filter for sine and square inputs.

3. Design an RC lowpass filter with a 3 dB break frequency of 120 kHz (or as near as you can get with the available resistors and capacitors).

4. Review Section 2-1 in [Couch] about normalized signal power, signal power into a load, and signal power in units of dBm.

IN LAB

1. As discussed in Appendix B, you need to let the spectrum analyzer warm up for 5 minutes, and go through its internal alignment procedure.

2. Record the answers to the following questions in your lab notebook:

- What is the frequency range that this spectrum analyzer will measure?
- What is the maximum DC level that can be applied to the RF input?
- What is the input impedance of the RF input?
- What is the maximum signal power, in dBm and in Watts, that can be applied to the RF input?

Before you connect any signal to the RF input, be sure that its amplitude or power does not exceed the maximum rated input. If you are unsure, measure the signal with the oscilloscope.

3. Given your answers to the questions in Item 2, calculate

   - the maximum amplitude sine wave (with zero DC offset) that can be applied to the RF input,
   - the maximum amplitude square wave (with zero DC offset) having 50% duty cycle that can be applied to the RF input.

   (When doing these calculations, don't forget what the input impedance of the analyzer is.)

   - What is the center frequency and the frequency span on power-up?
   - What is the resolution bandwidth on power-up?
   - What is the reference level and the amplitude scale in dB/division?
   - What is the attenuation? What is the purpose of the internal attenuator?

4. With no signal applied and with the analyzer in its default configuration (if you changed any of the settings you can get back to the default state by pressing the PRESET button), you will see the display of the *noise floor*. This noise is approximately *white noise*, meaning its power spectral density (which is what you are looking at on the screen) is approximately constant for all frequencies. Measure the power level in dBm and in W of this noise.

5. Use the 33120A function generator to produce a 1 MHz sine wave of amplitude $200\,\mathrm{mV_{p\text{-}p}}$. (Remember that you can set the function generator output impedance to high or to $50\,\Omega$—make sure you have it set

appropriately.) Get a good display of the spectrum on the analyzer. Measure the input power in dBm (don't forget that you are not measuring normalized power) of the lines and compare with theory. Make sure that you look for lines other than the ones you expect to see, and that you record their frequencies and amplitudes.

6. Change the vertical unit from dBm to mV and repeat item 5.

7. Adjust the resolution bandwidth (RBW) up and down and observe the effect on the displayed spectrum. Explain the appearance of the spectrum as you change the RBW, especially when you set the RBW to 1 MHz and 3 MHz

8. Use the Sweep control to obtain a single sweep and a continuous sweep (the default). What is the purpose of single sweep?

9. With the sine wave spectrum displayed, become familiar with using the FREQUENCY, SPAN, AMPLITUDE, and Res BW controls. Become familiar with the Marker controls for frequency and amplitude measurements, including the difference markers and the Peak Search control. What is the function of the Signal Track control?

10. Investigate the effect of the Video BW (video filter bandwidth) button on the display of the calibration signal. The video filter is a post-detection filter used to reduce noise in the displayed spectrum to its average value, making low-level signals easier to detect. Note: you should use the reduced VF bandwidth with care—it will reduce the indicated amplitudes of wideband signals, such as video modulation and short duration pulses. When you have finished this item, put the spectrum analyzer back in its default configuration with the PRESET button.

11. Use the function generator to produce a 100 kHz square wave of amplitude 200 $mV_{\text{p-p}}$, with 50% duty cycle and zero dc offset. Get a good display of the fundamental and the first several (at least out to the 5^{th}) harmonics. Which harmonics do you *expect* to see, and what do you *observe*? Explain. Measure how far below the fundamental the harmonics are, in dBm. Comment on the difference in amplitude between the even and odd harmonics. Compare with the theoretical values.

12. Get the display of the square wave spectrum the way you want it; print it and include it in your notebook. Explore the File control menus.

Note that, as with the oscilloscope, you can save the screen or the instrument configuration internally or on a floppy, you can organize the file structure (create directories, rename files), etc.

13. Build an RC lowpass filter having 3 dB bandwidth 120 kHz. Use the square wave from Item 11 as the input to the RC filter, and observe the spectrum of the output on the analyzer. Measure the fundamental and at least out to the 5^{th} harmonic of the output. Compare with theory.

    Also print out the filter output and include in your notebook.

    Note: depending on how you connect the function generator to your circuit, and how you connect the output of the circuit to the RF input of the analyzer (you will probably use the cables that have a BNC connector on one end and alligator clips on the other), your amplitude measurements may not be accurate due to impedance mis-matches. But your *relative* amplitude measurements will be accurate—i.e., the amplitude values of the lines in dBm may not agree with theory, but the *differences* between the lines in dB should.

Remarks: In this lab we of course have not used the spectrum analyzer to its full advantage—we did nothing here that could not have been done with the FFT feature of the oscilloscope. The purpose of this lab was simply to introduce you to the spectrum analyzer and its basic operation. In future you will be expected to be able to set the analyzer controls to get a good display of the spectrum of any signal, and to be able to read the frequencies and amplitudes of the spectral components from the display and convert the amplitudes into voltage levels or normalized powers.

References

[Couch] Leon W. Couch, II, *Digital and Analog Communication Systems*, 6^{th} ed., Prentice-Hall (2001)

LABORATORY 3
FREQUENCY RESPONSE OF SYSTEMS AND DISTORTION

OBJECTIVE

To measure the frequency response of a linear filter and to investigate linear and nonlinear distortion.

PRELAB

1. Read the following: in [Couch], Section 2-6, subsection on distortionless transmission, and Section 4-9 on nonlinear distortion; or in [Carlson], Section 3.2.

2. A popular type of Butterworth second-order lowpass filter is the Sallen-Key circuit shown in Figure 1.[1] Assuming an ideal op-amp, show that the transfer function of this linear system is

$$H(s) = \frac{V_{\mathrm{out}}(s)}{V_{\mathrm{in}}(s)} = \frac{1}{R_1 R_2 C_1 C_2 s^2 + (R_1 + R_2) C_2 s + 1}. \tag{1}$$

   A useful assumption for design is $R_1 = R_2 = R$ and $C_1 = C_2 = C$; under this assumption obtain an expression, in terms of R and C, for the 6 dB break frequency. (The 6 dB frequency is simply more convenient to deal with than the usual 3 dB frequency.)

[1] You will learn about Butterworth filters in Electronics 2. The Sallen-Key circuit was invented around 1955 (by Sallen and Key, surprisingly), and it is popular because it requires only one op-amp, hence it is inexpensive and does not consume much power. Its Q factor is, however, more sensitive to component tolerances than other configurations, especially for large Q. But in lowpass filters, Q is not large and the sensitivity problem is not a concern. See Sec. 11.8 in [Sedra/Smith]

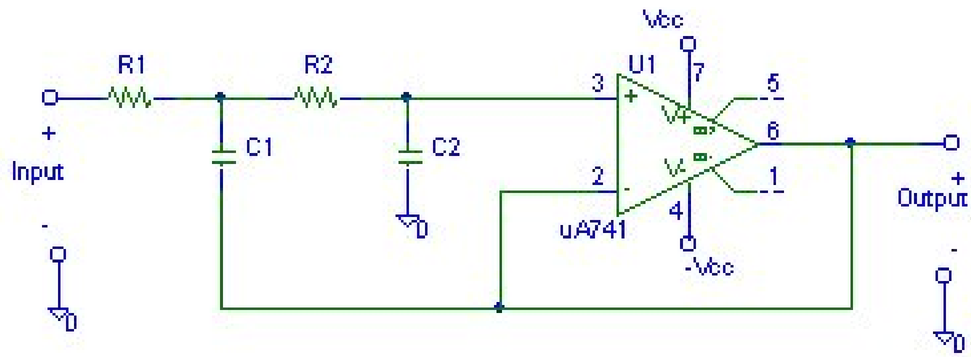


Figure 1: Sallen-Key Lowpass Filter

3. Find the 6 dB break frequency for the values $R = 8.2\,\mathrm{k}\Omega$ and $C = 0.01\,\mu\mathrm{F}$.

4. Using Mathcad or Matlab, obtain a plot of the amplitude gain and phase shift of the Sallen-Key filter using Equation (1). It is best to make Bode plots—frequency on a logarithmic scale and amplitude gain in dB.

5. Using the R and C values from Item 3, simulate the Sallen-Key filter in PSpice and obtain a Bode plot of the amplitude gain (in dB) over the frequency range 1 Hz to 100 kHz. Determine the slope, in dB per decade, of the high frequency asymptote. Be sure to choose V_{cc} and the input ampltitude so that the op-amp does not saturate—i.e., make sure the circuit is operating as a *linear* system. In lab you will use $V_{cc} = 5\,\mathrm{V}$, so choose the input amplitude appropriately.

   Hint: Recall that to get a frequency response plot in PSpice, use the VAC source for the input and in the simulation setup set the paramters under AC Sweep. It is convenient to use a voltage dB marker or phase marker at the ouput, depending on which part of the frequency response you want.

6. Compare your theoretical Bode plot from Item 4 with the circuit simulation result from Item 5. They should of course be close. Your theoretical anlaysis was based on an ideal op-amp and your simulation

uses the Spice model of the op-amp, so supposedly the simulation is more accurate to some degree. (This should always be your procedure. You do some analysis and design based on a simplified mathematical model. Now you have some idea of how the system should behave. Next you verify your analysis by doing as accurate a simulation as you can. Now you are pretty sure how the system should behave, and you are ready to build the prototype in the lab and make some measurements. Here is where you will discover effects that your modeling did not accurately take into account, and the loop returns to the beginning—you try to model these effects, then run a simulation, and so forth.)

7. Our theoretical analysis of the filter assumes a *linear* model—the system from input to output is assumed to be a linear system. But as you know, there is really no such thing as a perfectly linear system. As you know from the reading you did for Item 1, one way to measure how close a system is to being truly linear is to apply a sinusoid and look for harmonics in the output. If the system is truly linear it cannot introduce any harmonics in the output signal. But a nonlinear system does introduce harmonics of the input frequency—in fact, we could take this as the definition for nonlinear system. If the added harmonic components are small in amplitude, or in other words if the total harmonic distortion is small, then to that extent the system is "close" to linear, at least for that test frequency.

   For the Sallen-Key circuit, use the R and C values from Item 3 and set $V_{cc} = 5\,\mathrm{V}$. Do a PSpice simulation to see if there is any harmonic distortion. You need do this at only one test frequency; try one well below the 6 dB break frequency, say 500 Hz. Apply a sinusoid of this frequency to the input, keeping its amplitude small enough so that the op-amp does not saturate, and observe the output voltage. Observing the output *waveform* is not good enough—just because it "looks" like a sine wave does not make it a sine wave. You have to look at its spectrum. Make a Probe plot of the output waveform, then use the FFT tool in Probe to get the spectrum of the output. Measure the amplitudes of any harmonics and calculate the total harmonic distortion (THD).

8. You should have found from your simulation in Item 7 that, provided you do not saturate the op-amp, the system is indeed linear—there is zero THD.

As you know, it is possible to operate the system non-linearly by applying a large enough input signal to cause the op-amp to saturate. An input amplitude of 6 V should do. (Since the gain at 500 Hz is approximately 1, an input amplitude of slightly more than V_{cc} will cause saturation, and the larger the input is, the further into saturation the op-amp will go—i.e., the more nonlinear the circuit becomes.) You will now find the output to be distorted. Use the FFT in Probe to display the output spectrum and calculate the THD.

IN LAB

1. Build the Sallen-Key filter using the values of R and C that you used for the prelab calculations and simulations: $R = 8.2\,\mathrm{k}\Omega$ and $C = 0.01\,\mu\mathrm{F}$. Set $V_{cc} = 5\,\mathrm{V}$. By applying test input sinusoids at properly chosen frequencies, verify the prelab calculations and simulations for the frequency response (amplitude and phase) of the filter.

   Hint. The frequency response of a linear filter can be expressed as

   $$H(f) = |H(f)|e^{j\theta(f)},$$

   where $|H(f)|$ is the magnitude response and $\theta(f)$ is the phase response. If a sinusoid, say

   $$x(t) = A\cos 2\pi f_0 t,$$

   is the input, then the output will be the sinusoid

   $$\begin{aligned} y(t) &= \big(A|H(f_0)|\big)\cos(2\pi f_0 t + \theta(f_0)) \\ &= \big(A|H(f_0)|\big)\cos\left(2\pi f_0\left(t + \frac{\theta(f_0)}{2\pi f_0}\right)\right). \end{aligned}$$

   Hence, by observing the input and output sinusoids simultaneously (remember that your oscilloscope has two analog channels) we can measure the amplitude gain $|H(f_0)|$ of the filter at frequency f_0, and the time shift between input and output at f_0 from which we can calculate the phase shift $\theta(f_0)$. Take a sufficient number of data points so that you can produce plots of the amplitude and phase responses. You may produce the plots on graph paper, or you may read the data into Mathcad or Matlab to make the plots. (If you make the plots by hand I suggest you make Bode plots since the amplitude Bode plot should consist, except near the break points, of straight line segments.) Be sure that the theoretical 6 dB frequency is one of your test signals.

Remark. You will probably want to set the function generator to high impedance output termination, but do not rely on the function generator readout for an accurate value of amplitude. Instead, *measure* the function generator amplitude with the oscilloscope.

2. Verify your calculation of THD in the linear system from the Prelab. Apply a sine wave of frequency 500 Hz and small amplitude. Observe the output of the circuit on the oscilloscope and display its FFT. Calculate the THD.

   Caution: Your input in the simulation was a *pure* sine wave, and that should be your test signal in this Item. If your function generator contains spurious frequencies (record its FFT) you will need to account for them.

3. You have now verified that the Sallen-Key circuit does in fact behave as the linear model predicts. But, as you know from the lecture class and from your reading in Item 1 of the Prelab, a linear system can distort a signal—it causes *linear distortion* if $|H(f)|$ is not constant or if $\theta(f)$ is not linear. Does the Sallen-Key circuit satisfy the conditions for distortionless transmission? Does it satisfy the conditions over a small range of f? Perform the following two tests:

   - Apply a 100 Hz square wave (without causing saturation) and observe the input and output on the oscilloscope.
   - Apply a 1000 Hz square wave and observe the input and the output.

   Explain the differences in the two outputs in reference to linear distortion caused by the circuit.

4. Now drive the circuit with a large enough sine wave (6 V amplitude at 500 Hz) so that it operates non-linearly. Verify your THD calculation from Prelab.

References

[Carlson] A. Bruce Carlson, Paul B. Crilly, and Janet C. Rutledge, *Communication Systems: An Introduction to Signals & Noise in Electrical Communication*, 4$^{\text{th}}$ ed., McGraw-Hill (2002)

[Couch] Leon W. Couch, II, *Digital and Analog Communication Systems*, 6$^{\text{th}}$ ed., Prentice-Hall (2001)

[Sedra/Smith] Adel S. Sedra and Kenneth C. Smith, *Microelectronic Circuits*, 4$^{\text{th}}$ ed., Oxford University Press (1998)

LABORATORY 4
SINUSOIDAL OSCILLATORS

OBJECTIVES

To become familiar with two kinds of feedback oscillators used to produce sinusoidal signals: the Wien bridge oscillator and a phase shift oscillator.

PRELAB

1. Read Appendix C of this manual and Sections 12.1–12.3 of [Sedra/Smith].
2. Design a Wien bridge circuit having an oscillation frequency of 10 kHz with amplitude stabilization; use the circuit in Figure 12.6 in [Sedra/Smith] as your template. What value of resistance (from the tap to point b) of the potentiometer P will just sustain oscillations?
3. Verify your design in PSpice; look at the output at both points a and b. (Use a 741 op amp. You may use the generic breakout diode, `Dbreak`. There is a `POT` part in the Spice library.) Make sure to run your simulation for a long enough time that you can verify that oscillation is sustained, and that the amplitude is stabilized.
4. Verify the purity of the ouput waveform by looking at its FFT. Calculate the THD if there are measureable harmonics present.
5. For the basic Wien bridge oscillator without the amplitude stabilization circuit (i.e., Figure 8 in Appendix C), calculate the frequency stability factor S_F. Comment.

IN LAB

1. Build the Wien bridge with amplitude stabilization that you designed in Prelab.

- Record the oscilloscope display of the output (point b). Measure the oscillation frequency.
- Measure the potentiometer resistance required to sustain oscillation, and compare with your Prelab calculation.
- Record the FFT of the output on the oscilloscope. Compare with Prelab.

2. Vary the potentiometer resistance up and down and record your observations. What should happen to the output as you increase and decrease the resistance and what do you observe?

3. Build the op amp phase shift oscillator shown in Figure 1. This is just the phase shift oscillator of Figure 5 in Appendix C with the same simple amplitude stabilization used in the Wien bridge. The left-hand resistance of the POT (between the tap and C_3) is R in Figure 5 of Appendix C, and the right-hand resistance plus R_2 is the same as the feedback resistor R_1 in Figure 5 of Appendix C.

   - Adjust the potentiometer until oscillation is sustained. Record the oscilloscope display of the output. Measure the oscillation frequency.
   - Measure the potentiometer resistance required to sustain oscillation. Compare with the theoretical values calculated in Appendix C: if R_l is the resistance between C_3 and the tap and R_r is the resistance to the right of the tap, then R_l should be $10\,\mathrm{k}\Omega$, $(R_r + R_2)/R_l$ should be greater than 29, and under these conditions the frequency of oscillation is $f_0 = 1/(2\pi RC\sqrt{6})$.
   - Record the FFT of the output on the oscilloscope.

References

[Sedra/Smith] Adel S. Sedra and Kenneth C. Smith, *Microelectronic Circuits*, 4^{th} ed., Oxford (1998)

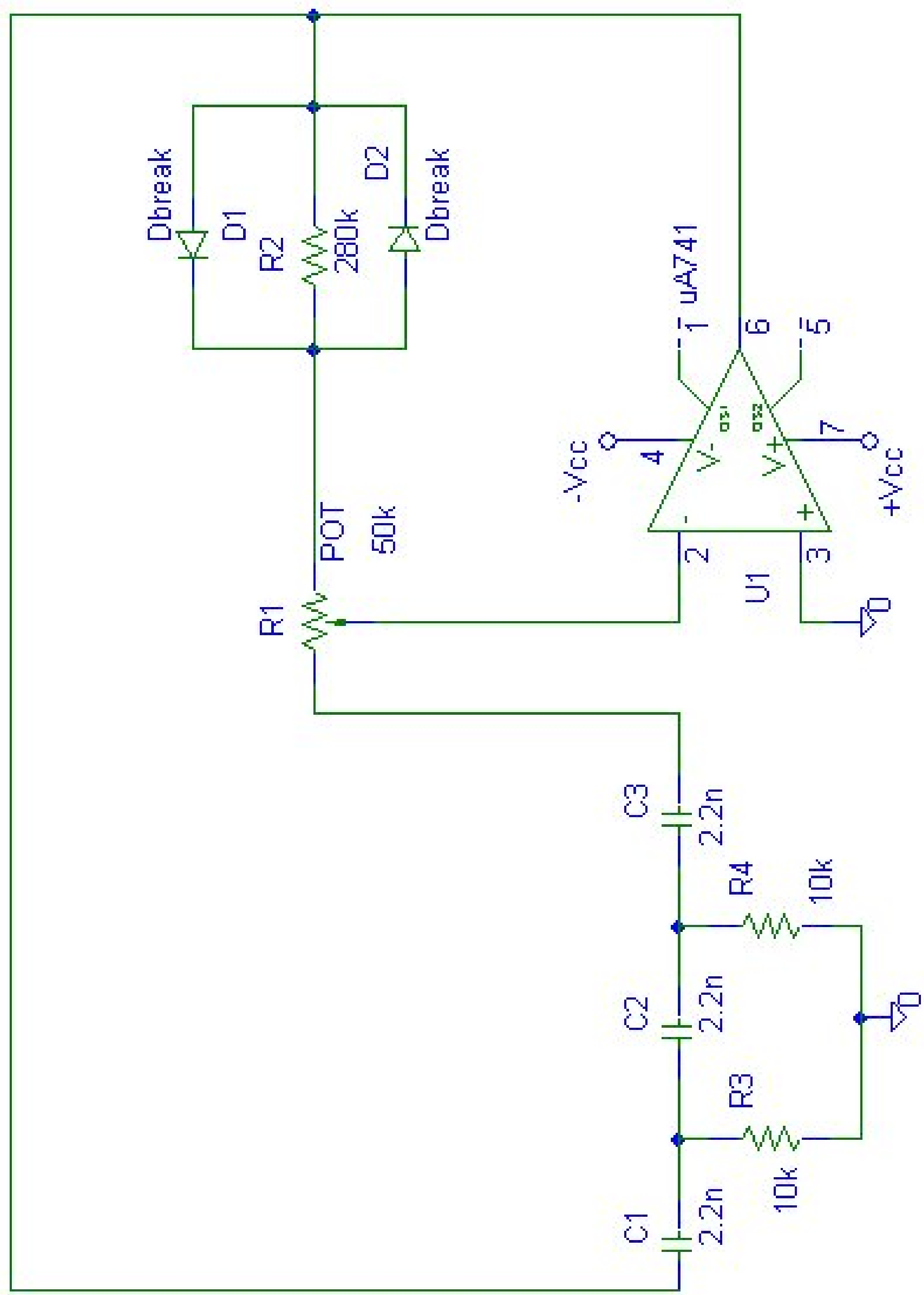


Figure 1: Phase Shift Oscillator With Amplitude Stabilization

LABORATORY 5
AMPLITUDE MODULATED SIGNALS AND ENVELOPE DETECTION

OBJECTIVES

To take measurements of AM signals in the time and frequency domains, and to investigate envelope detection of AM signals.

PRELAB

1. Read Section 5-1 (Amplitude Modulation) and Section 4-13 (Detector Circuits; read "Envelope Detector" subsection) in [Couch], or Section 4.2 (Double-Sideband Amplitude Modulation) and Section 4.5 (especially the subsection on Envelope Detection) in [Carlson].

2. An AM signal is written as

$$x_c(t) = A_c(1 + \mu x(t)) \cos 2\pi f_c t,$$

   where f_c is the carrier frequency, A_c is the carrier amplitude, μ is the modulation index, and $x(t)$ is the baseband message signal. We assume that $x(t)$ has absolute bandwidth $W \ll f_c$, and that its amplitude has been normalized so that $|x(t)| \leq 1$.

   If $x(t)$ is a cosine of amplitude 1 and frequency $f_m \ll f_c$:

   - Obtain an expression for the amplitude spectrum $X_c(f)$ of the AM signal $x_c(t)$.
   - Determine the power in the carrier and in the sidebands. Express the powers in units of dBm into a $50\,\Omega$ load. (Remember that the spectrum analyzer input impedance is $50\,\Omega$.)

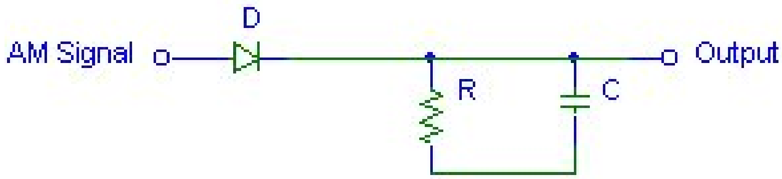


Figure 1: Simple Envelope Detector

- Determine the ratio of the power in the sidebands to the power in the carrier.

3. Obtain numerical values in Item 2 if $f_m = 15\,\text{kHz}$, $\mu = 1/2$, and the carrier amplitude and frequency are $A_c = 1$ and $f_c = 300\,\text{kHz}$. Also, use Mathcad or Matlab to plot the AM signal $x_c(t)$.

4. Repeat Item 2 for a message $x(t)$ which is a square wave of amplitude 1, zero dc level, 50% duty cycle, and fundamental frequency f_m.

5. Obtain numerical values in Item 4 if $f_m = 15\,\text{kHz}$, $\mu = 1/2$, and the carrier amplitude and frequency are $A_c = 1$ and $f_c = 300\,\text{kHz}$. Also, use Mathcad or Matlab to plot the AM signal $x_c(t)$.

6. In lab you will display the AM signal on the oscilloscope. Devise a way to *measure* the modulation index μ from the plot of the AM signal. (Hint: consider the maximum and minimum peak-to-peak swings of the AM signal—look at Figure 5-1(b) in [Couch] or Figure 4.2-1(b) in [Carlson].)

7. As explained in Section 4-13 of [Couch] or Section 4.5 of [Carlson], an AM signal with less than 100% modulation (i.e., with $\mu < 1$) can be easily demodulated using an envelope detector, shown in Figure 1. In fact, this is the reason for AM—we transmit a large amount of wasted power in the carrier, but we can use a non-synchronous detector. In practice, the situation is more complicated: the envelope detector has very low input impedance, so we need a large resistor at the input; then voltage division between the input resistor and the envelope detector causes the output signal level to be unacceptably small, and so we need to amplify it. The envelope detector circuit you will use in lab is shown in Figure 2. The resistor R_1 raises the input impedance to

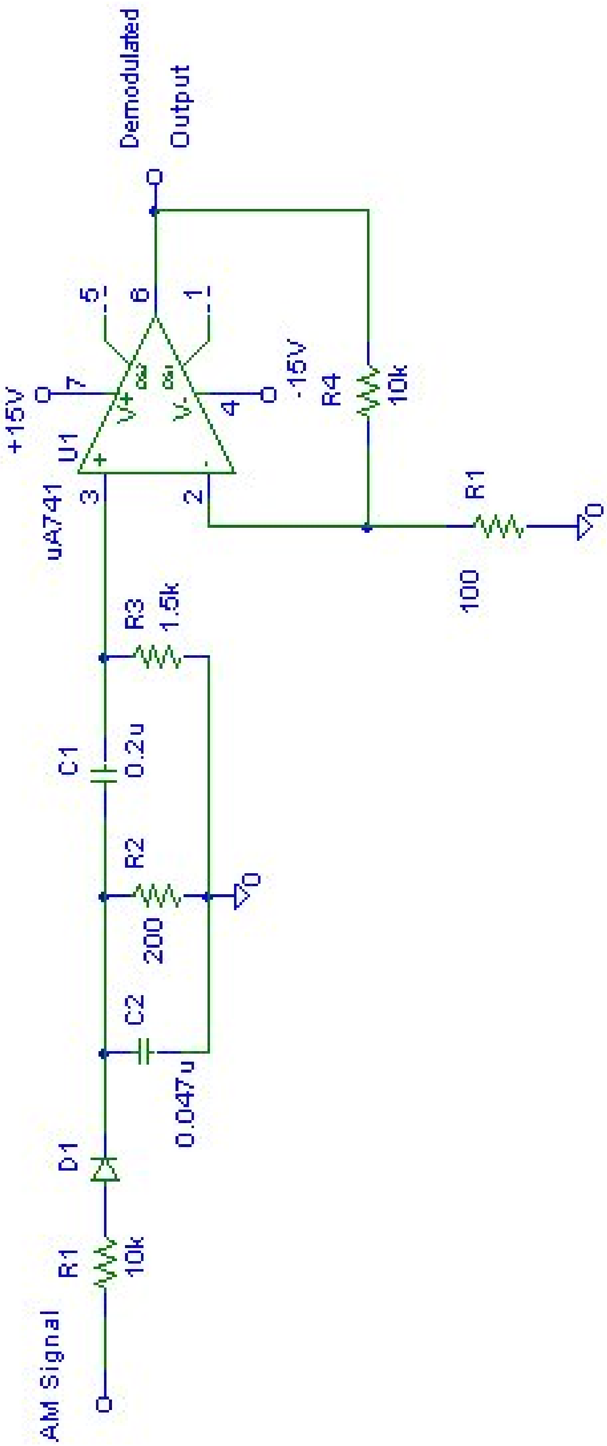


Figure 2: Envelope Detector To Be Used In Lab

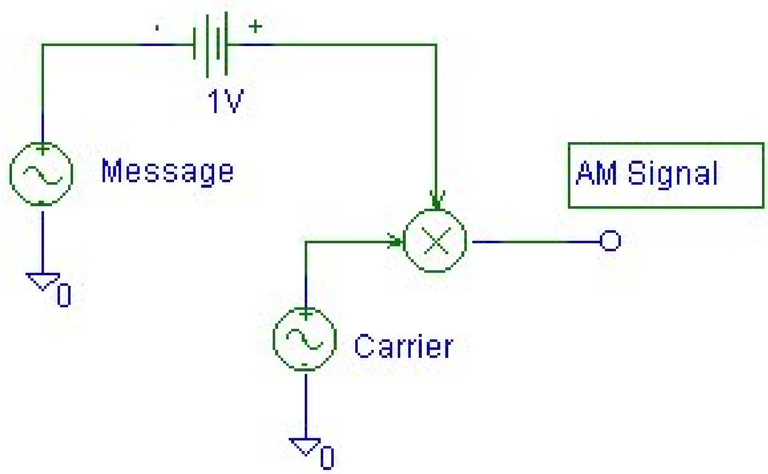


Figure 3: Using the MULT Part to Generate an AM Signal in PSpice

at least R_1. The envelope detector consists of D_1, R_2, and C_2. The amplifier is required to overcome voltage division between R_1 and the envelope detector. The R_3-C_1 circuit is a high-pass filter to block any dc in the signal coming from the envelope detector. Suppose that the AM input signal to the demodulator of Figure 2 is the signal from Items 2 and 3, in which the message is a cosine wave.

- Show that the bandwidth of the R_2-C_2 lowpass filter is appropriate for this AM signal.
- Show that the bandwidth of the R_3-C_1 highpass filter is appropriate.
- Calculate the gain of the op amp stage.
- Simulate the demodulator circuit in PSpice. (**Hint**: You can generate an AM signal by using the `MULT` part in the evaluation library. See Figure 3.)

IN LAB

1. Set the HP/Agilent function generator to produce the AM signal of Items 2 and 3 in the Prelab. Display the AM signal on the oscilloscope (watch your impedances).

Notes: (1) In AM mode the carrier amplitude is reduced to half the set value, so you will need to set the carrier amplitude to $4\,V_{p\text{-}p}$.

(2) You may find it useful to use the SYNC output of the function generator as a trigger source. The SYNC output is a TTL high pulse (look at it on the oscilloscope) produced at each zero crossing of the *modulating* signal. See the 33120A User's Guide for more information about the SYNC output.

2. Measure the modulation index (Item 6 in the Prelab) and check against the set value on the function generator.

3. Display the spectrum of the AM signal on the spectrum analyzer, in units of dBm into $50\,\Omega$. Measure the power level of the carrier and of the sideband line. How many dB below the carrier is the sideband line? Compare your measurements to your Prelab calculations.

4. Investigate the effect on the AM spectrum of varying the modulating frequency (i.e., message frequency) and the modulation index. In particular, investigate the effect on the sideband power of varying the modulation index.

5. Set the function generator so that the message is the square wave of Items 4 and 5 from the Prelab. Display the AM signal on the DSO and measure the modulation index.

6. Display the AM signal on the spectrum analyzer. Measure the carrier and at least five sideband pairs. How many dB below the carrier are the sideband lines? Compare your measurements to your Prelab calculations.

7. Build the envelope detector of Figure 2. Apply the AM signal of Item 1 (sinusoidal message) and display the demodulated output on the DSO. Compare the demodulated signal to the message signal, and comment on any discrepancies. Investigate the effect of varying the message frequency and the modulation index.

8. Repeat for the AM signal of Item 5 (square wave message).

References

[Carlson] A. Bruce Carlson, Paul B. Crilly, and Janet C. Rutledge, *Communication Systems: An Introduction to Signals & Noise in Electrical Communication*, 4$^{\text{th}}$ ed., McGraw-Hill (2002)

[Couch] Leon W. Couch, II, *Digital and Analog Communication Systems*, 6$^{\text{th}}$ ed., Prentice-Hall (2001)

LABORATORY 6
AM MODULATORS

OBJECTIVES

To simulate, build, and test an unbalanced AM modulator, and to simulate one kind of doubly balanced modulator.

PRELAB

1. Read Section 4.3 in [Carlson] (especially Square Law and Balanced Modulators), Section 4.11 in [Couch], and Appendix D of this lab manual.

2. You are going to build and test the very simple unbalanced diode AM modulator shown in Figure 1. In this circuit, the message is a 30 kHz sinusoid and the carrier is a 200 kHz sinusoid. The R_1-R_2-R_3 network adds the carrier and the modulating signal, the square-law device is the 1N4148 diode, and the L_1-C_1-R_4 network is the bandpass filter. The output is the voltage across L_1-R_4 to ground, as indicated.

3. Verify that the filter is a bandpass filter (the input is the current into the filter and the output is the voltage across it), and that its resonant frequency is the carrier frequency.

4. Simulate the circuit of Figure 1. Run the simulation for a long enough time that the FFT of the output voltage will be accurate. Reasonable values for the amplitudes of the sinusoids are 0.8 V for the message and 1.0 V for the carrier.

5. Display the FFT of the output voltage; include the printout in your notebook.

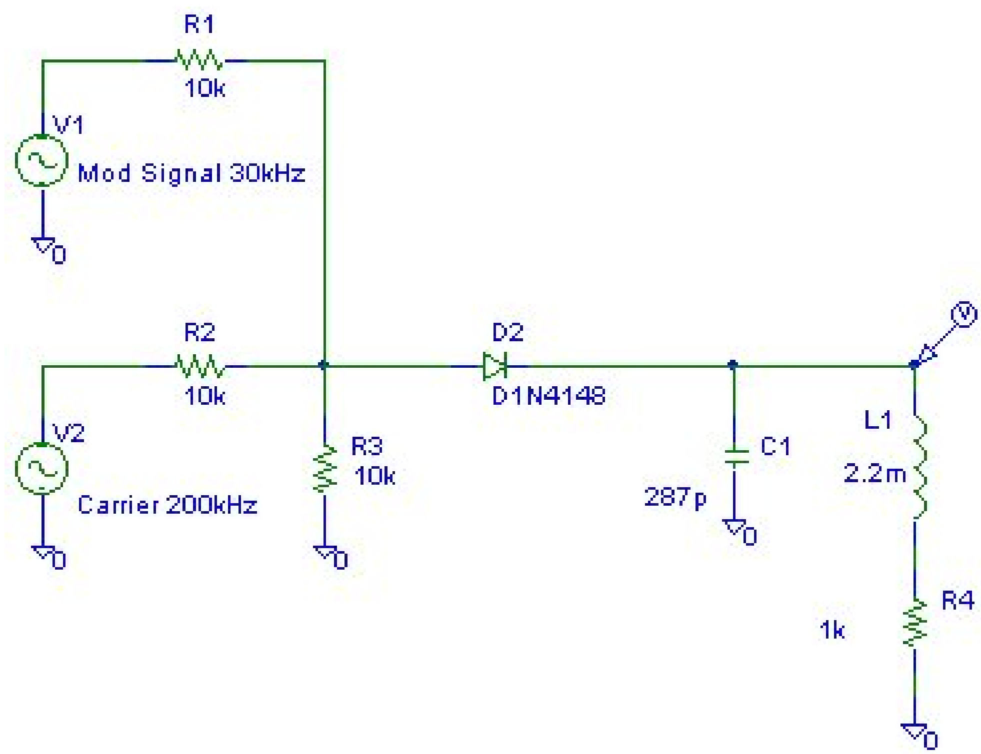


Figure 1: AM Modulator

6. Your FFT should show an AM signal at 200 kHz with the sideband lines 30 kHz above and below. But you will also see other smaller components. What is their origin? (Two hints: What is the frequency response of your bandpass filter? Is the diode *exactly* a square-law device?)

7. Calculate how many dB below the carrier line (200 kHz) the spurious lines in the spectrum are.

8. In Item 4 of the In Lab portion you will simulate a doubly-balanced modulator. You should have time to do that part in lab, but you may do it as a prelab if you wish.

IN LAB

1. Build the AM modulator of Figure 1. **Note**. The 2.2 mH inductors are available, but you cannot get exactly the 287 pF capacitors. But you can get close by using series or parallel combinations of capacitors that are available. The resonant frequency of the bandpass filter will be slightly off. (You may adjust the carrier frequency to match the resonant frequency of your filter if you like.)

2. Display the output voltage signal on the oscilloscope, and display its FFT on the oscilloscope.

3. Display the output spectrum on the spectrum analyzer. Compare the frequencies of the lines you observe with your prelab simulation, and compare the differences (in dB) of the line amplitudes from the carrier with your prelab simulation.

4. In this part you will *simulate*, but not build, one type of doubly balanced mixer for generation of DSB. Layout the circuit of Figure 2 in Schematics. (This type of doubly-balanced mixer is discussed in Section 4.11 of [Couch].) The message and the carrier are the same as in the preceding parts.

5. Run the simulation for what you think would be a good time to get an accurate FFT. Display the FFT.

6. You should see a prominent carrier line. But isn't this circuit supposed to produce DSB? This simulation demonstrates a phenomenon apparent only in the simulation. PSpice starts the simulation at $t = 0$,

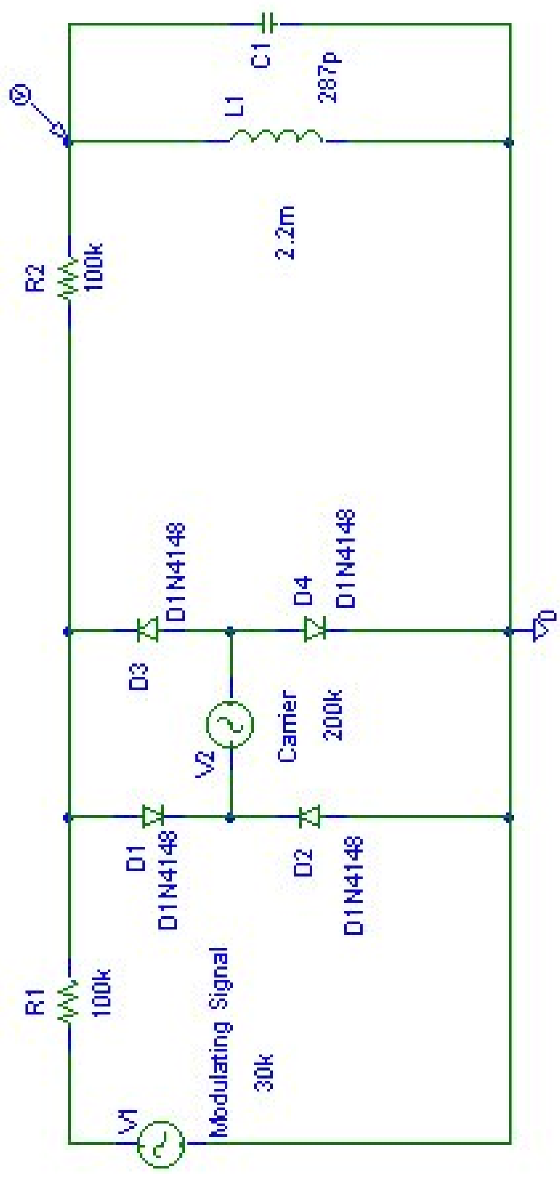


Figure 2: DSB Modulator

and so the circuit experiences a transient. In this circuit, the BPF resonates at $f_c = 200\,\text{kHz}$ and it is seeing $A_c \sin(2\pi f_c t)u(t)$ at the start of the simulation. As a result, the filter "rings" for a short time and so a significant line at 200 kHz is seen.

7. You can run a more accurate simulation as follows. (1) From your simulation, estimate how long the transient lasts. (In my simulation it lasts about 150–200 μs.) Run the simulation for much longer so that the output is mostly steady-state. Now look at the FFT. (2) Better still, in the simulation setup enter a no-print delay large enough so that the the initial transient data is not collected. Display the output voltage and its FFT. You should find that the carrier line is suppressed.

8. The moral of this little exercise is that you have to pay attention to transients in simulations. Sometimes you want to see the transient. But sometimes it is unimportant, and if you don't set up your simulation appropriately, you may be misled when you go to make steady-state measurements on the circuit.

9. One final point. Why did you not build this circuit? (It seems to be simple enough.) Answer: look at how the carrier must be connected. Can you connect the function generator this way? The answer is no. The function generator produces a *single-ended* output, meaning that it must be connected between a node and ground. The carrier generator called for in Figure 2 must have a *differential* output. (It's the same sort of reason that you cannot use the oscilloscope probe to measure the voltage across two nodes—you must always measure from a node to ground. To measure across nodes you need a differential probe—they are available, but expensive. A 20 MHz differential probe for our oscilloscopes costs around $500.)

References

[Carlson] A. Bruce Carlson, Paul B. Crilly, and Janet C. Rutledge, *Communication Systems: An Introduction to Signals & Noise in Electrical Communication*, 4$^{\text{th}}$ ed., McGraw-Hill (2002)

[Couch] Leon W. Couch, II, *Digital and Analog Communication Systems*, 6$^{\text{th}}$ ed., Prentice-Hall (2001)

LABORATORY 7
THE PHASE-LOCKED LOOP AND FREQUENCY MODULATION AND DEMODULATION

OBJECTIVES

To investigate FM signals in the time and frequency domains; to measure the characteristics of a phase-locked loop (PLL); to use a PLL for frequency modulation and demodulation.

PRELAB

Prelab

1. Read Section 5-6 (Phase Modulation and Frequency Modulation) and Section 4-14 (Phase-Locked Loops and Frequency Synthesizers) in [Couch], or Sections 5.1 (Phase and Frequency Modulation) and 5.2 (Transmission Bandwidth and Distortion) and Section 7.3 (Phase-Lock Loops) in [Carlson], and Appendix E (The Phase-Locked Loop) in this manual.

2. Obtain an expression for the spectrum of an FM signal with single-tone modulation, where the carrier amplitude is A_c, the carrier frequency is f_c, the message frequency is f_m, and the modulation index is β.

   - For such an FM signal, what is the smallest value of β for which the carrier spectral component is zero?
   - Plot the FM spectrum for the following values: $A_c = 100\,\text{mV}$, $f_c = 100\,\text{kHz}$, $f_m = 10\,\text{kHz}$, and $\beta = 1$. Express the amplitudes of the lines in units of dBm into $50\,\Omega$.
   - For these values, use Carson's rule to estimate the FM bandwidth.

- Determine the 99% power bandwidth of the FM signal. (That is, the frequency band containing 99% of the total power.)
- Finally, plot the FM signal in the time domain. **Hint**: In Mathcad, use the following to calculate the Bessel functions: `J0(x)` returns $J_0(x)$, `J1(x)` returns $J_1(x)$, and `Jn(m,x)` returns $J_m(x)$ for $0 \leq m \leq 100$. In Matlab, use `BESSELJ`.
- Repeat for $\beta = 3.25$.

3. Design an RC lowpass filter having half-power bandwidth between 1.5 kHz and 2.5 kHz (the lower the cutoff frequency the better), and having $R \geq 10\,\text{k}\Omega$. You will use this filter in the PLL demodulator part of the lab.

IN LAB

1. Use the function generator to produce a tone-modulated FM signal with a sine wave carrier having the following parameters: carrier frequency $f_c = 100\,\text{kHz}$, carrier amplitude $A_c = 100\,\text{mV}$, message frequency $f_m = 10\,\text{kHz}$, and modulation index $\beta = 1$. (You set β by setting the peak frequency deviation on the function generator.)
2. Display the FM signal on the DSO.
3. Display the FM signal on the spectrum analyzer.
   - Measure the frequencies and power levels (in dBm) of the carrier and the first five lines above the carrier. Compare with your prelab.
   - Use the spectrum analyzer to measure the 99% power bandwidth of the FM signal. Compare with your prelab bandwidth calculations and with the Carson's rule bandwidth.
4. Repeat items 1, 2, and 3 with an FM signal having modulation index $\beta = 3.25$.
5. Keeping the carrier frequency and the message frequency fixed, investigate the effect on the FM spectrum of changing the modulation index. Determine the smallest frequency deviation for which the carrier power is zero and compare to your prelab.

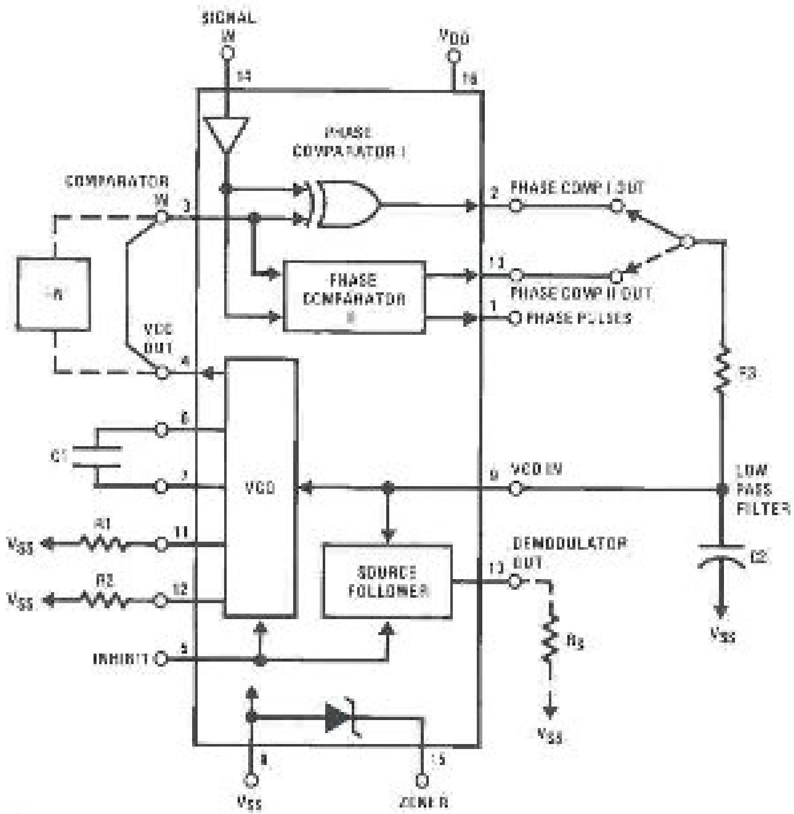


Figure 1: Block Diagram of the CD4046 PLL

6. We shall now study the characteristics of a particular phase-locked loop. The CD4046 is a digital PLL chip implemented with CMOS technology; the block diagram of the chip is shown in Figure 1.[1] Any PLL consists of three blocks: a phase detector (or phase comparator), a low-pass filter, and a voltage-controlled oscillator (VCO). (See Figure 4-19 in [Couch] or Figure 7.3-2 in [Carlson], and Figure 2 in Appendix E of this manual.) The CD4046 provides two different phase detectors and the VCO; the lowpass filter must be connected exter-

[1]Specification data for the CD4046 PLL, National Semiconductor Corp., Document no. RRD-B30M115, (1995).

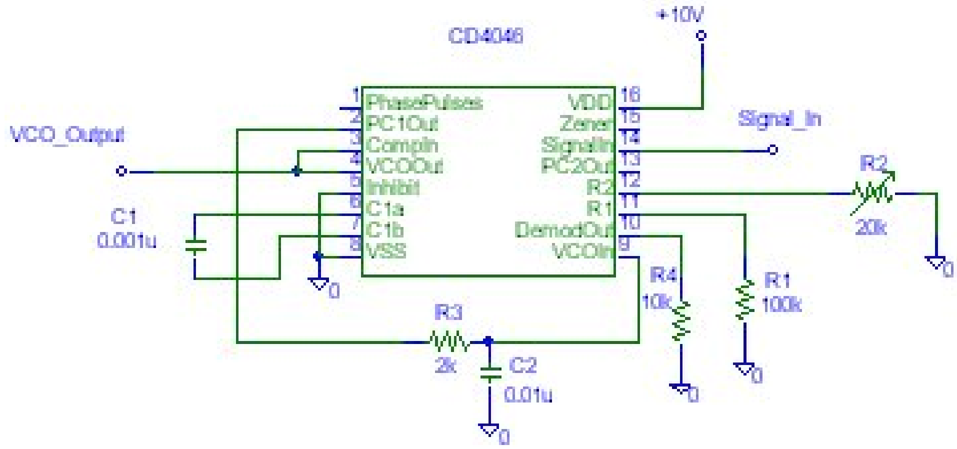


Figure 2: PLL Circuit

nally by the user. (That is, the user can design the filter to obtain the desired PLL behavior.) Phase detector I is an exclusive OR gate phase detector, which provides a triangle characteristic , and phase detector II is an edge controlled memory network (essentially, it is a flip-flop phase detector) which provides a sawtooth characterisitic; see Figure 4-20 in [Couch] or Figure 7.3-1 in [Carlson]. In this lab we shall use phase detector I.

- Build the PLL circuit shown in Figure 2.
- Note that Signal In (pin 14), VCO Out (pin 4), and PC1 Out (pin 2) are digital signals—i.e., they are square waves with LOW = 0 V and HIGH = 10 V.

7. Set Signal In equal to zero. (Connect pin 14 to ground.) Set the free-running frequency of the VCO to $f_0 = 100\,\text{kHz}$ by adjusting the $20\,\text{k}\Omega$ potentiometer until you see a 100 kHz square wave at the VCO Out (pin 4) and a symmetric error voltage (i.e. equal LOW and HIGH durations) at the Phase Comparator I output (pin 2). Display both signals on the DSO.

8. Use the function generator to generate a 100 kHz square wave that switches between 0 V and 10 V. Disconnect pin 14 from ground, and use the function generator as Signal In. (**Note**: Pin 14 of the CD4046

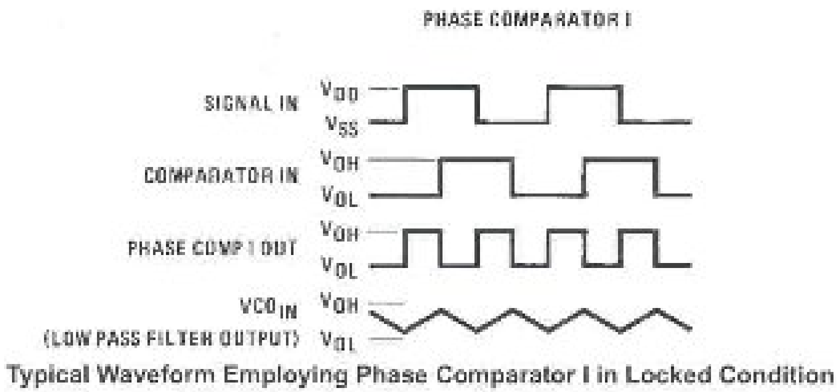


Figure 3: Typical PLL Waveforms in Locked Condition

is a high impedance input.) Display and print the signals at PC1 Out (pin 2), Comparator In (pin 3), VCO In (pin 9), and Signal In (pin 14). (Typical waveforms that you should see are shown in Figure 3.) Be sure to record the voltage levels and frequencies of the signals. **Note**: You may use the Signal In to trigger the DSO.

9. We shall next measure the hold-in and pull-in ranges of the PLL. (Refer to Figure 4-23 and the accompanying discussion in [Couch].) The *hold-in* range is the range of frequencies about f_0 over which a locked loop will remain in lock; the *pull-in* range is the range of frequencies over which a loop will acquire lock.[2] The pull-in range is never larger than the hold-in range; see Figure 4.

   - Verify that the VCO output (pin 4) and the input signal (pin 14) are both at $f_0 = 100\,\text{kHz}$.
   - Set the input frequency to a value below f_0 such that the PLL is out of lock; when the loop is out of lock the VCO output signal will be unstable.
   - Slowly increase the input frequency until the VCO output becomes stable. This is the lower frequency of the pull-in range—the PLL has just pulled-in the input frequency.
   - Slowly increase the input frequency until the VCO output becomes unstable. The PLL has now lost lock; this is the upper

[2] The hold-in range is also called the lock range, and the pull-in range is sometimes called the acquisition range or capture range.

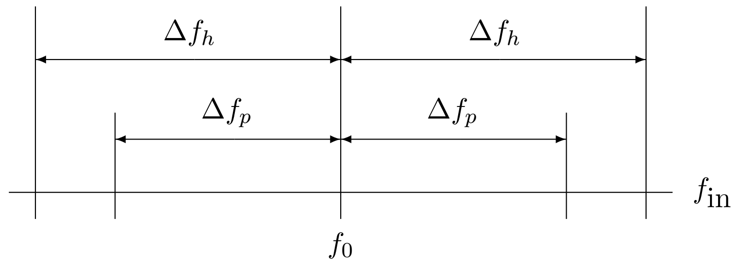


Figure 4: Pull-in and Hold-in Ranges: Pull-in = $2\Delta f_p$, Hold-in = $2\Delta f_h$

frequency of the hold-in range.

- Slowly decrease the input frequency until the PLL again acquires lock—this is the upper frequency of the pull-in range.
- Continue decreasing the input frequency until the PLL loses lock—this is the lower end of the hold-in range.
- The device manufacturer gives the following approximate relationship between the hold-in and pull-in ranges[3]:

$$2\Delta f_p \approx \sqrt{\frac{2\Delta f_h}{\pi R_3 C_2}}.$$

Compare your measured values to this formula.

10. We shall now use the PLL as an FM modulator; build the circuit of Figure 5. Set the free-running frequency of the VCO (pin 4) to 100 kHz; see item 7.
    - Use one function generator to produce a 100 kHz square wave that switches between 0 V and 10 V. Use this for the Carrier In signal (pin 14).
    - Use your second function generator to produce a 1 kHz, $5\,\mathrm{V_{p\text{-}p}}$ sine wave. Use this for the Message Signal.
    - Display the FM signal (the VCO output at pin 4) on the DSO.
    - Systematically investigate the effect on the FM signal of varying the amplitude and frequency of the message signal. Explain your observations.
11. We shall now use the PLL as an FM demodulator; build the circuit of Figure 6. Set the free-running frequency of the VCO to 100 kHz.

[3]Specification data for the CD4046, *op. cit.*, p.11

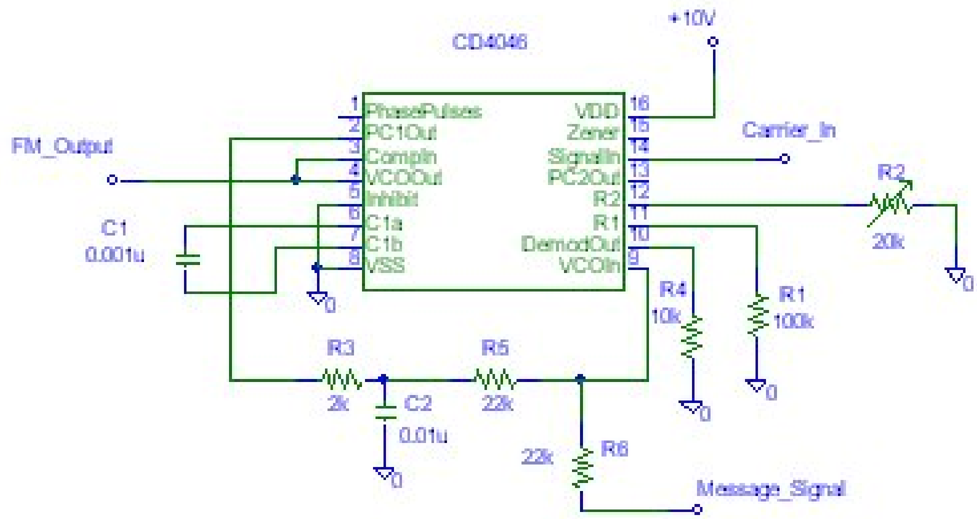


Figure 5: FM Modulator Circuit

- Use the function generator to produce the following FM signal. Carrier: sine wave at 100 kHz, 10 $V_{\text{p-p}}$ with 5 V dc offset; this is the Carrier In signal on pin 14. (The dc offset must be present because the pin 14 signal must have LOW level 0 and HIGH level 10 V.) Message: sine wave at 1 kHz. Peak frequency deviation: 1 kHz. This is the Message Signal input in Figure 6. Connect the RC lowpass filter from the second week prelab to pin 2 as indicated in Figure 6.
- Display the demodulated signal (output of the LPF) on the DSO. **Hint**: Use the SYNC output of the function generator for your trigger.
- Investigate the effect of varying the frequency of the message signal, and explain your observations.

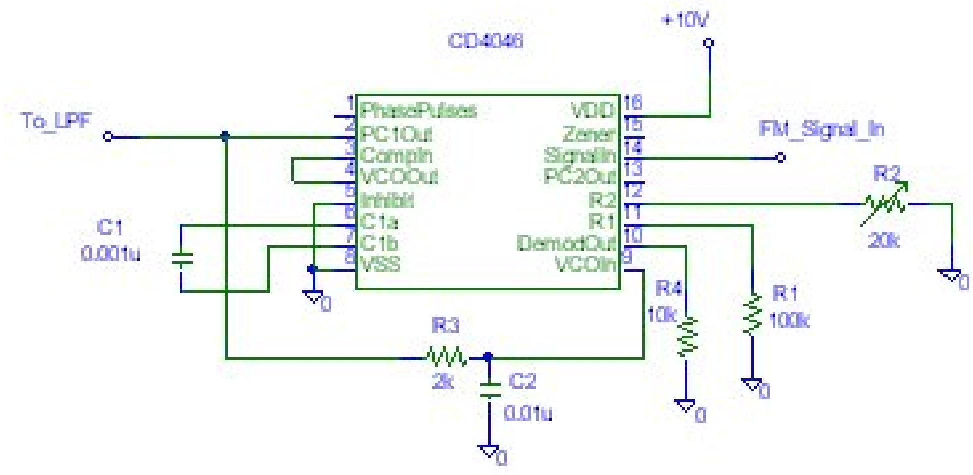


Figure 6: FM Demodulator Circuit

References

[Carlson] A. Bruce Carlson, Paul B. Crilly, and Janet C. Rutledge, *Communication Systems: An Introduction to Signals & Noise in Electrical Communication*, 4th ed., McGraw-Hill (2002)

[Couch] Leon W. Couch, II, *Digital and Analog Communication Systems*, 6th ed., Prentice-Hall (2001)

LABORATORY 8
MORE FREQUENCY MODULATION/DEMODULATION

OBJECTIVES

To investigate direct FM using a VCO and slope detection of FM.

PRELAB

1. Read Sections 4-13 and 5-6 in [Couch], or Section 5.3 in [Carlson] on direct generation of FM, and slope detection of FM.

2. There are many ways of generating and detecting FM; we saw one in Laboratory 7 using a PLL. In this lab we shall consider one method of direct FM using a voltage-controlled oscillator (VCO). A VCO is also an integral part of the PLL. We shall use the popular 555 timer IC as the VCO in this lab. The 555 is basically a multivibrator; it can be operated in monostable mode (i.e., as a "one-shot") or in astable mode as an oscillator. When used as an oscillator it of course provides a square wave output.[1]

   The FM modulator is shown in Figure 1. The message is the sinusoidal source labeled `VMod`; it has an amplitude of 1 V and a frequency of 5 kHz. The DC offset `Voff` must be present because the 555 control input must always be positive. (You may of course set the offset in the sinusoidal source.) Simulate the modulator and display the output and its spectrum. (Remember that you are looking at tone modulation of a *square* carrier.) Is the spectrum what you expect?

[1]See the 555 data sheet for further details: *LM555 Timer Specifications, National Semiconductor Corp., February 2000.*

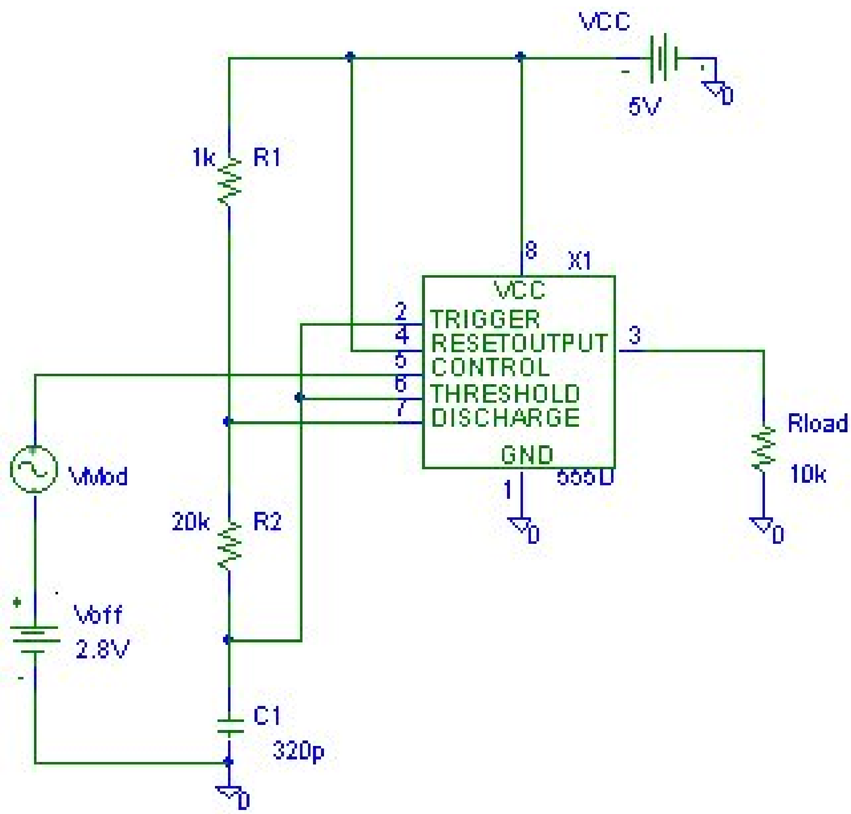


Figure 1: FM Modulator

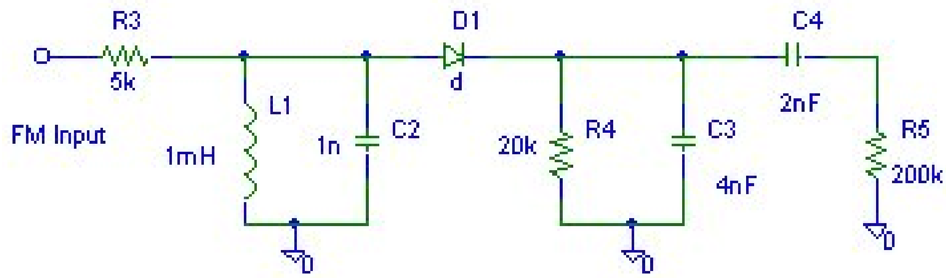


Figure 2: FM Slope Detector

3. The demodulator is the simple slope detector of Figure 2. This is an FM-to-AM converter—it differentiates the FM signal and passes the resulting mixed FM/AM signal through an envelope detector. The front end is a tuned bandpass filter; its resonant frequency is slightly higher than the carrier frequency so that the incoming FM signal lies on the left side of the filter frequency response so that it acts as a differentiator. The diode and R_4-C_3 circuit is of course the envelope detector, and the C_4-R_5 circuit is the highpass filter (DC block).

   - Calculate the resonant frequency of the bandpass filter.
   - Calculate the time constant of the lowapss filter in the envelope detector and show that it is appropriate for the message and carrier frequencies.
   - Calculate the time constant of the highpass filter and show that it is appropriate.

4. Connect the FM modulator of Figure 1 to the slope detector of Figure 2 and simulate the whole system. (Remove the load resistor in Figure 1—connect the output directly to the FM Input in Figure 2.) Display the output and its FFT. (**Note**: As always, you will want to run the simulation for a long enough time to get good FFT; you will also see that the output has a transient before it settles into a steady-state that you will probably not want to include in the FFT. But, if you try to run the simulation for too long, you will encounter a limitation of the evaluation version of PSpice—the 555 is a mixed analog/digital part and if you try to run the simuation for too many periods of the square wave output you will find a limitation on the

number of transitions allowed in the digital circuit. You will have to find a good compromise for the simulation time.)

IN LAB

1. Build the circuit of Figure 1 *without* the modulating signal and its DC offset—replace them with a small capacitor. This is the free-running astable circuit; the output across the load resistor will be a square wave. Display the output and its spectrum and measure its fundamental frequency. This square wave is the carrier.

2. Compare the measured frequency against the theoretical value

$$f_c = \frac{1.44}{(R_1 + 2R_2)C_1}.$$

   (See the 555 data sheet.)

3. Now connect the message (with DC offset) as in Figure 1. Display the output and its spectrum; compare with your prelab simulation.

4. Build the slope detector of Figure 2. Test the slope detector by using the function generator to provide an FM signal of the same carrier frequency and tone modulating frequency as your 555 FM modulator. Choose the frequency deviation to give you approximately the same FM bandwidth as your 555 modulator. You can test with sinusoidal and square carriers. Display the demodulated output and its spectrum.

5. Now connect the output of the 555 modulator to the input of the slope detector. (Remove the load resistor in Figure 1.) Display the demodulated output and its spectrum.

6. Explain sources of distortion in the detector.

References

[Carlson] A. Bruce Carlson, Paul B. Crilly, and Janet C. Rutledge, *Communication Systems: An Introduction to Signals & Noise in Electrical Communication*, 4th ed., McGraw-Hill (2002)

[Couch] Leon W. Couch, II, *Digital and Analog Communication Systems*, 6th ed., Prentice-Hall (2001)

LABORATORY 9
SAMPLING AND PULSE AMPLITUDE MODULATION

OBJECTIVES

To investigate the time- and frequency-domain properties of PAM signals with natural sampling.

PRELAB

1. Review the discussion of the sampling theorem in Section 2-7 of [Couch] and Section 6.1 of [Carlson].

2. Read Section 3-2 in [Couch] about flat-top sampled PAM and naturally sampled PAM. (Section 6.2 in [Carlson] discusses only flat-top PAM, but naturally sampled PAM was discussed in lecture.)

3. Consider a sinusoidal message signal

$$x(t) = A_0 \cos(2\pi f_0 t).$$

   Suppose we create a naturally sampled PAM waveform, $x_s(t)$, using a sampling waveform having sampling frequency f_s and duty cycle $d = \tau/T_s$. (See Figure 3-1 in [Couch].) Assume that f_s exceeds the Nyquist rate for $x(t)$.

   If $f_0 = 500\,\text{Hz}$, $f_s = 5\,\text{kHz}$, $\tau = 40\,\mu\text{s}$, and $A_0 = 1\,\text{V}$:

   - Calculate and plot the PAM signal $x_s(t)$,
   - Calculate and plot the magnitude spectrum $|X_s(f)|$.

You may of course make the plots carefully and to scale by hand on graph paper, but it will be much easier and more efficient to use Mathcad or Matlab. You should use the FFT function in these programs to obtain the plot of $|X_s(f)|$.

4. In lab you will implement naturally-sampled PAM using an electronic switch. Specifically, you will use the CD4016 CMOS quad bilateral switch. Simulate the circuit of Figure 1 in PSpice. The part CD4016BD is available in the EVAL library of Microsim PSpice—the message is applied to pin 1, the sampling waveform is applied to pin 13, $+V_{cc}$ is applied to pin 14, $-V_{cc}$ is applied to pin 7, and the PAM output is on pin 2. Use a sinusoidal message and a sampling waveform as in Item 3. Set the amplitude of the sampling waveform for a $\pm V_{cc}$ swing. Plot the PAM output signal and its spectrum using the FFT in Probe.

   Hint: I suggest using the VPULSE part in PSpice to generate the sampling waveform. You need to specify the rise time and fall time of this square wave. You can also specify a maximum time step in the Analysis Setup. Your PAM signal will probably look "spikey". This is caused partly by how you adjust the rise and fall times relative to the maximum step size. You can reduce this effect (but you may not be able to eliminate it) by making the maximum step size small relative to the rise and fall times of the sampling waveform. Of course, the smaller you make the step size the longer the simulation will take. As the engineer on this project, you will have to reach a reasonable compromise.

5. The message $x(t)$ can be recovered from the PAM signal by ideal low-pass filtering. (This is explained in [Couch] and in lecture.)

   Of course we do not have an ideal LPF. Suppose that we recover the sinusoidal signal from the PAM signal in Item 3 by means of a non-ideal low-pass filter. To be exact, we shall use the Sallen-Key circuit from Laboratory 3 with $R = 30\,\text{k}\Omega$ and $C = 0.01\,\mu\text{F}$ (which result in 6 dB break frequency of 530 Hz). Calculate and plot the signal and its amplitude spectrum at the filter output. (Again, you should use Mathcad or Matlab.)

   The demodulated output of the filter will not be precisely the sinusoidal message that you started with—there will be some other frequency components present. In other words, the demodulated output signal will be *distorted.* This distortion is not nonlinear distortion,

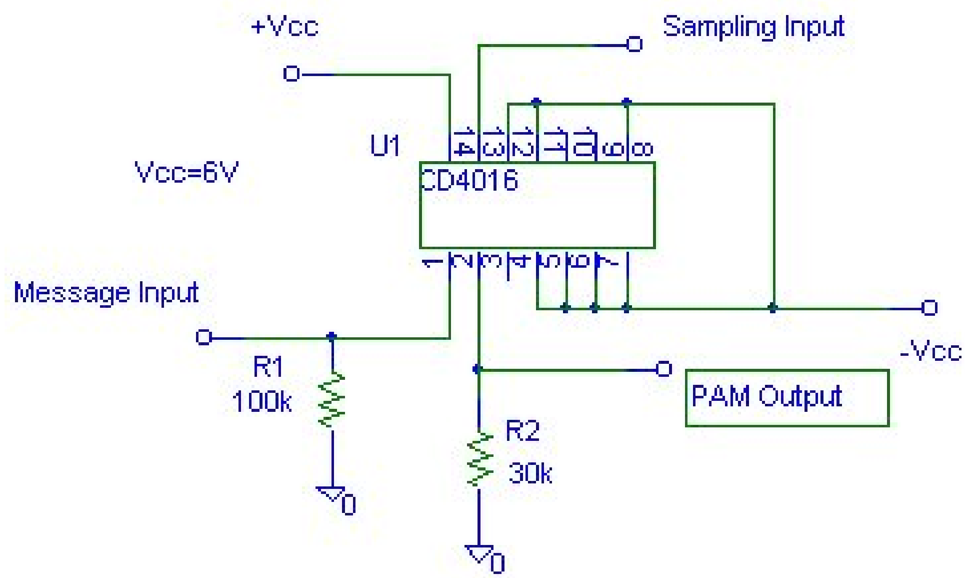


Figure 1: Generation of naturally sampled PAM

but is present simply because the filter is not an ideal LPF—it passes some unwanted frequency components. As we have seen (in Lab 3), one fairly quick way to quantify the distortion is to calculate the total harmonic distortion. Calculate the THD of the demodulated signal at the output of the filter.

6. Simulate the demodulation of the PAM signal in PSpice: connect the output of the PAM circuit to the input of the Sallen-Key circuit. Plot the demodulated output of the filter in Probe and its spectrum.

   Also calculate the THD of the demodulated signal in this simulation.

IN LAB

1. Build the circuit shown in Figure 1. This circuit implements the naturally sampled PAM system shown in Figure 3-2 of [Couch]—the signal to be sampled is the input to a switch, the opening and closing of which is controlled by the sampling signal consisting of a sequence of rectangular pulses. The CD4016 is a quad analog CMOS bilateral

switch. (That is, there are four switches on the chip, and on each switch the signal flow can be in either direction.) Pins 1, 2, and 13 constitute one switch: pins 1 and 2 are the input and output, and pin 13 is the on/off control signal. The other pins that are tied low (pin 7 is ground) are the inputs and controls of the other three switches; the open pins are the outputs of the other three switches. These pins must be connected as indicated to prevent crosstalk. You should understand that the device is a switch, but it is not an *ideal* switch. In particular, it has a non-zero propagation delay from input to output (approximately 15 ns), non-zero "on" resistance (approximately 215 Ω), and its frequency response is not flat (it has a 3 dB break frequency of about 150 MHz).[1]

2. Set one function generator to produce the rectangular sampling waveform with parameters f_s and τ from item 3 of the Prelab. Set the amplitude of the sampling signal for $\pm V_{cc}$ swing. Pay attention to how you connect the function generator to the circuit—what should you set the output impedance of the function generator to?

3. Use the other function generator for the sinusoidal message signal to be sampled—set $f_0 = 500\,\mathrm{Hz}$ and $A_0 = 1\,\mathrm{V}$.

4. Display the PAM signal on the oscilloscope. (It may help to get your trigger off the message signal.) Is its amplitude what you expect? Explain. (Remember: the switch is not ideal.) To make measurements easier, adjust the message signal amplitude so that the PAM signal has swing $2\,\mathrm{V_{p\text{-}p}}$. Include a printout of the PAM signal in your notebook.

5. Display the spectrum of the PAM signal on the oscilloscope. Include a printout of the spectrum in your notebook.

6. Record the magnitudes of the spectral components of the PAM signal, and record the ratios (or differences in dB) between adjacent peaks. Compare with your prelab item 3 PAM spectrum.

7. Now connect the PAM output signal to the input of the Sallen-Key filter with cutoff $f_c = 530\,\mathrm{Hz}$. Display the demodulated signal on the oscilloscope, and include a printout in your notebook. Is it what you expect?

[1] "Motorola Semiconductor Technical Data: MC54/74HC4016", Motorola, Inc., 1995.

8. Display the spectrum of the demodulated signal on the oscilloscope and include a printout in your notebook. Measure the magnitudes, and differences in magnitudes, of any spectral peaks, and compare with your calculations from item 5 of the prelab.

9. Calculate the THD of the demodulated signal and compare with your prelab.

10. Investigate *systematically* the effect of sampling pulse duration τ (or duty cycle d) and sampling rate f_s on the PAM signal and on the demodulated signal. Record your observations *systematically* and *quantitatively*. Compare your observations to what you should expect the effects to be in theory. Be sure to decrease f_s below the Nyquist rate so that you can observe aliasing.

References

[Carlson] A. Bruce Carlson, Paul B. Crilly, and Janet C. Rutledge, *Communication Systems: An Introduction to Signals & Noise in Electrical Communication*, 4$^{\text{th}}$ ed., McGraw-Hill (2002)

[Couch] Leon W. Couch, II, *Digital and Analog Communication Systems*, 6$^{\text{th}}$ ed., Prentice-Hall (2001)

LABORATORY 10
ISI and Eye Patterns

Overview

Prelab

- The goal of the prelab will be to use simulation to generate an eye pattern for a binary or 4-ary PAM signal. The eye pattern will be observed for several different roll-off factor values. This will be a multi-step problem:
    1. Generate a random PAM signal
    2. Generate a Raised Cosine filter (pulse)
    3. Run the PAM signal through the Raised Cosine filter
    4. Plot the Eye Pattern
    5. Display the Fourier transform of the output

In-Lab

- The goal of the in-lab portion of the experiment is to observe an eye pattern on the oscilloscope that is formed by running a PAM signal through a low-pass filter. This is also a multi-step problem:
    1. Generate a pseudo-random PAM signal using the arbitrary function generator
    2. Build an RC filter
    3. Run the PAM signal through the RC filter
    4. Plot the output eye pattern onto the oscilloscope
    5. Display the PSD of the output.

Prelab

1. **Read the section in the book pertaining to ISI and eye pattern diagrams. (Carlson/Crilly/Rutledge Secs. 11.1 and 11.3, Couch Sec. 3-6)**

2. **Generate a random 4-ary PAM signal (at least 100 symbols). Display the random PAM sequence on a stemplot.**
    - **The following Matlab code will do this, or you can write your own to achieve the same result:**

```
a=[-3 -1 1 3];                   %Create the 4-ary constellation
ind=floor(4*rand(100,1))+1;      %Create a Random bit Sequence
PAM=a(ind);                      %Random 4-PAM sequence
stem(PAM);                       %Plot Sequence
```

3. **Generate a raised cosine filter impulse response. The bandwidth is 6kHz. What condition must we impose on the sampling frequency and why? We will use a sampling frequency of 27kHz. Assume that we use a**

symbol rate of 9000symbols/s. What is the rolloff factor (α)? Consider the raised cosine from -5T to 5T where T is the symbol period. Plot the raised cosine filter. *Note:* the rolloff factor α is a parameter between 0 and 1. Carlson, et. al., (Sec. 11.3) use parameters β and r to define the raised cosine filter: the relationship is $\alpha=2\beta T$, and r =1/T is the rate. Couch (Sec. 3-6) uses a parameter f_Δ in the raised cosine definition; his f_Δ is the same as Carlson's β, or $\alpha=2f_\Delta T$.

Matlab code:

```
Fs = 27000;                     %Sampling frequency is 27kHz
T = 1/9000;                     %Symbol period
t=-5*T:1/Fs:5*T;                %Set time scale
t=t+1e-10;                      %So that t=0 is not included
alpha=0.5;                      %Set roll-off factor
p=(sin(pi*t/T)./(pi*t/T).*cos(alpha*pi*t/T)./(1-(2*alpha*t/T).^2));
%p is the raised cosine pulse
clf;
plot(t,p);                      %plot the filter
hold on;
stem(t,p); xlabel('Time [s]'); ylabel('Amplitude');
hold off;
```

4. **Run the PAM sequence through the raised cosine filter. Remember that to use the 'filter' function in Matlab, the two vectors must have the same sampling frequency, so it will be necessary to upsample the PAM vector (i.e., [a1 a2 a3] becomes [a1 0 0 a2 0 0 a3 0 0]).**

```
N=length(PAM);
r=Fs*T;
pams=zeros(size(1:r*N));
pams(1:r:r*N) = PAM;            % upsampled version of PAM
xn=filter(p,1,pams);            %runs vector pams through filter p
figure; plot(xn(1:200));        %plots a portion of the filter output
clf;
hold on;
```

5. **Generate the eye pattern. Remember that eye patterns are typically shown over a time period of 2T. Is there a delay to the signal? If so, why? Now change α (rolloff) to various values between 0 and 1. Make eye diagrams for several different rolloff factors. How does the rolloff factor affect the ISI as seen through the eye diagram? How does the eye diagram show the effect of ISI on sensitivity to timing error and the noise margin? What is the primary negative effect of high ISI?**

```
d=5*T*Fs+1;                     %calculating delay
for i=d:6:300-6                 %start from point 16 (delay)
```

```
plot(xn(i:i+6))                    %plot the first 7 samples (2T)
end                                %the loop will plot on top of itself
```

6. **Experiment with the spectral characteristics of the system. Using 2048 samples, generate frequency spectrum plots for both the filter and the output signal. Print out both the Signal spectrum of the output and the filter (in amplitude and dB). How does changing the roll-off factor of the pulse-shaping filter affect the signal spectrum of the output and the filter? Make printouts to substantiate your assertions.**

```
Nfft=2048;
P=fftshift(fft(p,Nfft));                    %Displays the fft of p
X=fftshift(fft(xn,Nfft));              %Displays the fft of xn
f=-Fs/2:Fs/(Nfft-1):Fs/2;                   %Frequency axis scale
figure;
subplot(211);plot(f,abs(P));grid;title('Signal Spectrum of P');
xlabel('Frequency [Hz]');
subplot(212);plot(f,20*log10(abs(P)));grid;title('Signal Spectrum in dB');
xlabel('Frequency [Hz]');
figure;
subplot(211);plot(f,abs(X));grid;title('Signal Spectrum of X');
xlabel('Frequency [Hz]');
subplot(212);plot(f,20*log10(abs(X)));grid;title('Signal Spectrum in dB');
xlabel('Frequency [Hz]');
```

In-Lab

1. **Using Matlab, generate a random binary bit pattern with length 15.**

2. **Use the function generator to create and store this signal. Because we can not create a truly random signal, the idea is that we will create a 'pseudo-random' signal. By using 15 random bit values repeated at the proper frequency, we will be able to control the symbol rate. Instructions on how to create an arbitrary function can be found in the instruction manual.**

3. **Design and build an RC filter with the same bandwidth as the raised cosine filter of the prelab, 6kHz. Record the values of your Resistor and Capacitor.**

4. **The symbol frequency is still 9kHz. Set the output of the function generator accordingly and attach the signal to the filter.**

5. **Display the eye pattern on the oscilloscope. What effect does changing the symbol rate have on ISI? Demonstrate your results with experimentation and commentary.**

APPENDIX A
BASICS OF THE DIGITAL STORAGE OSCILLOSCOPE

1 Introduction

This appendix contains basic information about digital storage oscilloscopes in general, and some specific information about the Agilent 54622D oscilloscope that you will use in the communication laboratory.

The function of any oscilloscope of course is to provide a visual display of a time-varying signal (i.e. voltage). In an analog oscilloscope the signal is directly displayed on a cathode ray tube—an internally generated ramp causes the electron beam to scan horizontally across the CRT, and the signal being measured is applied across the vertical deflection plates. In a digital storage oscilloscope (DSO), the signal is acquired in an entirely different way. In order to understand the advantages and limitations of a DSO, we must begin with an understanding of the way in which the oscilloscope acquires the signal.

2 Signal Acquisition in a DSO

In broad terms, a DSO first samples the input signal and then displays the waveform that is *reconstructed* from the samples. This analog-to-digital conversion performed by the DSO results in several advantages over an analog oscilloscope. The major advantage is that we can perform signal processing operations on the sampled signal, such as differentiation, integration, addition of signals, calculation of Fourier transforms, as well as storage of the waveform in memory. At the same time, the digital-to-analog conversion has its limitations, and you must be aware of these as you make measurements; if you are not careful, the signal you display (i.e. the reconstructed signal) may not bear any resemblance to the true signal.

2.1 Sampling

The idea of sampling is simple: take samples of a signal at discrete time instants, and if the samples are "close enough" in time the signal can be reconstructed (at least approximately) by interpolating between the samples. The famous sampling theorem[1] tells us precisely how close is "close enough": if the samples occur at a rate at least twice the highest frequency contained in the signal, then the signal can be *exactly* reconstructed from its samples by passing the sampled signal through an ideal lowpass filter whose bandwidth is equal to the highest frequency contained in the signal.

The sampling theorem implies the trade-off that we always face in analog-to-digital conversion: we want closely spaced samples, but the more closely they are spaced, the more samples we need, and hence the more memory we need to hold them. In most practical problems, memory is the critical limitation. Hence, we usually start not by specifying the sampling rate, but by specifying the total number of samples that we will take; this is the *record length*. We then select (or more accurately, the oscilloscope selects) the appropriate sampling rate for the signal. Note how the sampling trade-off appears now: with a fixed record length, we will only acquire a short time duration of a rapidly varying signal, while we will acquire a longer duration of a slowly varying signal.

The Agilent 54622D has a maximum sampling rate of 200 Msamples/sec for one channel and 100 Msamples/sec for two channels, and a maximum memory of 4 Mbytes. When you set the horizontal time base, the oscilloscope chooses the sampling rate and the record length (hence the memory depth). There are some complications in the relationship—under some circumstances the sampling rate can be faster than the rate at which samples are stored. This is handled internally by a smoothing operation. The signal frequencies used in this lab should not cause any difficulties. But you should always be aware of the sampling rate that the instrument is using. (Press the Main/Delayed button to see it.)

2.2 Acquisition Modes

Now you have the basic idea of the operation of a DSO: take a finite record length of samples of a signal, and display the time signal reconstructed from the samples.

There are many aspects of the signal acquisition that you can control, the main being the choice of acquisition mode. See the manual for more

[1]The sampling theorem is discussed in detail in the lecture course, EEL 4514.

information.

- **Normal Mode**. This is the default. The oscilloscope creates a record by saving the first sample (of perhaps several) during each acquisition interval.
- **Peak Detect Mode**. Any signal wider than 5 ns will be displayed regardless of sweep speed.
- **Average Mode**. The DSO acquires data after each trigger using Normal mode, and then averages the record point from the current acquisition with those stored from previous acquisitions. This mode helps reduce random noise.
- **Real Time Mode**. The oscilloscope produces the waveform from samples collected during one trigger. It should only be necessary at sweep speeds of 200 ns/div or faster.

You also control signal acquisition with the RUN/STOP and SINGLE buttons.

3 Triggering

Another important function that you need to learn how to control is triggering; basically, triggers determine when the DSO will start acquiring and displaying a waveform. That is, the trigger determines the time-zero point. Once a trigger occurs, the DSO acquires samples to construct the post-trigger (to the right, or after in time) part of the waveform. (The DSO automatically acquires enough samples to fill in the pre-trigger part of the waveform.) The oscilloscope will not recognize another trigger until the acquisition is complete.

3.1 Trigger Source

You can obtain your trigger from one of the input channels, from the AC power line (useful for testing signals related to the power line frequency, such as when you are testing a power supply), or from an externally supplied source (for example, you can use the SYNC signal produced by the function generator as the trigger source).

3.2 Trigger Types

The DSO has several types of triggers that you can use. The default type, and the only type you will need in this course, is the **Edge** type. An edge trigger occurs when the trigger *source* passes through a specified voltage level in a specified direction (i.e. slope).

The other trigger types available are pulse, pattern, CAN, duration, I^2C, sequence, SPI, TV, and USB. You can find details in the DSO user manual.

3.3 Trigger Modes

The mode determines what the DSO will do in the absence of a trigger. There are three modes.

- **Normal**. In this mode the DSO will acquire a waveform only when the trigger conditions are met.

- **Auto**. This mode will allow the DSO to acquire a waveform even if a trigger does not occur. In auto mode, a timer starts after a trigger occurs; if another trigger is not detected before the timer runs out, the oscilloscope forces a trigger. The duration of the timer depends on the time base setting. Note that if triggers are being forced, successive acquisitions will not be triggered at the same point on the waveform, and so the waveform will not be synchronized on the screen—it will roll across.

- **Auto Level**. Works only when edge triggering on analog channels or external trigger. The oscilloscope first tries to Normal trigger. If no trigger is found, it searches for a signal at least 10% of full scale on the trigger source and sets the trigger level to the 50% amplitude point. If there is still no signal present,the oscilloscope auto triggers. This mode is useful when moving a probe from point to point on a circuit board.

3.4 Other Aspects of Triggering

Holdoff When the DSO sees a trigger, it disables the trigger system until the acquisition is complete. Some repetitive signals, especially digital pulses, contain many valid trigger points; a simple trigger might result in a series of waveforms on the screen. You can set the holdoff time to be longer than the acquisition interval to get a stable display.

Coupling Coupling determines what part of the trigger signal is passed to the trigger circuit. Your choices are DC (all of the signal), AC (the dc part is blocked), low frequency rejection (frequencies below 50 kHz are blocked), TV, high frequency rejection (frequencies above 50 kHz are blocked), and noise rejection (makes the trigger circuit less sensitive to noise, but may require a higher amplitude signal to trigger).

4 Signal Spectra on the DSO

As we have said, one very useful feature of the DSO is its ability to display the results of mathematical operations on the signals. Your Agilent 54622D can display the product of the two channels, the difference between the channels, the derivative of a signal, the integral of a signal, and the amplitude spectrum of a signal. Here we shall discuss the display of the spectrum.

The DSO calculates the spectrum by calculating the discrete Fourier transform (DFT) of the signal.[2] To be precise, the oscilloscope calculates the fast Fourier transform (FFT), which is just an efficient algorithm for the DFT. It is important that you have a basic understanding of how the DSO calculates the FFT, because it is possible that it will display utter gibberish (as any computer will) if you do not understand its limitations. The basic idea is this[3]: we wish to calculate the Fourier transform of a continuous-time signal on a digital computer, so we first truncate the signal to a finite-time duration by multiplying it by a "window" function, and then we sample the windowed time function at an appropriate rate to create a finite-length record of samples. Suppose that $x(t)$ is the waveform, and after windowing and sampling we have N samples, say $x_0, x_1, \ldots, x_{N-1}$. The DFT of the samples is defined by the equation

$$X(k) = \frac{1}{N} \sum_{n=0}^{N-1} x_n e^{-j2\pi nk/N}, \quad k = 0, 1, \ldots, N-1,$$

and the inverse DFT is

$$x_n = \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N}, \quad n = 0, 1, \ldots, N-1.$$

[2] The DFT is similar, but not identical, to the discrete-time Fourier transform for discrete-time signals that you learned about in EEL 3135.

[3] See Section 2–8 of Couch's book.

Note that the DFT $X(k)$ is a discrete-frequency function; if we select the windowing function correctly and if we sample at the appropriate rate, $X(k)$ will be a good approximation to the Fourier transform $X(f)$.

We shall not go into details about the DFT here; for now we shall merely state some of the limitations about using the DFT as an approximation to the continuous Fourier transform that you should keep in mind. These limitations are summarized in the conditional statement we made earlier: **if** we select the windowing function correctly and **if** we sample at the appropriate rate, $X(k)$ will be a good approximation to the Fourier transform $X(f)$.

- Regardless of the number of points in the waveform record, the Agilent DSO uses 2048 points for the FFT.
- Three windows are available: Hanning, rectangular, and flat-top. See the User's Guide for advice on using windows.
- Note that the vertical units for the FFT display are dBV.
- It would be to your benefit to read "FFT Measurement Hints" on pages 5-29–5-30 in the User's Guide, especially the discussion of frequency resolution.

Always remember: Every time you make a measurement with an oscilloscope, you must know how the input is coupled, how the waveform is acquired, how the oscilloscope is triggered, and the sampling rate being used.

APPENDIX B BASICS OF THE SPECTRUM ANALYZER

1 Introduction

This appendix contains some general information about spectrum analyzers, and some specfic information about the Agilent E4411B spectrum analyzer that you will use in the communication laboratory. Remember that the spectrum analyzer User's Guide is included in the "Equipment Manuals" folder on the PC desktop at your lab station.

Like an oscilloscope, a spectrum analyzer produces a visible display on a screen; the Agilent spectrum analyzer has a VGA screen rather than a CRT screen. Unlike an oscilloscope, however, the spectrum analyzer has only one function—to produce a display of the frequency content of an input signal. (But it is possible to display the waveform on the spectrum analyzer screen with the proper settings.) And also like an oscilloscope, the spectrum analyzer will always produce a picture on the screen; but if you do not know how to properly use the spectrum analyzer, that picture may be complete gibberish.

CAUTION: The input of the spectrum analyzer cannot tolerate large signals; before you connect a signal to the input, be sure you know that the signal will not exceed the maximum allowable input rating of the spectrum analyzer. (The maximum signal input is printed right on the front panel, near the input connector.)

2 Signal Acquisition in a Spectrum Analyzer

Most spectrum analyzers (including the Agilent models in the communication lab) are *heterodyne*[1] spectrum analyzers (also called *scanning* spec-

[1]Heterodyne is derived from the Greek, meaning mixing different frequencies

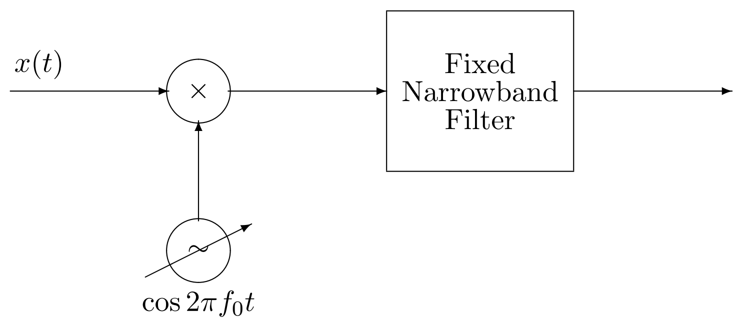


Figure 1: Frequency Mixing, or Heterodyning

trum analyzers). A heterodyne analyzer is essentially a radio receiver (a very sensitive and selective reciever). Radio receivers, including those based on the heterodyne principle, are covered in some detail in the lecture course (see Section 4–16 in [Couch] or Section 7.1 in [Carlson]); for now we shall content ourselves with a simple description of the basic ideas.

Given a voltage signal $x(t)$, how do we resolve it into its frequency components for display on a screen? As we know, one solution is provided by the digital storage oscilloscope—calculate the FFT of the signal from its internally stored samples. Another solution would be to pass $x(t)$ through a bank of very narrow bandpass filters, having adjacent passbands, and then plot the amplitudes of the filter outputs. That is, if filter 1 has passband $f_1 - B/2 \leq f \leq f_1 + B/2$, and filter 2 has passband $f_2 - B/2 \leq f \leq f_2 + B/2$, where $f_1 + B/2 = f_2 - B/2$, and so on, and if B is small enough, then the filter outputs give us the frequency components $X(f_1), X(f_2), \ldots$. This is, of course, not a practical solution. A better solution is suggested by a simple property of Fourier transforms: recall that if we multiply (in the time domain) a signal by a sinusoid the spectrum of the signal is shifted in frequency by an amount equal to the frequency of the sinusoid. That is,

$$x(t)\cos 2\pi f_0 t \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0).$$

Now instead of a bank of narrow filters, we shall have **one** narrow filter centered at a **fixed** frequency, say f_I, and we shall scan the signal spectrum across this filter by multiplying $x(t)$ by a sinusoid of varying frequency f_0. See Figure 1. The filter is a narrow bandpass filter at a fixed center frequency, f_I, (called the intermediate frequency); in a spectrum analyzer, its bandwidth is selected by the user. The oscillator frequency, f_0, is adjustable, as indicated in Figure 1. In an ordinary AM or FM radio, when you tune the receiver you are selecting this frequency so that the desired signal will pass through the filter; in a spectrum analyzer, this frequency is automatically scanned (repeatedly) over a range, which must be selected so that the frequency component $X(f)$ is shifted to f_I and passed by the filter. For example, if we want to view the frequency content of $x(t)$ from f_1 to f_2, then we must select f_0 to scan from $f_1 + f_I$ to $f_2 + f_I$.

Of course, much more signal conditioning is going on inside the spectrum analyzer than is indicated in Figure 1; but the frequency mixing is the fundamental step. In particular, the signal first is passed through a lowpass filter whose bandwidth is chosen to eliminate *image frequencies.* (Once again, see the section on the superhet in [Couch] or in [Carlson].) Also, most scanning spectrum analyzers are *multiple conversion* analyzers—they have two to four intermediate frequency stages, at successively lower frequencies. The reason is that we have two conflicting goals to achieve; we would like to have the filter bandwidth as small as feasible, and we would like to be able to scan over large frequency ranges. It is hard to build sharp narrow filters at high frequencies, but it is also hard to build multipliers that will work over large frequency ranges. Therefore, we achieve narrow filters at low intermediate frequencies by shifting the frequency down in several steps.

You may naturally ask why we have a spectrum analyzer if the oscilloscope will display an FFT of a signal. The DSO's display of the FFT has the advantage of capturing one-shot events, as well as being able to store the FFT in memory or on a floppy. But the scanning spectrum analyzer usually holds the advantage over the FFT in frequency range, sensitivity, and dynamic range. If you find yourself working in communications, especially in RF and microwave communications, you will probably find that you will frequently be using a spectrum analyzer for spectral measurements.

3 Spectrum Analyzer Controls

In this section we shall describe some of the basic controls on the spectrum analyzer that you will frequently use. More details on these, and descriptions of the more obscure controls, can be found in the user manual. Mainly, you will use the three large buttons labeled FREQUENCY, SPAN, and AMPLITUDE, the various MARKER buttons for making measurements, and the BW/Avg button for selecting the resolution bandwidth. In addition, you will use the control knob, the up and down buttons labelled with large arrows (above the control knob), and the numerical keypad for entering values that will control the display.

When you use the spectrum analyzer, always pay attention to the information about the instrument state given in the top, left, and bottom margins of the screen.

Calibration. The manufacturer recommends a 5 minute warm-up for the analyzer.

When the spectrum analyzer is turned on, it goes through an internal alignment, or calibration, procedure. You will hear clicking and see the alignment screens flash by. This procedure only takes a couple of minutes. The analyzer then continuously runs its alignment check—you will hear occasional noises as this goes on, but it will not interrupt your measurements. You can also manually run the alignment, but this should never be necessary.

FREQUENCY control. In normal operation the frequency control selects the range that the variable oscillator in Figure 1 sweeps through. Pressing the FREQUENCY button causes the frequency menu to appear at the right side of the screen. You can select the center frequency (CF) and the start and stop frequencies. You select the numerical values by turning the control knob, pressing the up/down arrows (the step size is controlled by the CF Step entry in the menu), or by entering the value with the numerical keypad.

SPAN Control. Pressing the SPAN button brings up the frequency span menu. Here you select the frequency span displayed on the screen (as opposed to selecting start and stop frequencies), and you can select span zoom, zero span, and full span.

AMPLITUDE control. Pressing this button displays the amplitude menu. Here you select the reference level, whether the amplitude units are power (dBm) or linear (mV), and the scale in dB/division (when using the logarithmic scale). Here is where the spectrum analyzer seems strange compared to an oscilloscope: you measure signal levels *from the top* of the screen, or down from the reference level. For example, on power-up, the reference level is 0 dBm, meaning that the *top* line on the screen is at 0 dBm and you measure the amplitudes of lines in the spectrum down from that level.

Once again, you are cautioned to be careful about applying signals to the spectrum analyzer; it is easy to cause extensive and expensive damage.

Resolution Bandwidth control. The resolution bandwidth is essentially the bandwidth of the fixed narrowband filter in Figure 1. (In reality, there are several stages of filtering.) Pressing the BW/Avg button displays the menu from which you can select the resolution bandwidth, the video bandwidth, and associated controls. Note that you cannot select a continuous range of RBW—there is only a finite selection available.

The resolution bandwidth determines how close frequency components in the signal spectrum can be and still be displayed as distinct components on the screen.

Sweep control The sweep time determines how often the input signal is scanned through the analyzer. Note that you can select continuous sweep and single shots, just as you can with the oscilloscope.

Markers. Just as the oscilloscope has markers, the spectrum analyzer has four markers to help you make measurements. You select markers, difference markers, or no markers with the MARKER control buttons and their menus.

File control Like the oscilloscope, this spectrum analyzer has the capability of storing screen captures and instrument states internally or on an external floppy disk. You access this through the file menus.

References

[Carlson] A. Bruce Carlson, Paul B. Crilly, and Janet C. Rutledge, *Communication Systems: An Introduction to Signals & Noise in Electrical Communication*, 4^{th} ed., McGraw-Hill (2002)

[Couch] Leon W. Couch, II, *Digital and Analog Communication Systems*, 6^{th} ed., Prentice-Hall (2001)

APPENDIX C
SOME BACKGROUND ON OSCILLATORS

1 Introduction

In this appendix we present a brief background on sinusoidal oscillator circuits, which you will investigate in Laboratory 4. Oscillators are ubiquitous in communications—we need to generate carrier signals, normally sinusoids, in both the transmitter and receiver. We shall discuss only *sinusoidal* oscillators. One way to obtain a sinusoid is to produce some easily generated periodic waveshape, such as a square wave by means of a multivibrator circuit, and then to filter out all of the frequency components except the fundamental. Another way is to generate a triangle wave (again with a multivibrator) and to use a waveshaping circuit to produce a sine wave. (This is the way in which many function generators work since they are designed to produce several types of waveforms.) But in communications circuits we need just a sine wave, not a function generator.

There are many factors that need to be taken into account when designing an oscillator, such as its physical size, power consumption, fabrication cost and complexity, and so on, but every oscillator is meant to provide a *sine wave* at a *fixed frequency* and with a *fixed amplitude*. That is, whatever else the design engineer needs to worry about, there are three fundamental measures of merit for any oscillator:

- The purity of the sine wave—its spectrum should consist of one line. Every circuit is going to produce some harmonics, of course, but the power contained in the harmonics should be small relative to the fundamental. (One way to quantify this is with the THD.)

- The *frequency stability* of the oscillator should be good. That is, the frequency of the sine wave should not drift, both in the short term and the long term.

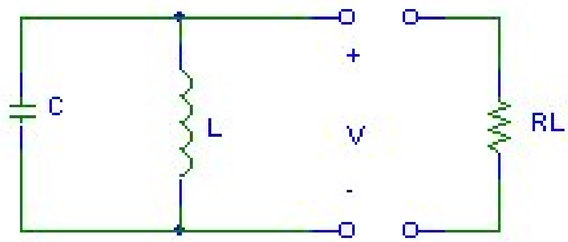


Figure 1: Parallel Resonant Circuit

- The *amplitude stability* of the sine wave should also be good.

The subject of oscillators is quite large, and no single reference covers everything. The main reason is the huge range of operating frequencies—oscillators find application in circuits operating over the whole of the electromagnetic spectrum, from tens of Hz (the low end of the audio range) up to around 300 GHz (the upper end of the microwave range). The devices and circuit design techniques become quite different as we move into higher and higher frequencies. Nevertheless, some general classifications of oscillator types can be made and the aim of this appendix is to outline these for you. The References list contains some titles that will help you pursue your own research into this area.

2 The Negative Resistance Oscillator

As you know from your basic circuits course, the voltage across a parallel resonant LC circuit with no resistance will oscillate sinusoidally when an initial condition is applied. To review quickly, consider the LC circuit in Figure 1, without the load resistor connected. Suppose that the initial voltage across the capacitor is V_0 and the initial current through the inductor is 0. Then the initial-value problem describing this circuit is

$$\ddot{v}(t) + \omega_n^2 v(t) = 0$$

with initial conditions $v(0) = V_0$ and $\dot{v}(0) = 0$, and where $\omega_n = 1/\sqrt{LC}$. The solution for the voltage is

$$v(t) = V_0 \cos(\omega_n t), \quad t \geq 0.$$

Voilà! A sinusoidal oscillator. But there is a problem—this circuit cannot deliver power to an external circuit, which an oscillator must do of course. Suppose that the external circuit has equivalent resistance R_L; i.e., consider the circuit in Figure 1 with the load resistor connected and with the same initial conditions. Now we have the differential equation

$$\ddot{v}(t) + 2\zeta\omega_n \dot{v}(t) + \omega_n^2 v(t) = 0,$$

with $v(0) = V_0$ and $\dot{v}(0) = -V_0/R_L C$, and where

$$\zeta = \frac{1}{2R_L}\sqrt{\frac{L}{C}}$$

is the damping ratio. The differential equation is also written in terms of the Q factor of the circuit:

$$\ddot{v}(t) + \frac{\omega_n}{Q}\dot{v}(t) + \omega_n^2 v(t) = 0,$$

where $Q = 1/2\zeta$. If $\zeta \geq 1$, the voltage response is critically damped or overdamped and there is no oscillation. If $0 < \zeta < 1$, the voltage response is underdamped—it tries to oscillate, but the power consumption of the resistor causes the oscillations to be exponentially damped:

$$v(t) = V_0 e^{\sigma t}\left[\cos\omega t - \left(\frac{\sigma}{\omega} + \frac{1}{R_L C\omega}\right)\sin\omega t\right],$$

where

$$\sigma = -\zeta\omega_n \quad \text{and} \quad \omega = \omega_n\sqrt{1-\zeta^2}.$$

(σ and ω are just the real and imaginary parts of the characteristic roots of the differential equation.)

The idea of the negative resistance oscillator is very simple: design the resonant circuit with a *negative* resistance of value $R_{\text{neg}} = -R_L$ so that when the load is connected to the oscillator the LC circuit sees an equivalent resistance of infinity, and so the output voltage will be sinusoidal.[1] See Figure 2. The question arises: where do we get a negative resistance? Certain semiconductor devices, such as tunnel diodes, Gunn diodes, and IMPATT diodes have *i*–*v* characteristics that have negative slope (hence negative resistance) over part of the curve; see Figure 3. These devices can

[1] It is probably better to say that we design the circuit with a negative conductance $G_{\text{neg}} = -G_L$ so that the conductance seen by the LC circuit is zero, and to call it a negative conductance oscillator.

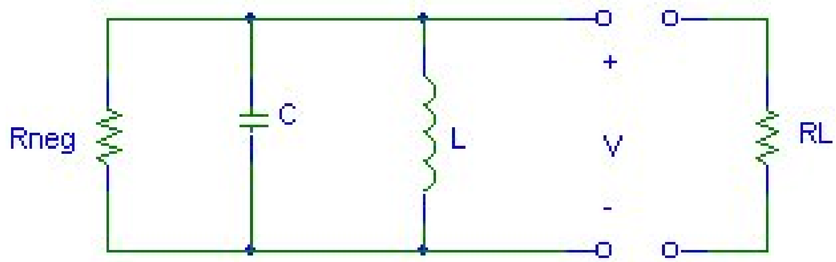


Figure 2: Negative Resistance Oscillator

provide oscillation frequencies in the range from 1 GHz up to 100 GHz. Note that a DC voltage must be supplied—the semiconductor device must be biased so as to operate on the negative resistance part of its characteristic. It should be pointed out that at microwave frequencies the resonator is not a simple lumped parallel LC circuit. The resonator may consist of waveguide cavities, microstrip transmission lines, and dielectric resonators.

We shall not be using these oscillators in lab and so we shall not pursue the analysis of them further.

3 Feedback Oscillators

The basic idea in generating sinusoidal oscillations electronically is that *positive* feedback around a linear amplifier, when chosen with appropriate gain, will cause the amplifier output to oscillate sinusoidally.[2] Remember that if the input to a linear circuit is a sinusoid, then the output is also a sinusoid; hence if a linear feedback amplifier (without input-signal excitation) oscillates, the output waveform must be sinusoidal. Consider Figure 4. The output of the amplifier is $X_0(s) = A(s)X_i(s)$, and the output of the feedback network is

$$X_f(s) = \beta(s)X_0(s) = A(s)\beta(s)X_i(s).$$

Hence the open loop gain is

$$L(s) = \frac{X_f(s)}{X_i(s)} = A(s)\beta(s).$$

[2]In amplifier design we usually try to *avoid* oscillation. There is an old saw in electronic design that says an oscillator is just a badly designed feedback amplifier.

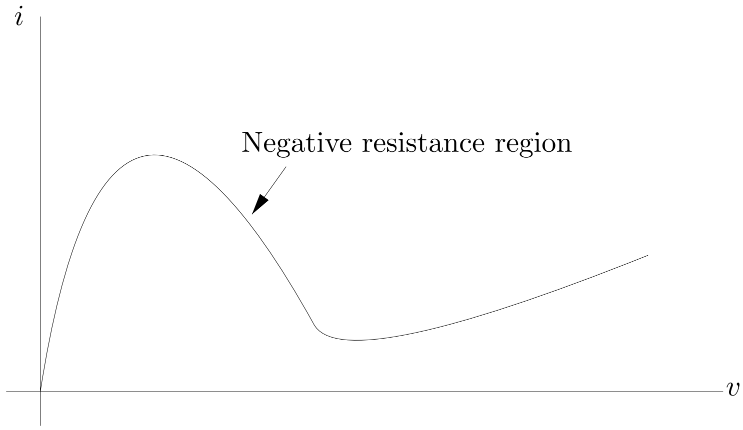


Figure 3: i–v characteristic of a negative resistance device

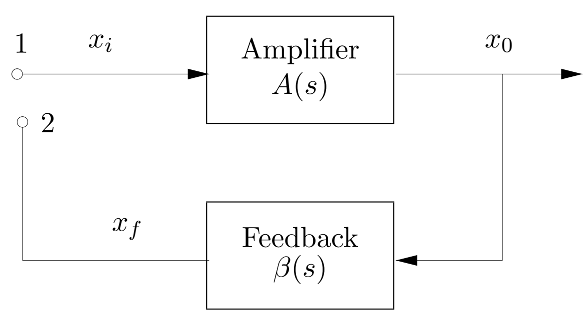


Figure 4: An amplifier and feedback network not yet connected to form a closed loop

Suppose that we could have $x_i(t) \equiv x_f(t)$ (i.e., the instantaneous values are equal for all t). Since the amplifier cannot distinguish the source of the input signal applied to it, it would appear that if we connect points 1 and 2 the amplifier would continue to provide the same output signal $x_0(t)$. Since $x_i(t) = x_f(t)$ is equivalent to $A(s)\beta(s) = 1$, we come to the following conclusion.

> **The Barkhausen Criterion**. A feedback amplifier with no external input signal will oscillate at frequency f_0 if the loop gain at f_0 is unity: $A(f_0)\beta(f_0) = 1$.

Note that the Barkhausen criterion really implies two conditions for oscillation: (1) the magnitude of the loop gain must be 1, and (2) the phase of the loop gain must be 0 (or an integral multiple of 2π).

Remarks. (1) The Barkhausen criterion requires that the closed loop phase shift be zero at the frequency of oscillation f_0. Hence the *frequency stability* of the oscillator is determined by the slope of the phase of $L(f)$ near f_0. Component characteristics (especially those of the transistors making up amplifiers) drift with temperature, age, voltage level, etc. A large slope in the phase of $L(f)$ at f_0 implies a more stable frequency of oscillation because any change in phase from 0 due to drift in amplifier parameters results in a small change in frequency; see Figure 12.2 in [Sedra/Smith]. We shall consider frequency stability in Section 5.

(2) The Barkhausen criterion also requires that $|A(f_0)\beta(f_0)|$ be exactly 1. If $|A\beta| < 1$, then oscillations will be damped out; if $|A\beta| > 1$, then the amplitude of the oscillations will continue to increase. Of course, such an increase can continue only until it is limited by the onset of nonlinearity in the active devices constituting the amplifier. In fact, this onset of nonlinearity is an essential feature of practical oscillators. Suppose that we initially have $|A(f_0)\beta(f_0)| = 1$. As the circuit characteristics drift, we soon have $|A(f_0)\beta(f_0)|$ either smaller or bigger than 1; in the former case the oscillation stops, in the latter it increases until limited by the onset of nonlinearity. Hence, in order to make sure that oscillations are sustained, we always design a practical oscillator to have $|A(f_0)\beta(f_0)|$ slightly greater than 1 (say by 5%), and let nonlinearity limit the amplitude of the oscillations. In fact, most practical oscillators are designed with a limiting circuit of some kind on the output; see Section 12.1 in [Sedra/Smith], especially Figure 12.3. As a result, we have to accept a small amount of distortion in the output sinusoid.

In practical feedback oscillator circuits, the amplifier $A(s)$ is an active device, such as an op amp or an FET, with high input impedance; sometimes, at least at low frequencies, a BJT amplifier is used. The feedback system $\beta(s)$ is usually a passive resonant network. In principle, any network can be used as the feedback as long as the Barkhausen criterion is satisfied. In the following subsections we discuss some commonly used configurations.

3.1 The Phase Shift Oscillator

A simple example of the ideas we have discussed is the phase shift oscillator shown in Figure 5 in both FET and op amp versions. (For simplicity the amplitude limiting circuit is not shown.) The phase shift oscillator consists of an inverting amplifier with a three section RC ladder network as feedback. The amplifier causes a 180° phase shift (it has negative gain), so in order to satisfy the Barkhausen criterion the feedback must provide another 180° shift; three RC sections is the minimum number that will work at a finite frequency.

Consider the FET version shown in Figure 5. The transfer function of the RC network from V_d (the voltage from drain to ground) to V_o, which is the negative of the feedback factor, is

$$-\beta(s) = \frac{V_o(s)}{V_d(s)} = \frac{(RC)^3 s^3}{(RC)^3 s^3 + 6(RC)^2 s^2 + 5RCs + 1}.$$

Hence

$$-\beta(\omega) = \frac{-j(RC)^3\omega^3}{-j(RC)^3\omega^3 - 6(RC)^2\omega^2 + j5RC\omega + 1} = \frac{1}{1 - 5\gamma^2 + j(\gamma^3 - 6\gamma)},$$

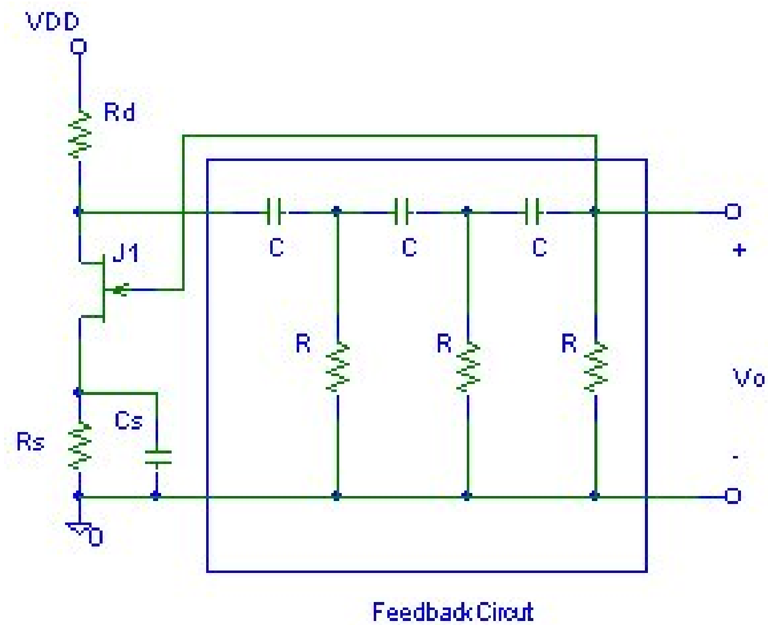
where $\gamma = 1/(RC\omega)$. The phase shift of V_o/V_d is 180° when $\gamma^2 = 6$, or when

$$f = \frac{1}{2\pi RC\sqrt{6}}.$$

At this frequency of oscillation $\beta = 1/29$. Hence, in order to satisfy the amplitude half of the Barkhausen criterion $|A|$ must be 29. ($|A|$ must be a little larger than 29 in practice.)

In the op amp version, the virtual ground between the + and − terminals means that the phase shift network is the same as the one in the FET oscillator, and so the frequency of oscillation is the same. Since the op amp gain is $-R_1/R$, we require R_1/R to be slightly greater than 29.

This oscillator is usually used in the range from several Hz to several hundred kHz, and so includes the range of audio frequencies.



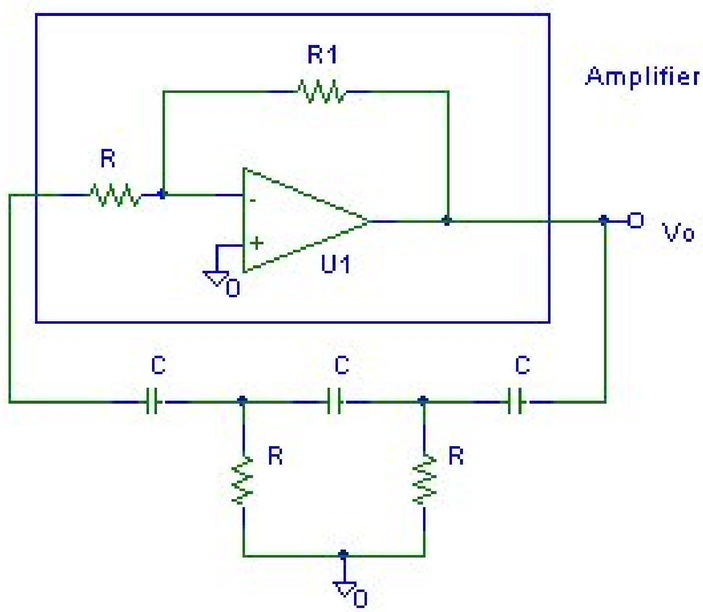


Figure 5: Phase Shift Oscillators

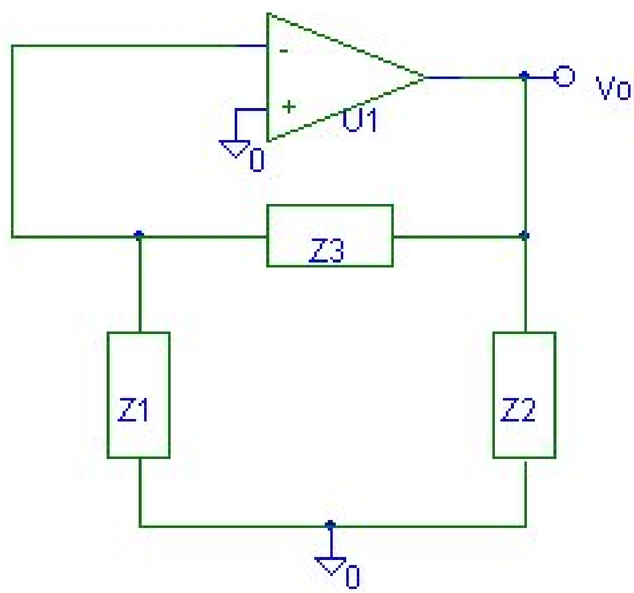


Figure 6: Oscillator With π Network Feedback

3.2 Oscillators With π Network Feedback

Many oscillator circuits use impedances arranged in a π network as the feedback; the op amp version is shown in Figure 6. Assuming the standard op amp model shown in Figure 7, it is easy to calculate the loop gain. Without feedback we have a load Z_L on the output consisting of Z_2 in parallel with the series combination of Z_1 and Z_3:

$$Z_L = \frac{Z_2(Z_1 + Z_3)}{Z_1 + Z_2 + Z_3}.$$

The open loop gain (i.e., without feedback) is

$$A = \frac{-A_v Z_L}{Z_L + R_o}.$$

The feedback factor is

$$\beta = \frac{Z_1}{Z_1 + Z_3}.$$

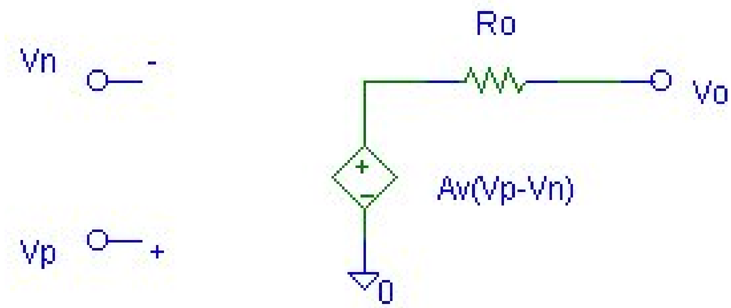


Figure 7: The Standard Op Amp Model

Hence the loop gain is

$$L = A\beta = \frac{-A_v Z_1 Z_2}{Z_2(Z_1 + Z_3) + R_o(Z_1 + Z_2 + Z_3)}.$$

Given a desired frequency of oscillation f_0, we need to choose the impedances so as to satisfy the Barkhausen criterion.

Suppose that the impedances are purely reactive (either inductive or capacitive) so that $Z = jX$. Then we have

$$L = \frac{A_v X_1 X_2}{-X_2(X_1 + X_3) + jR_o(X_1 + X_2 + X_3)}.$$

The Barkhausen criterion implies first that the loop phase shift be zero; in this case,

$$X_1 + X_2 + X_3 = 0.$$

Then we have

$$L = \frac{-A_v X_1}{X_1 + X_3},$$

or, since $X_1 + X_3 = -X_2$,

$$L = \frac{A_v X_1}{X_2}.$$

The Barkhausen criterion also implies that $L = 1$, and so X_1 and X_2 must have the same sign; i.e., they must be the same kind of reactance, either inductive or capacitive. It follows that $X_3 = -(X_1 + X_2)$ must be the other type of reactance. If X_1 and X_2 are capacitors and X_3 is an inductor, the circuit is called a *Colpitts* oscillator; if X_1 and X_2 are inductors and X_3 is a capacitor, the circuit is called a *Hartley* oscillator. Other combinations are also used. For example, if Z_1 and Z_2 are capacitors and Z_3 is a series combination of an inductor and a capacitor, then the circuit is called a *Clapp* oscillator.

Remark. Transistor versions of the Colpitts and Hartley oscillators are possible; see Section 12.3 in [Sedra/Smith]. Qualitatively, the operation of the circuits is the same as the op amp versions, but the detailed analysis is more difficult, for two reasons. First, the low input impedance of the transistor shunts Z_1, and so the equation for the loop gain is more complicated. Second, if the frequency of oscillation is beyond the audio range, the simple h-parameter model is not valid, and the hybrid-π model of the transistor must be used.

3.3 The Wien Bridge Oscillator

A Wien bridge oscillator uses a balanced bridge as the feedback network. The circuit is shown in Figure 8 at the top; below it the feedback network is redrawn to show explicitly that it is indeed a bridge. (Again the limiter used for amplitude stabilization is omitted; see Figures 12.5 and 12.6 in [Sedra/Smith].) Note that there are two feedback paths—a positive feedback through Z_1 and Z_2 which determines the frequency of oscillation, and negative feedback through R_1 and R_2 which determines the amplitude of oscillation. We have

$$\beta = \frac{Z_2}{Z_1 + Z_2} \quad \text{and} \quad A = 1 + \frac{R_1}{R_2},$$

and therefore

$$L(s) = A(s)\beta(s) = \left(1 + \frac{R_1}{R_2}\right) \frac{RCs}{(RC)^2 s^2 + 3RCs + 1}.$$

Hence

$$L(\omega) = \left(1 + \frac{R_1}{R_2}\right) \frac{1}{3 + j(RC\omega - \frac{1}{RC\omega})}$$

By the Barkhausen criterion, the frequency of oscillation is

$$f_0 = \frac{1}{2\pi RC}$$

and the oscillations will be sustained if

$$\frac{R_1}{R_2} = 2.$$

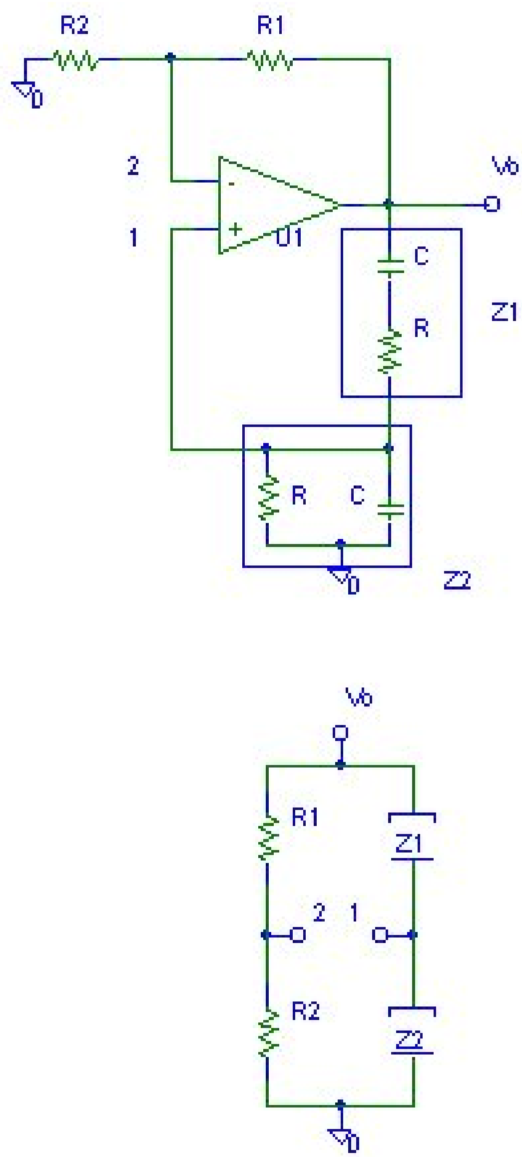


Figure 8: The Wien Bridge Oscillator

4 Crystal Controlled Oscillators

We have already alluded to the major concern with electronic oscillator circuits—frequency stability. As component characteristics change with age, temperature, signal level, etc., the oscillation frequency drifts. A crystal oscillator is often used in those cases in which the frequency drift must be kept small.

The crystals used in oscillators are usually quartz, although other materials can be used in specialized applications. The property that quartz possesses that we use is the *piezoelectric effect*[3]: electrical stresses (i.e., voltages) applied across the crystal in certain directions produce mechanical stresses (i.e., deflections) in other directions, and conversely, mechanical stresses produce voltages. We take advantage of the back-and-forth transfer of electrical and mechanical energy to produce very stable oscillations.

Piezoelectric quartz crystals are grown in the form of a rod having a hexagonal cross section. (See Figure 9.) The longitudinal Z axis is called the optical axis; electrical stresses applied in this direction produce no piezoelectric effect. Consider now a slice of the crystal perpendicular to the optical axis. Axes passing through the corners of the hexagon (such as the X axis in Figure 9) are called the electrical axes, and axes perpendicular to the faces of the hexagon (such as the Y axis in Figure 9) are called the mechanical axes. A flat section cut from the crystal in such a way that the flat sides are perpendicular to an electrical (X) axis is called an X cut; see Figure 9. Likewise a section cut with the flat sides perpendicular to a mechanical (Y) axis is called a Y cut. A mechanical stress in the direction of a Y axis produces an electrical stress in the direction of the X axis that is perpendicular to that Y axis, and conversely an electrical stress in the direction of an X axis produces a mechanical stress in the direction of the perpendicular Y axis. For example, in the X cut crystal shown in Figure 9, a mechanical stress along the Y axis causes charges to accumulate on the flat sides of the crystal, positive charges on one face and negative on the other (and so a voltage is developed across the faces). If the direction of the mechanical stress is reversed from tension to compression, or vice versa, the polarity of the charges (and hence the polarity of the voltage) on the faces reverses. Conversely, a voltage applied across the faces causes a mechanical stress along the Y axis.

When an alternating voltage is applied across the crystal in the direction of an electrical axis, alternating mechanical stresses will be produced in the

[3] Piezo comes from the Greek; it means "to press".

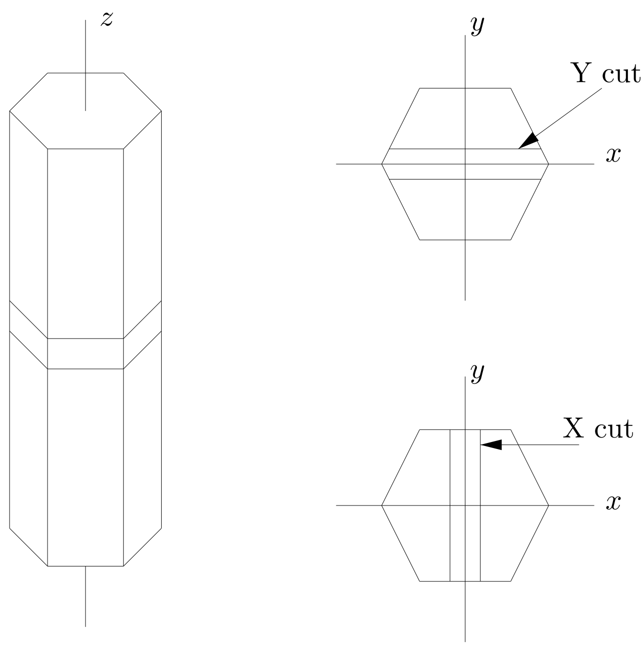


Figure 9: Quartz crystal showing X and Y cuts

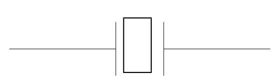


Figure 10: Crystal circuit symbol

direction of the perpendicular mechanical axis. The crystal will therefore vibrate, and if the frequency of the applied voltage is close to a frequency at which mechanical resonance can exist in the crystal, the amplitude of the vibrations will be large. Many other cuts at different angles are also used to obtain different resonant frequencies; in fact, X and Y cuts are rarely used in crystals today. Physically, a crystal oscillator consists of a flat section cut from a quartz crystal sandwiched between two electrodes, with leads for connection to an external circuit. The circuit symbol, shown in Figure 10, is a representation of this construction.

The crystal can be modeled with the electrical equivalent shown in Figure 11. Here C_1 models the electrostatic capacitance between the electrodes

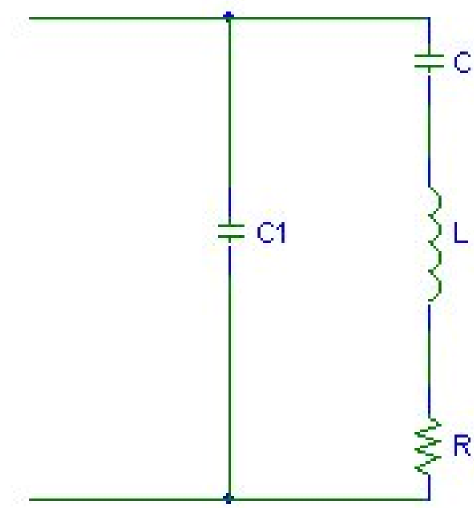


Figure 11: Equivalent circuit model of a crystal

when the crystal is not vibrating, and the series LCR circuit represents the electrical equivalent of the vibrational characteristics. The inductance L models the crystal mass, C models the mechanical compliance, and R models the mechanical friction. Typical values for a quartz crystal are listed in Table 1.[4]

It is a simple matter to calculate the impedance of the crystal modeled by the equivalent circuit of Figure 11:

$$Z(s) = \frac{s^2 + \frac{R}{L}s + \frac{1}{LC}}{C_1 s\left(s^2 + \frac{R}{L}s + \frac{C_1 + C}{LCC_1}\right)}, \tag{1}$$

or

$$Z(j\omega) = \frac{\omega^2 - \frac{R}{L}j\omega - \frac{1}{LC}}{C_1 j\omega\left(\omega^2 - \frac{R}{L}j\omega - \frac{C_1 + C}{LCC_1}\right)}.$$

Note that $Z(s)$ has a pole at $\omega = 0$—at dc the crystal is just a piece of rock,

[4] From [Terman].

Mechanical characteristics		Electrical characteristics
Length	2.75 cm	$L = 3.3\,\mathrm{H}$
Width	3.33 cm	$C = 0.042\,\mathrm{pF}$
Thickness	0.636 cm	$C_1 = 5.8\,\mathrm{pF}$
Resonant frequencies:		$R = 4518\,\Omega$
Series	427.50 kHz	
Parallel	429.05 kHz	

Table 1: Characteristics of a typical quartz crystal

and its impedance is infinite. We shall not concern ourselves further with the dc pole.

In Equation (1), the numerator of $Z(s)$ is just the impedance of the series RLC branch. The zeros are

$$s_1,\, s_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}},$$

or, defining the natural frequency ω_1 and damping ratio ζ of the series circuit in the usual way,

$$\omega_1 = \frac{1}{\sqrt{LC}}, \quad \zeta = \frac{R}{2}\sqrt{\frac{C}{L}},$$

the zeros of the numerator of $Z(s)$ are

$$s_1,\, s_2 = -\zeta\omega_1 \pm \omega_1\sqrt{\zeta^2 - 1}.$$

Likewise, the second degree polynomial in the denominator of $Z(s)$ has zeros which (along with $s = 0$) are the poles of $Z(s)$:

$$s_1,\, s_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{L}\left(\frac{1}{C} + \frac{1}{C_1}\right)},$$

or, if we define

$$\omega_2 = \sqrt{\frac{1}{L}\left(\frac{1}{C} + \frac{1}{C_1}\right)},$$

then the denominator zeros are

$$s_1,\, s_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \omega_2^2}.$$

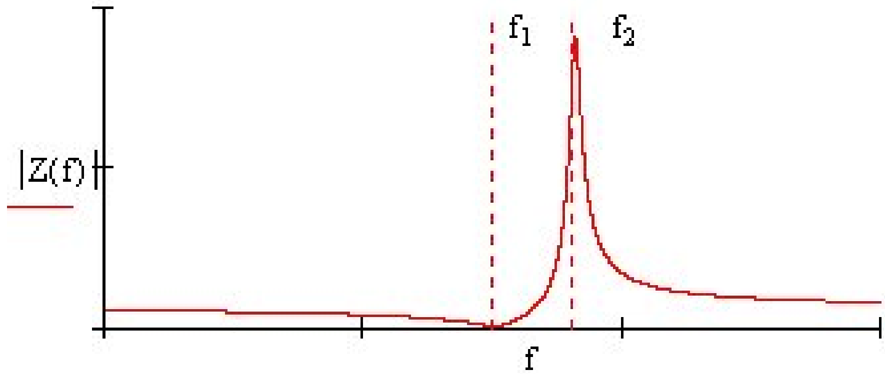


Figure 12: Impedance magnitude near the two resonant frequencies

We have two resonant frequencies, namely ω_1 (the series resonance) and ω_2 (the parallel resonance). At ω_1 the series LCR circuit is in resonance and its impedance is R, which is small compared to the impedance of C_1. (Look at Table 1.) At ω_2, we have parallel resonance—both branches have high impedance and so $|Z(j\omega_2)|$ is high. A typical plot of the magnitude of the equivalent impedance for a quartz crystal is shown in Figure 12. See also Figure 12.15 in [Sedra/Smith]. Note from the equations defining the two resonant frequencies that $\omega_2 > \omega_1$, but usually $C_1 \gg C$, and so the parallel resonant frequency is only slightly greater than the series resonant frequency; look at Table 1 again. Note that the damping ratio is very small, or in other words the Q factor of the parallel resonance is very high; this is reflected in the very narrow peak at f_2 in Figure 12.

The high Q of the parallel resonance peak means that the parallel resonant frequency of the crystal is very stable. We take advantage of this by using the crystal in the feedback section of an oscillator circuit. For example, a crystal can replace the inductor in a Colpitts oscillator; an example of this kind of crystal oscillator is shown in Figure 12.16 in [Sedra/Smith]. As another example, consider our general oscillator configuration, Figure 6, with the op amp replaced by a FET (which also has a high input impedance). We can use a crystal for Z_1, a tuned LC tank for Z_2, and the capacitance, C_{dg} between drain and gate for Z_3.

We conclude with two remarks. (1) The oscillation frequency of a crystal is very stable, but remember that it is also *fixed*—you have to change the crystal to change the frequency. (2) The oscillation in a crystal is due to mechanical vibrations, which can be longitudinal, flexural, or shear. As with all mechanical vibrations, there is a fundamental frequency, and its harmonics; the word "overtones" is preferred instead of harmonics because the overtone frequencies are usually not exact integer multiples of the fundamental. Hence, we can have oscillations at overtone frequencies. The Q at the overtones can be as high as it is at the fundamental, but the magnitude of the piezoelectric effect gets progressively smaller at the overtones.

5 Frequency Stability

As we have said, frequency stability is of prime importance in oscillator design. We are not using "stability" in the control theory sense of location of poles of a transfer function. Rather, a better term would be "frequency sensitivity" to changes in circuit parameters.

The study of frequency stability can get quite complicated, and each oscillator presents its own problems, but we can say something useful by considering a very simple example. The (feedback) oscillators that we considered involved an active amplifier with a passive resonant feedback network; the frequency of oscillation ω_0 is determined by the feedback network. Suppose that the phase changes by $\Delta\phi$; then the frequency of oscillation must change by a $\Delta\omega$ to cause a phase shift of $-\Delta\phi$ to maintain zero phase shift around the closed loop. (Remember that the phase of $L(s)$ must be zero.) Thus the greater the magnitude of the phase change $\Delta\phi$ for a change $\Delta\omega$ from ω_0, the greater the frequency stability. (I.e., a smaller $\Delta\omega$ will be required to bring the loop back to zero phase.) Hence, we define the frequency stability as

$$S_F = \frac{\Delta\phi}{\Delta\omega/\omega_0} \to \omega_0 \frac{d\phi}{d\omega}\bigg|_{\omega=\omega_0}.$$

For a simple example, consider a parallel RLC circuit. The impedance is

$$Z(s) = \frac{\frac{1}{C}s}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}$$

where $\omega_n = 1/\sqrt{LC}$ and $Q = R\sqrt{C/L}$ as usual. Then

$$Z(j\omega) = \frac{\frac{1}{C}j\omega}{\omega_n^2 - \omega^2 + j\frac{\omega_n}{Q}\omega}.$$

The phase angle of the impedance as a function of frequency is

$$\phi(\omega) = \arg Z(j\omega) = \frac{\pi}{2} - \arctan \frac{\frac{\omega_n}{Q}\omega}{\omega_n^2 - \omega^2}.$$

The derivative of the phase is

$$\frac{d\phi}{d\omega}(\omega) = \frac{-Q\omega_n(\omega_n^2 + \omega^2)}{(\omega_n\omega)^2 + Q^2(\omega_n^2 - \omega^2)}.$$

At the resonant frequency, this becomes

$$\frac{d\phi}{d\omega}(\omega_n) = -\frac{2Q}{\omega_n}.$$

Hence, the frequency stability is

$$S_F = \omega_n \frac{d\phi}{d\omega}(\omega_n) = -2Q.$$

(The negative sign merely means that $\Delta\phi < 0$ for $\Delta\omega > 0$.)

This result should not be surprising—it simply says that the higher the Q of the resonant circuit, the higher the frequency stability of the oscillator. Although the details differ for each oscillator, the general conclusion is the same. This is why we want the resonators in oscillator circuits to have a high Q. (Another reason is that a high Q circuit will do a better job of filtering out harmonics and noise.)

6 Variable Frequency Oscillators

As you know from the lecture course, it is often necessary to have a variable frequency oscillator. For example, in the superheterodyne receiver, the local oscillator must tune over an appropriate range so that the mixer will shift the incoming RF signal down to the intermediate frequency. In this section we shall only comment on some of the ways of obtaining a VFO; you are left to pursue the references for details.

There are several ways of varying the frequency of an oscillator; which to use depends on the application. One obvious way is to simply use a variable capacitor or inductor in the resonant circuit, and to manually adjust it. This is in fact how all AM and FM radios used to work—when you turned the tuning knob you were actually turning the adjustment on a variable capacitor and thereby adjusting the frequency of the local oscillator.

At microwave frequencies, the mechanically tunable elements are YIG[5] elements, dielectric resonators, and waveguide cavities.

In many applications, such as direct FM or in phase locked loops, we need the tuning of the oscillator to be automatic. One way to achieve this is with a *voltage-controlled oscillator* (VCO). A device that can be used in a VCO is the *varactor diode.* Any diode is a PN junction, and so has a junction capacitance. A varactor diode is designed so that the junction capacitance can be controlled by the reverse bias voltage across the junction:

$$C(V) = C_0 \left(1 - \frac{V}{V_d}\right)^{-\frac{1}{2}},$$

where V is the reverse bias, C_0 is a constant, and V_d is the diffusion barrier voltage of the junction.

Another technique that is becoming more and more common is *direct digital synthesis.* The basic idea is to store samples of the desired waveform (such as a sinusoid) in a microprocessor memory, produce the PCM data for these samples, and use a D/A converter to produce the analog waveform. Most of your radio and TV sets now use this technique, and the tuning is done by pushing a button. Many arbitrary function generators used in labs use this technique to produce a variety of waveforms, as well as allowing the user to enter his own data (samples) and letting the instrument produce the analog waveform.

References

[Clarke/Hess] Kenneth K. Clarke and Donald T. Hess, *Communication Circuits: Analysis and Design*, Addison-Wesley (1971) (Reprinted by Krieger Publishing Co., 1994)

[Collin] Robert E. Collin, *Foundations for Microwave Engineering*, 2nd ed., McGraw-Hill (1992)

[Couch] Leon W. Couch, II, *Digital and Analog Communication Systems*, 6th ed., Prentice-Hall (2001)

[5] YIG stands for yttrium-iron-garnet, a magnetic crystal material with frequency of oscillation proportional to an applied bias magnetic field.

[Millman] Jacob Millman, *Microelectronics: Digital and Analog Circuits and Systems*, McGraw-Hill (1979)

[Rohde/Whitaker/Bucher] Ulrich L. Rohde, Jerry C. Whitaker, & T.T.N. Bucher, *Communications Receivers: Priciples and Design*, 2nd ed., McGraw-Hill (1997)

[Sedra/Smith] Adel S. Sedra and Kenneth C. Smith, *Microelectronic Circuits*, 4th ed., Oxford (1998)

[Smith] Jack R. Smith, *Modern Communication Circuits*, 2nd ed., McGraw-Hill (1998)

[Terman] Frederick Emmons Terman, *Electronic and Radio Engineering*, McGraw-Hill (1955)

APPENDIX D AMPLITUDE MODULATORS, MIXERS, AND FREQUENCY CONVERSION

1 Introduction

As we know from the communications course, amplitude modulation consists essentially of frequency translation—a lowpass message spectrum is shifted up to a high carrier frequency. The frequency translation is accomplished by multiplying the message signal by a sinusoid at the carrier frequency.

This frequency conversion operation is not limited to AM—there are many times when we wish to shift a bandpass spectrum, regardless of its origin, to another frequency. For example, in the superheterodyne receiver the incoming modulated signal at carrier f_c is shifted to the intermediate carrier frequency f_I and then demodulated.[1]

In principle, the idea of frequency conversion is very simple. It is based on the Fourier transform property

$$x(t)\cos 2\pi f_0 t \longleftrightarrow \frac{1}{2}X(f+f_0)+\frac{1}{2}X(f-f_0).$$

Hence, if $x(t)$ is a lowpass (or baseband) signal, then $v(t)=x(t)\cos 2\pi f_c t$ is a bandpass signal at f_c; see Figure 1. (This is just double sideband supressed carrier modulation.) If $x(t)$ is a bandpass signal at f_c, then $w(t)=x(t)\cos 2\pi f_0 t$ contains bandpass spectra at $f_c \pm f_0$. We then obtain the desired bandpass signal $v(t)$ by passing $w(t)$ through a bandpass filter. If the BPF is at f_c+f_0, we have an "up converter", and if the filter is at f_c-f_0, we have a "down converter". Figure 2 illustrates an up converter.

[1] You will learn about the operation of the superheterodyne receiver in the communications class.

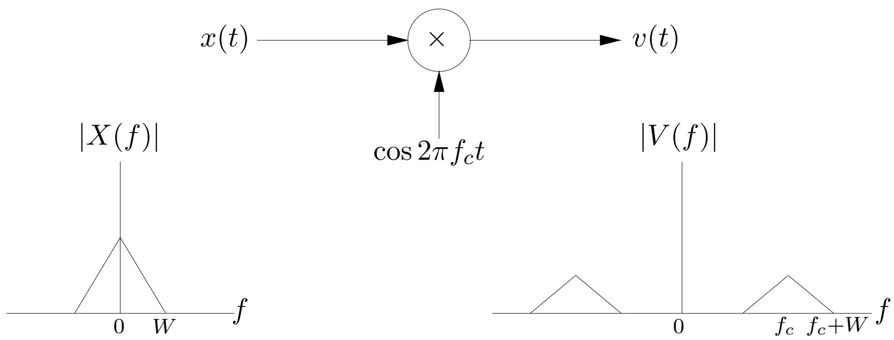


Figure 1: Lowpass-to-bandpass conversion—DSB modulation

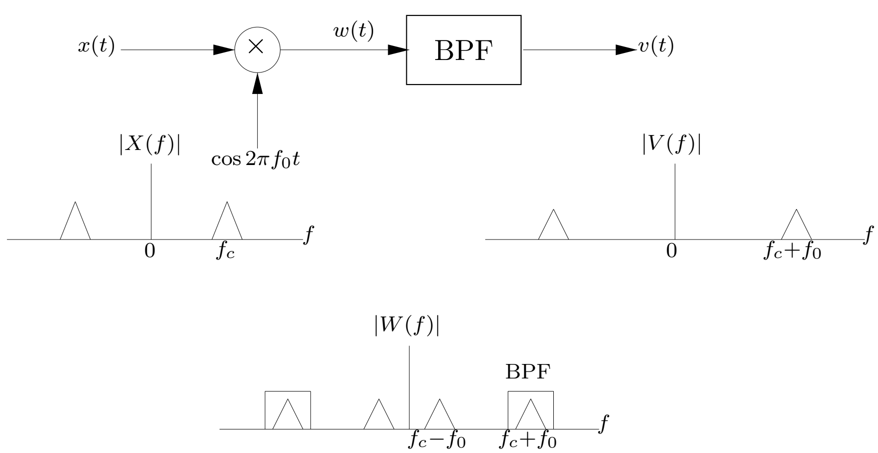


Figure 2: Up converter—bandpass-to-bandpass conversion

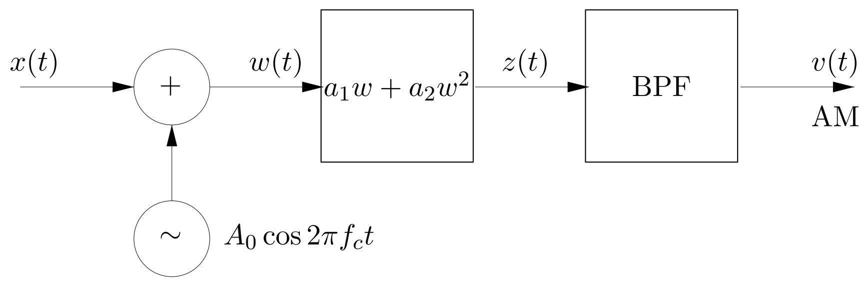


Figure 3: Square law AM modulator

Note that in this communications application, we do not multiply two arbitrary signals—we multiply a signal by a *sinusoid*. That is, the multiplier blocks in Figures 1 and 2 are not general multipliers. In communications, a device that multiplies by a sinusoid is called a *mixer*, and the whole system, consisting of mixer and filter (if the filter is needed), is the up/down converter.

2 Amplitude Modulators

2.1 Double sideband AM with carrier

Let us begin with the simpler case of amplitude modulation, or up conversion of a baseband signal. There are several ways to realize the mixer (multiplier) electronically, but the most common is with a nonlinear device that has a square law characteristic. Consider the system shown in Figure 3. The signal $x(t)$ is a baseband message signal having absolute bandwidth W (as in Figure 1). The *local oscillator* produces the carrier. Then the sum of the message and carrier is the input to the nonlinear device. The output is

$$\begin{aligned}
z(t) &= a_1\Big(x(t) + A_0 \cos 2\pi f_c t\Big) + a_2\Big(x(t) + A_0 \cos 2\pi f_c t\Big)^2 \\
&= a_1\Big(x(t) + A_0 \cos 2\pi f_c t\Big) \\
&\quad + a_2\Big(x^2(t) + 2x(t)A_0 \cos 2\pi f_c t + A_0^2 \cos^2 2\pi f_c t\Big) \\
&= \underbrace{\left(a_2 x^2(t) + a_1 x(t) + \frac{a_2 A_0^2}{2}\right)}_{\text{lowpass}} + \underbrace{A_0\Big(a_1 + 2a_2 x(t)\Big)\cos 2\pi f_c t}_{\text{AM at } f_c} \\
&\quad + \underbrace{\frac{a_2 A_0^2}{2}\cos 4\pi f_c t}_{\text{line at } 2f_c}\,.
\end{aligned} \tag{1}$$

The spectrum $Z(f)$ of $z(t)$ given by Eq. (1) consists of three parts, as indicated:

- The first term has spectrum

$$a_2(X \star X)(f) + a_1 X(f) + \frac{a_2 A_0^2}{2}\delta(f),$$

  which is a lowpass signal with bandwidth $2W$.

- The second term,

$$A_0 a_1\left(1 + \frac{2a_2}{a_1}x(t)\right)\cos 2\pi f_c t,$$

  is an AM signal at carrier frequency f_c with modulation index $\mu = (2a_2)/a_1$.

- The third term is a line at frequency $2f_c$.

Hence, if $z(t)$ is passed through a BPF centered at f_c and having bandwidth $2W$, the output will be the AM signal given by the second term in Eq. (1).

Figure 3 is the basic AM modulator circuit. It is called an *unbalanced* modulator or mixer, or a *single-ended* modulator. In Section 4 we shall see what kind of electronic devices can be used for the square law device, but first let us continue investigating frequency conversion systems.

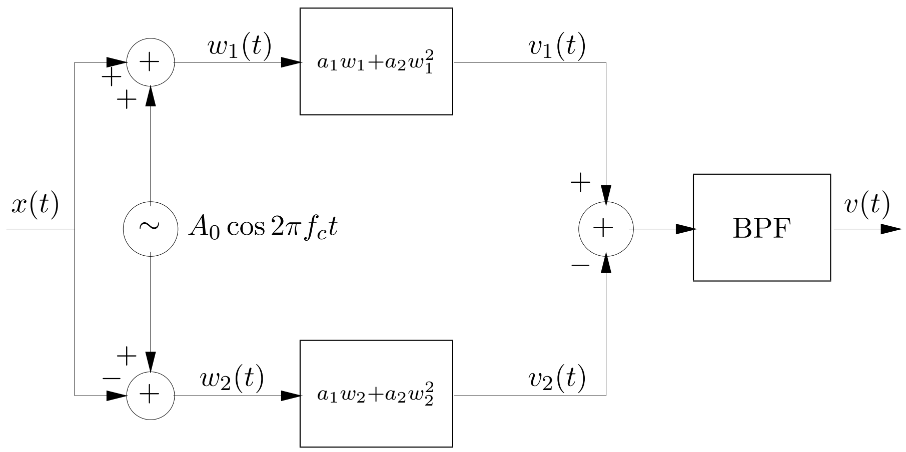


Figure 4: Balanced modulator for DSB

2.2 Double sideband suppressed carrier AM

To *suppress* the carrier line and thereby generate DSB modulation, we can use two identical square law devices in a *balanced* configuration—we generate two AM signals, subtract them, and the carrier line is suppressed. The block diagram for the *balanced modulator* (also called a *singly* balanced modulator) is shown in Figure 4. We have

$$\begin{aligned} v_1(t) &= a_1\big(x(t) + A_0\cos 2\pi f_c t\big) + a_2\big(x(t) + A_0\cos 2\pi f_c t\big)^2 \\ &= a_1 x(t) + a_1 A_0 \cos 2\pi f_c t + a_2 x^2(t) + \frac{a_2 A_0^2}{2} + \frac{a_2 A_0^2}{2}\cos 4\pi f_c t \\ &\quad + 2a_2 A_0\, x(t)\cos 2\pi f_c t \end{aligned}$$

and

$$\begin{aligned} v_2(t) &= a_1\big(-x(t) + A_0\cos 2\pi f_c t\big) + a_2\big(-x(t) + A_0\cos 2\pi f_c t\big)^2 \\ &= -a_1 x(t) + a_1 A_0 \cos 2\pi f_c t + a_2 x^2(t) + \frac{a_2 A_0^2}{2} + \frac{a_2 A_0^2}{2}\cos 4\pi f_c t \\ &\quad - 2a_2 A_0\, x(t)\cos 2\pi f_c t. \end{aligned}$$

The input to the BPF then is

$$v_1(t) - v_2(t) = 2a_1 x(t) + 4a_2 A_0\, x(t)\cos 2\pi f_c t.$$

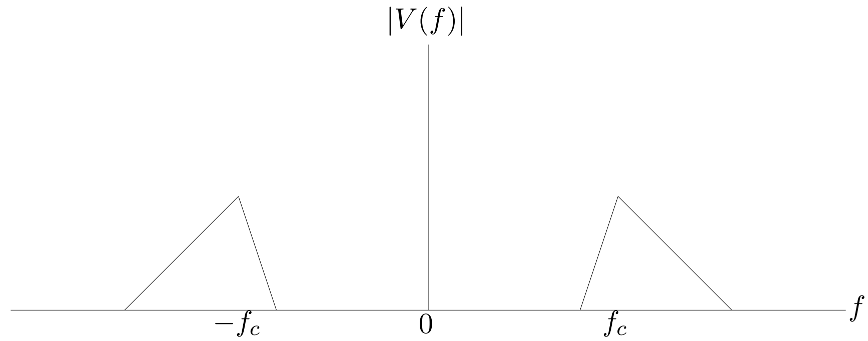


Figure 5: A bandpass spectrum at f_c to be shifted to f_N

Therefore, with a BPF centered at f_c and having bandwidth $2W$, the output $v(t)$ is a DSB supressed carrier signal:

$$v(t) = 4a_2 A_0 \, x(t) \cos 2\pi f_c t.$$

3 Frequency Conversion

The same basic systems that we considered in Section 2, the unbalanced mixer and the singly balanced mixer, can be used to move a bandpass spectrum from one carrier frequency to another, but we have to be careful about the details of the analysis.[2]

Suppose that we have a bandpass signal $v(t)$ at some carrier frequency f_c, and we wish to move this spectrum to a new carrier $f_c + f_0$ (up conversion) or $f_c - f_0$ (down conversion). We assume that $v(t)$ is a real signal so that $|V(f)|$ is an even function of f and $\arg V(f)$ is an odd function, but $v(t)$ can be any type of modulated signal and so the spectrum need not be symmetric about f_c; see Figure 5.

3.1 Conversion with an unbalanced mixer

Consider the unbalanced square law mixer of Figure 3 with the following modifications: the input is the bandpass $v(t)$ at f_c (Figure 5) rather than

[2]The conversion of a bandpass spectrum to 0 frequency is rarely encountered—that is after all the function of the demodulator in the receiver. Therefore we shall not discuss bandpass-to-lowpass conversion.

a lowpass signal, and the local oscillator produces $A_0 \cos 2\pi f_0 t$, where we shall assume that $f_0 < f_c$ for now. The output of the square-law device is:

$$\begin{aligned} z(t) &= a_1\Big(v(t) + A_0 \cos 2\pi f_0 t\Big) + a_2\Big(v(t) + A_0 \cos 2\pi f_0 t\Big)^2 \\ &= \underbrace{a_1 v(t)}_{\text{I}} + \underbrace{a_1 A_0 \cos 2\pi f_0 t}_{\text{II}} + \underbrace{a_2 v^2(t)}_{\text{III}} + \underbrace{2a_2 A_0 v(t) \cos 2\pi f_0 t}_{\text{IV}} \\ &\quad + \underbrace{a_2 A_0^2 \cos^2 2\pi f_0 t}_{\text{V}}. \end{aligned} \tag{2}$$

If you sketch a picture of the spectrum of the signal in Eq. (2), you will find the following frequency components:

Term I This is just the input bandpass signal at f_c.

Term II This is of course a line at f_0.

Term III We need to do a little work to see what the spectrum of $v^2(t)$ is. Since $v(t)$ is a bandpass signal, it has a quadrature-carrier description:

$$v(t) = v_i(t) \cos 2\pi f_c t - v_q(t) \sin 2\pi f_c t,$$

where $v_i(t)$ and $v_q(t)$ are lowpass signals. Then

$$\begin{aligned} v^2(t) &= v_i^2(t) \cos^2 2\pi f_c t - 2v_i(t)v_q(t) \cos 2\pi f_c t \cdot \sin 2\pi f_c t + v_q^2(t) \sin^2 2\pi f_c t \\ &= \frac{1}{2}v_i^2(t) + \frac{1}{2}v_q^2(t) + \frac{1}{2}v_i^2(t) \cos 4\pi f_c t - \frac{1}{2}v_q^2(t) \cos 4\pi f_c t \\ &\quad + v_i(t)v_q(t) \sin 4\pi f_c t. \end{aligned}$$

The first two terms are lowpass signals and the last three terms are bandpass signals all at $2f_c$.

Term IV This is our desired signal—the bandpass $v(t)$ shifted up to $f_c + f_0$ and down to $f_c - f_0$.

Term V This consists of a line at 0 and one at $2f_0$.

Hence with the proper choice of f_c and f_0, the up and down conversion parts of the spectrum (Term IV) are isolated and we can select either one with a BPF at that frequency. That is, if the BPF in Figure 3 is at $f_c + f_0$, the system is an up converter, and with the filter at $f_c - f_0$, it is a down converter.

Remark. We assumed in this analysis that $f_0 < f_c$ so that the down conversion frequency is positive. It is left for you to show that if $f_0 > f_c$, the down conversion part of the spectrum has the upper and lower sidebands *reversed.* In normal applications, for down conversion we want $f_0 < f_c$. For up conversion, the down conversion spectrum is irrelevant anyway.

3.2 Conversion with a singly balanced mixer

The unbalanced mixer will in principle work as a bandpass-to-bandpass frequency converter, but as we saw (Eq. (2)), the spectrum is rather crowded. In particular, the unbalanced mixer has *input feedthrough* (i.e., the input $v(t)$ appears at the output), and there are lines at f_0 (this could be close to $f_c - f_0$) and at $2f_0$ (this could be close to $f_c + f_0$), and so heavy filtering may be required to block these lines.

A singly balanced mixer has the advantage that the output eliminates all of the output spectral components except the input feedthrough and the desired up and down conversion terms. Consider the singly balanced mixer of Figure 4, but again with the modifications that the input is a bandpass signal at f_c and the local oscillator produces f_0. The input $z(t)$ to the BPF is

$$z(t) = v_1(t) - v_2(t) = 2a_1 v(t) + 4a_2 v(t)\cos 2\pi f_0 t. \tag{3}$$

Compare this with Eq. (2)—now the only undesired term is the input feedthrough term.

Remarks. (1) The balanced mixer *in principle* eliminates the other spectral components. There is of course no such thing as perfectly matched square law devices. That is, one device will have coefficients a_1 and a_2, and the other will have a_1' and a_2'—the coefficients will be close, but not identically equal. The result is that there will be small unwanted components in the output.

(2) There is also such a thing as a *doubly balanced* mixer. This mixer eliminates the input feedthrough term as well as the local oscillator and its harmonics. You will *simulate*, but not build, one kind of doubly balanced mixer in lab, as a DSB modulator.

4 Square Law Devices

We now come to the question of how to realize a square law nonlinearity. Several devices can be used, but the most common ones are diodes and FETs.

4.1 Diode mixers

A p-n junction diode is modeled by the i-v equation

$$i(v) = I_s(e^{v/nV_T} - 1),$$

where I_s is the saturation current, $V_T = kT/q$ is the thermal voltage (at room temperature, $V_T \approx 25.2\,\mathrm{mV}$), and $1 \le n \le 2$, depending on the physical construction of the diode. For example, the PSpice model of the familiar 1N4148 signal diode uses $n = 2$ and $I_s = 2.682\,\mathrm{nA}$. If we expand the function $i(v)$ in a Taylor series about any $v = v_0$, we have

$$\begin{aligned} i(v) &= i(v_0) + i'(v_0)(v - v_0) + \frac{1}{2!}i''(v_0)(v - v_0)^2 + \frac{1}{3!}i'''(v_0)(v - v_0)^3 + \cdots \\ &= I_s e^{v_0/nV_T}\left(\left(1 - e^{-v_0/nV_T}\right) + \frac{1}{nV_T}(v - v_0) + \frac{1}{2!(nV_T)^2}(v - v_0)^2 \right. \\ &\qquad \left. + \frac{1}{3!(nV_T)^3}(v - v_0)^3 + \cdots\right) \end{aligned}$$

Thus, for v near v_0, or $|v - v_0| \ll 1$, we have

$$i(v) \approx I_s e^{v_0/nV_T}\left(\left(1 - e^{-v_0/nV_T}\right) + \frac{1}{nV_T}(v - v_0) + \frac{1}{2!(nV_T)^2}(v - v_0)^2\right). \tag{4}$$

In fact, since

$$\frac{2!(nV_T)^2}{3!(nV_T)^3} = \frac{1}{3nV_T} \le \frac{1}{6V_T} \approx 7,$$

we can say that Eq. (4) holds for $|v - v_0| \ll 7$.

In particular, near $v_0 = 0$, we have

$$i(v) \approx I_s\left(\frac{1}{nV_T}v + \frac{1}{2!(nV_T)^2}v^2\right). \tag{5}$$

That is, the diode acts as a square law device.

4.2 FET mixers

A JFET (junction FET) has the following i_D-v_{GS} characteristic:

$$i_D = \begin{cases} 0 & \text{if } v_{GS} < V_t \\ \beta\left[2(v_{GS} - V_t)v_{DS} - v_{DS}^2\right](1 + \lambda v_{DS}) & \text{if } v_{GS} > V_t \text{ and } v_{DS} \le v_{GS} - V_t \\ \beta(v_{GS} - V_t)^2(1 + \lambda v_{DS}) & \text{if } v_{GS} > V_t \text{ and } v_{DS} \ge v_{GS} - V_t \end{cases}$$

The first case is the cutoff region, the second is the triode region, and the third is the pinch-off or saturation region. (See [Sedra/Smith] for more details.) In these equations, $\beta = I_{DSS}/V_t^2$ is the transconduction coefficient, $I_{DSS} = i_D\Big|_{v_{GS}=0} = \beta V_t^2$, V_t is the pinch-off voltage, and $\lambda = 1/V_A$ is the channel-length modulation, and V_A is the Early voltage.[3]

Assuming that $|V_A| \gg 1$ (the Spice default is $V_A = -\infty$) so that $\lambda \approx 0$, we see that in saturation the FET is a square-law device:

$$\begin{aligned} i_D(v_{GS}) &\approx \beta(v_{GS} - V_t)^2 \\ &= \beta v_{GS}^2 - 2\beta V_t v_{GS} + I_{DSS}. \end{aligned} \tag{6}$$

In real devices we usually cannot say $\lambda \approx 0$; we must modify Eq. (6) slightly to account for the term $1 + \lambda v_{DS}$, and this correction term depends on the bias point (remember that we are operating in saturation). But the FET still is a square-law device.

The FET mixer is popular in a balanced configuration because very closely matched JFET's are commercially available. (The JFET's are built on a single substrate.)

References

[Clarke/Hess] Kenneth K. Clarke and Donald T. Hess, *Communication Circuits: Analysis and Design*, Addison-Wesley (1971) (Reprinted by Krieger Publishing Co., 1994)

[Collin] Robert E. Collin, *Foundations for Microwave Engineering*, 2nd ed., McGraw-Hill (1992)

[Couch] Leon W. Couch, II, *Digital and Analog Communication Systems*, 6th ed., Prentice-Hall (2001)

[Rohde/Whitaker/Bucher] Ulrich L. Rohde, Jerry C. Whitaker, & T.T.N. Bucher, *Communications Receivers: Priciples and Design*, 2nd ed., McGraw-Hill (1997)

[Sedra/Smith] Adel S. Sedra and Kenneth C. Smith, *Microelectronic Circuits*, 4th ed., Oxford (1998)

[3] I am using the standard Spice notation and terminology for these quantitites.

[Smith] Jack R. Smith, *Modern Communication Circuits*, 2$^{\mathrm{nd}}$ ed., McGraw-Hill (1998)

APPENDIX E
THE PHASE-LOCKED LOOP

1 Introduction

A *phase-locked loop* (PLL) is a feedback control system used to automatically adjust the phase of a locally generated signal to the phase of an incoming signal. The PLL is widely used for carrier synchronization in coherent demodulation of AM and PM signals, both digital and analog. The PLL is also widely used in FM demodulation—we can use it to recover the phase $\theta(t)$ in an angle modulated signal. It is probably accurate to say that almost all synchronous receivers built today—analog or digital, AM or FM—use the phase-lock principle.

It turns out that the PLL has one other important feature. The feedback structure of the loop results in improved performance in noise over slope detection of FM. Unfortunately, we shall be unable to pursue this. We shall concentrate on how the PLL recovers $\theta(t)$.

Historical note: a large part of PLL theory was worked out during the 1960's and 1970's, and it is still an active topic of research, but it was only recently that easy and inexpensive implementations became available.

2 The Basic Loop

Suppose that the incoming signal v_i is a narrowband signal with constant envelope (i.e., an angle modulated wave).

$$v_i(t) = A_i \cos\bigl(2\pi f_c t + \theta_i(t)\bigr),$$

where $\theta_i(t)$ is slowly varying with respect to f_c:

$$\frac{1}{2\pi}\left|\frac{d\theta_i}{dt}\right| \ll f_c.$$

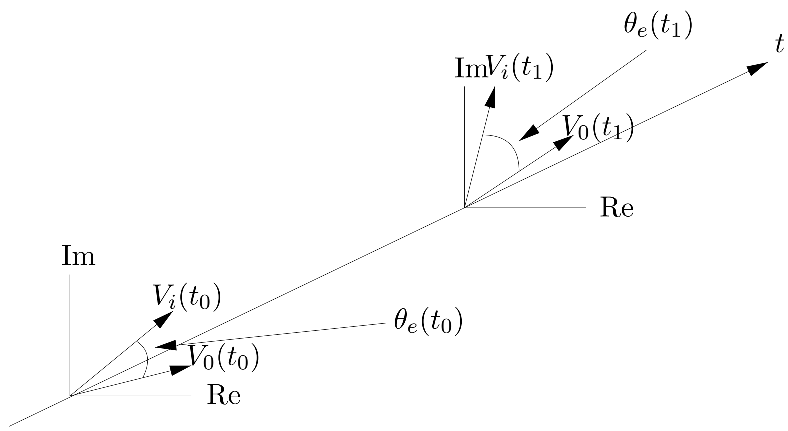


Figure 1: Phasor diagram

Suppose that we locally generate another signal v_0:

$$v_0(t) = A_0 \cos\big(2\pi f_c t + \theta_0(t)\big).$$

We want to adjust the local phase, $\Phi_0(t) = 2\pi f_c t + \theta_0(t)$, to that of the input, $\Phi_i(t) = 2\pi f_c t + \theta_i(t)$.

Note: we have assumed that v_i and v_0 have the same nominal carrier frequency, f_c. This entails no loss of generality because any difference in instantaneous frequency can be included in $\theta_0(t)$.

The situation is pictured in Figure 1. The phasor $V_0(t)$ makes an angle $\Phi_0(t)$ with the positive real axis, and $V_i(t)$ makes an angle $\Phi_i(t)$ with the axis. The two phasors rotate with instantaneous frequencies

$$\begin{aligned}\frac{1}{2\pi}\frac{d\Phi_i}{dt} &= f_c + \frac{1}{2\pi}\frac{d\theta_i}{dt},\\ \frac{1}{2\pi}\frac{d\Phi_0}{dt} &= f_c + \frac{1}{2\pi}\frac{d\theta_0}{dt}.\end{aligned}$$

Ideally the two phasors should coincide at every time t; the misalignment is described by the phase error

$$\begin{aligned}\theta_e(t) &= \Phi_i(t) - \Phi_0(t)\\ &= \theta_i(t) - \theta_0(t).\end{aligned}$$

We can adjust $\theta_e(t)$ to 0 by an automatic control system if we can generate a control signal as a function of $\theta_e(t)$. One way to do this is to multiply the two signals:

$$\begin{aligned} v_i(t)v_0(t) &= A_iA_0\cos\bigl(2\pi f_ct+\theta_i(t)\bigr)\cos\bigl(2\pi f_ct+\theta_0(t)\bigr) \\ &= \frac{A_iA_0}{2}\cos\bigl(\theta_i(t)-\theta_0(t)\bigr)+\frac{A_iA_0}{2}\cos\bigl(4\pi f_ct+\theta_i(t)+\theta_0(t)\bigr). \end{aligned}$$

The first term is what we want—it provides a measure of the phase difference. Since θ_i and θ_0 are slowly varying with respect to f_c, the second term is a narrowband signal at $2f_c$ which can be removed by a low-pass filter. There is, however, one difficulty. Because $\cos(\,\cdot\,)$ is an even function we cannot tell from $\cos(\theta_e(t))$ whether $\theta_i(t)$ is larger than $\theta_0(t)$ or the other way round. We need an error function which is an *odd* function of $\theta_e(t)$. This is easily obtained by advancing the locally generated signal by 90°. That is, the locally generated signal should be

$$v_0(t) = A_0\cos\bigl(2\pi f_ct+\theta_0(t)-\pi/2\bigr) = A_0\sin\bigl(2\pi f_ct+\theta_0(t)\bigr).$$

Then we have

$$v_i(t)v_0(t) = \frac{A_iA_0}{2}\sin\bigl(\theta_i(t)-\theta_0(t)\bigr)+\frac{A_iA_0}{2}\sin\bigl(4\pi f_ct+\theta_i(t)+\theta_0(t)\bigr).$$

Again the second term is eliminated by a low-pass filter and we are left with our desired error signal

$$\frac{A_iA_0}{2}\sin\theta_e(t) = \frac{A_iA_0}{2}\sin\bigl(\theta_i(t)-\theta_0(t)\bigr).$$

If $\theta_i(t)-\theta_0(t)\neq 0$ then an error signal with the same *sign* as the phase error is produced. Suppose that this error signal is filtered and applied to a device that produces a sinusoidal output whose instantaneous frequency varies according to the voltage applied to it. Such a device is called a *voltage controlled oscillator* (VCO). When the control voltage is 0 the VCO runs at its quiescent frequency f_c. A positive [negative] control voltage causes the VCO to increase [decrease] its instantaneous frequncy, thus forcing the control voltage to decrease [increase].

The block diagram of the system we have described is in Figure 2. This is the basic PLL. The input is

$$v_i(t) = A_i\cos\bigl(2\pi f_ct+\theta_i(t)\bigr).$$

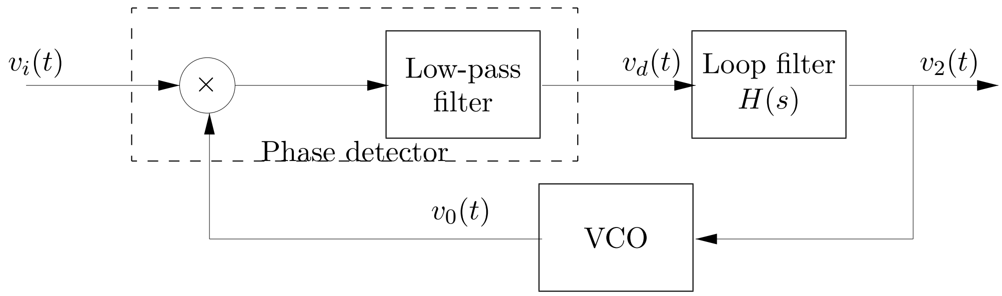


Figure 2: The basic phase-locked loop

The locally generated reference is the VCO output

$$v_0(t) = A_0 \cos\big(2\pi f_c t + \theta_0(t) - \pi/2\big).$$

The output of the multiplier and LPF is the error signal

$$v_d(t) = \frac{A_0 A_i}{2} K_m \sin\big(\theta_i(t) - \theta_0(t)\big) = \frac{A_0 A_i}{2} K_m \sin\theta_e(t).$$

The combination of multiplier and LPF is a product phase detector, and K_m is its gain. Phase detectors with non-sinusoidal characteristics are also available, but all are odd functions of θ_e; see Figure 4-20 in [Couch]. Define

$$K_d = \frac{A_0 A_i K_m}{2}$$

so that we may write

$$v_d(t) = K_d \sin\big(\theta_i(t) - \theta_0(t)\big).$$

The loop filter is a linear system with transfer function $H(s)$ and impulse response $h(t)$; we shall come back to it later. The output of the PLL, $v_2(t)$, is fed back into the VCO. As we have said, the VCO produces the reference

$$v_0(t) = A_0 \cos\big(2\pi f_c t + \theta_0(t) - \pi/2\big)$$

whose instantaneous frequency varies according to $v_2(t)$:

$$\frac{1}{2\pi}\frac{d\Phi_0}{dt} = f_c + \frac{1}{2\pi}\frac{d\theta_0}{dt} = f_c + K_v v_2(t),$$

where K_v is a constant, representing the VCO gain in units of Hz/V.

3 An Equivalent Model

In the analysis of the PLL we are not interested in the signals $v_i(t)$, $v_0(t)$, and $v_2(t)$ as much as in the phases $\theta_i(t)$ and $\theta_0(t)$ and the phase error $\theta_e(t) = \theta_i(t) - \theta_0(t)$. Therefore we shall replace the block diagram of the basic loop (Figure 2) by a mathematically equivalent one which operates on the phases. We do this as follows. First,

$$v_d(t) = K_d \sin\bigl(\theta_i(t) - \theta_0(t)\bigr) = K_d \sin\theta_e(t) \,.$$

The output $v_2(t)$ is the convolution of $v_d(t)$ and $h(t)$:

$$v_2(t) = \int_0^t h(t-\tau)v_d(\tau)d\tau = K_d \int_0^t h(t-\tau)\sin\bigl(\theta_i(\tau) - \theta_0(\tau)\bigr)d\tau \,.$$

The VCO is defined by

$$\frac{d\theta_0}{dt} = 2\pi K_v v_2(t) = 2\pi K_v K_d \int_0^t h(t-\tau)\sin\bigl(\theta_i(\tau) - \theta_0(\tau)\bigr)d\tau \,.$$

Finally, $\theta_e(t) = \theta_i(t) - \theta_0(t)$ and so $\theta_0(t) = \theta_i(t) - \theta_e(t)$:

$$\frac{d}{dt}\bigl(\theta_i(t) - \theta_e(t)\bigr) = 2\pi K_v K_d \int_0^t h(t-\tau)\sin\theta_e(\tau)d\tau \,,$$

and so we have the dynamic equation for the phase error,

$$\frac{d\theta_e}{dt} = \frac{d\theta_i}{dt} - 2\pi K_v K_d \int_0^t h(t-\tau)\sin\theta_e(\tau)d\tau \,. \tag{1}$$

The control system diagrammed in Figure 3 obeys this dynamic equation. This control system is the equivalent model of the PLL that we were looking for. The phase detector (multiplier and LPF) of Figure 2 is replaced by a subtractor and sinusoidal nonlinearity, and the VCO is replaced by an integrator. The phases $\theta_i(t)$ and $\theta_0(t)$ appear explicitly in this model. Note that the model is independent of f_c; referring to the phasor diagram of Figure 1, this model describes the *relative* motion of the two phasors.

We have obtained a model describing the PLL in terms of θ_i and θ_0, but we have introduced a new problem—the sinusoidal nonlinearity makes an exact analysis very difficult. We shall therefore have to content ourselves with an approximate analysis of the equivalent model.

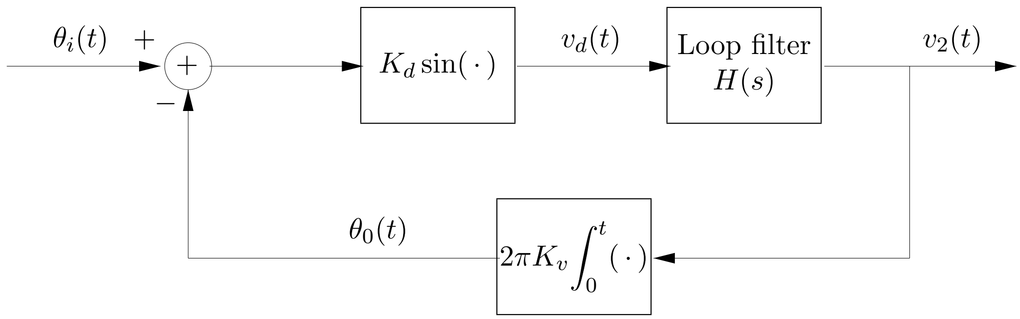


Figure 3: An equivalent model of the PLL

4 The Equivalent Linear Model

Let us assume that the phase error is small: $|\theta_e(t)| \ll 1\,\text{rad}$ for all t. Then $\sin\theta_e(t) \approx \theta_e(t)$, and we thus obtain an approximate *linear* model—the nonlinear box $K_d \sin(\,\cdot\,)$ in Figure 3 is replaced by a constant gain box K_d.

Let us now do our analysis in the Laplace transform domain:

$$\begin{aligned} V_d(s) &= \mathcal{L}[v_d](s) = \int_0^\infty v_d(t)e^{-st}dt\,, \\ V_2(s) &= \mathcal{L}[v_2](s)\,, \\ \Theta_i(s) &= \mathcal{L}[\theta_i](s)\,, \\ \Theta_0(s) &= \mathcal{L}[\theta_0](s)\,, \\ \Theta_e(s) &= \mathcal{L}[\theta_e](s)\,. \end{aligned}$$

Then the integration box $2\pi K_v \int_0^t(\,\cdot\,)$ of Figure 3 is replaced by the transfer function $(2\pi K_v)/s$. The PLL is therefore approximately modeled by the *linear* control system of Figure 4. We have

$$\begin{aligned} V_d(s) &= K_d\Theta_e(s) = K_d\big(\Theta_i(s) - \Theta_0(s)\big)\,, \\ \Theta_0(s) &= \frac{2\pi K_v}{s} V_2(s)\,, \\ V_2(s) &= H(s)V_d(s)\,. \end{aligned}$$

Let us first calculate the closed loop transfer function $G(s) = \Theta_0(s)/\Theta_i(s)$.

$$\Theta_0(s) = \frac{2\pi K_v}{s} V_2(s) = \frac{2\pi K_v}{s} H(s)V_d(s) = \frac{2\pi K_v K_d}{s} H(s)\big(\Theta_i(s) - \Theta_0(s)\big)\,.$$

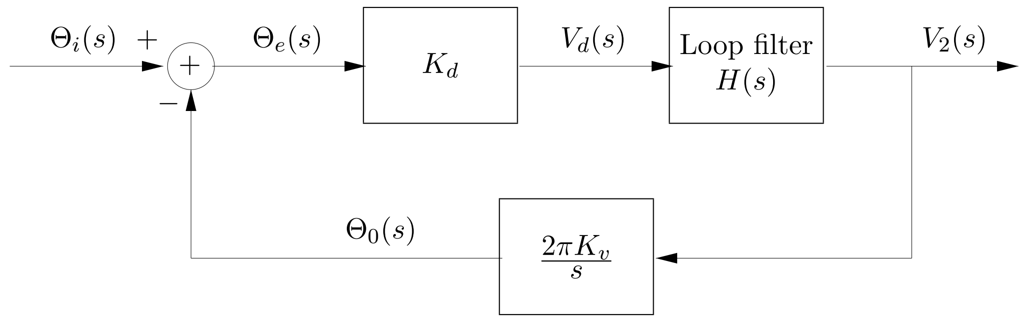


Figure 4: The approximate linear model of the PLL

Therefore,

$$\frac{2\pi K_v K_d}{s} H(s)\Theta_i(s) = \Theta_0(s) + \frac{2\pi K_d K_v}{s} H(s)\Theta_0(s) = \frac{s + 2\pi K_v K_d H(s)}{s}\Theta_0(s)\,.$$

Hence

$$\boxed{G(s) = \frac{\Theta_0(s)}{\Theta_i(s)} = \frac{2\pi K_v K_d H(s)}{s + 2\pi K_v K_d H(s)}} \tag{2}$$

We can also calculate the transfer function $\Theta_e(s)/\Theta_i(s)$ from input phase to phase error.

$$\Theta_e(s) = \Theta_i(s) - \Theta_0(s) = \Theta_i(s) - G(s)\Theta_i(s)\,,$$

and so

$$\boxed{\frac{\Theta_e(s)}{\Theta_i(s)} = 1 - G(s) = \frac{s}{s + 2\pi K_v K_d H(s)}} \tag{3}$$

Now we should like to have $\Theta_i(s) = \Theta_0(s)$ which implies $G(s) = 1$. But this implies that

$$2\pi K_v K_d H(s) = s + 2\pi K_v K_d H(s)\,,$$

and this in turn implies that $s = 0$. That is, it would appear that the PLL performs as we want it to only for *zero frequency*, and then $H(s)$ can be anything. This is, of course, unacceptable. It is true, however, that for many types of loop filter $H(s)$ we can show that $\Theta_0(s)$ is approximately $\Theta_i(s)$. We shall analyze the linear model for the two cases of $H(s)$ most commonly encountered in practice.

4.1 First order loop

The first order loop refers to the case $H(s) = 1$, which results in the closed-loop $G(s)$ being of first order.

$$G(s) = \frac{\Theta_0(s)}{\Theta_i(s)} = \frac{V_2(s)}{M(s)} = \frac{2\pi K_v K_d}{s + 2\pi K_v K_d} .$$

Or, in terms of frequency f (where $s = j2\pi f$),

$$G(f) = \frac{\Theta_0(f)}{\Theta_i(f)} = \frac{K_v K_d}{jf + K_v K_d} = \frac{1}{1 + jf/f_b} . \qquad (4)$$

where $f_b = K_v K_d$. Note that $G(f)$ is just the transfer function of an RC low-pass filter with 3dB bandwidth f_b. That is, the first order loop produces an output $\theta_0(t)$ which is essentially $\theta_i(t)$ passed through an RC low-pass filter. Hence if $f_b = K_v K_d$ is large enough (compared to the bandwidth of $\theta_i(t)$), we will have $\theta_0(t) \approx \theta_i(t)$. The parameter $f_b = K_v K_d$ is called the *loop gain* of the first order loop.

Example 1 Suppose that $\theta_i(t) = 2\pi K u(t)$. (That is, consider the step response of the first order linear model—if you have taken the controls course you know that the step response is a standard measure of control system performance.) Then

$$v_i(t) = A_i \cos\bigl(2\pi f_c t + 2\pi K u(t)\bigr) .$$

and $\Theta_i(s) = 2\pi K/s$. Then

$$\Theta_0(s) = G(s)\Theta_i(s) = \frac{2\pi K}{s} \cdot \frac{2\pi K_v K_d}{s + 2\pi K_v K_d} .$$

Hence

$$\theta_0(t) = 2\pi K\bigl[1 - e^{-t/\tau}\bigr]u(t) ,$$

where

$$\tau = \frac{1}{2\pi K_v K_d} = \frac{1}{2\pi f_b}$$

is the time constant. (Draw a sketch of $\theta_0(t)$.) Note that as the loop gain f_b is increased we have

$$\theta_0(t) \to 2\pi K u(t) = \theta_i(t) \quad \text{as} \quad f_b \to \infty.$$

Example 2 Let $\theta_i(t) = 2\pi K t u(t)$ and so $\Theta_i(s) = 2\pi K/s^2$. (That is, consider now the ramp response.) Therefore

$$\Theta_0(s) = G(s)\Theta_i(s) = \frac{2\pi K}{s^2} \cdot \frac{2\pi K_v K_d}{s + 2\pi K_v K_d}.$$

Hence

$$\theta_0(t) = 2\pi K(t + \tau e^{-t/\tau})u(t),$$

where $\tau = 1/(2\pi f_b)$ as before. Again we have $\theta_0(t) \to 2\pi K t u(t) = \theta_i(t)$ as $f_b \to \infty$.

4.2 A digression: validity of the linear model

The preceding analysis depends on the validity of the linear approximation $K_d \sin\theta_e(t) \approx K_d\theta_e(t)$. Before we analyze the second commonly encountered case of the PLL, let us investigate the validity of the linear model. We shall show that in the linear model the phase error does indeed tend to drive the loop into lock.

Consider the first order loop ($H(s) = 1$), but *without the assumption of linearity*. That is,

$$v_2(t) = K_d \sin\big(\theta_i(t) - \theta_0(t)\big)\,.$$

The frequency deviation of the VCO output is

$$\frac{d\theta_0}{dt} = 2\pi K_v v_2(t) = 2\pi K_v K_d \sin\big(\theta_i(t) - \theta_0(t)\big)\,.$$

Consider a very simple example: suppose that $\theta_i(t) = (2\pi K t)u(t)$. Then

$$\frac{d\theta_i}{dt} = 2\pi K u(t)\,.$$

Now $\theta_e(t) = \theta_i(t) - \theta_0(t)$, so

$$\frac{d\theta_0}{dt} = \frac{d\theta_i}{dt} - \frac{d\theta_e}{dt} = 2\pi K - \frac{d\theta_e}{dt} \quad \text{for } t \geq 0.$$

But also

$$\frac{d\theta_0}{dt} = 2\pi K_v K_d \sin\theta_e(t)\,,$$

and so, assuming the ramp $\theta_i(t)$, the phase error must satisfy the first-order differential equation

$$\frac{d\theta_e}{dt} + 2\pi K_v K_d \sin\theta_e(t) = 2\pi K\,, \quad t \geq 0\,. \tag{5}$$

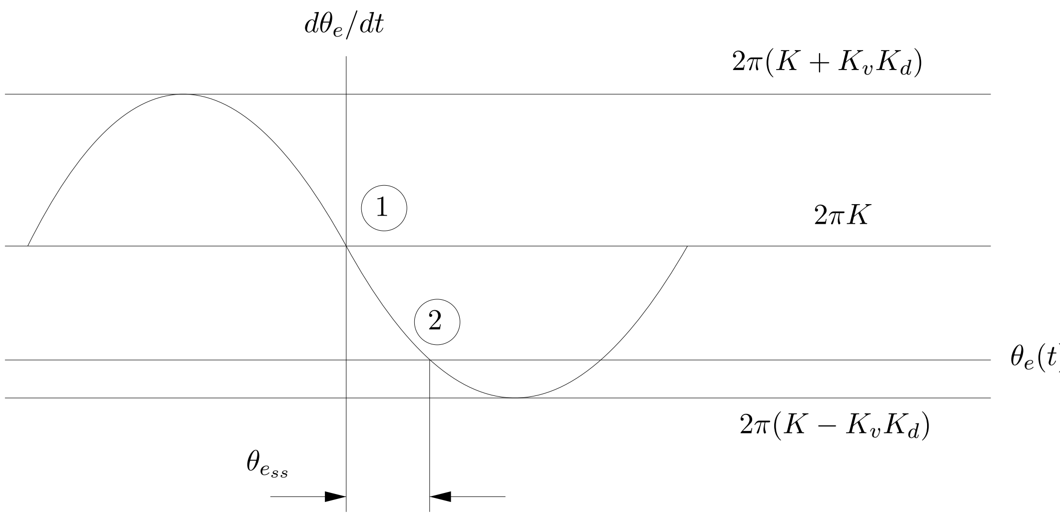


Figure 5: The phase-plane plot

A plot of $d\theta_e/dt$ vs. $\theta_e(t)$ is called the *phase-plane* plot, as shown in Figure 5. The phase error $\theta_e(t)$ and the frequency error $d\theta_e/dt$ must satisfy the differential equation (5)— i.e. they must both lie on the graph of Figure 5. Suppose that the initial condition in Eq. (5) is 0. Then at $t = 0+$ the frequency error is $d\theta_e/dt = 2\pi K$. So we start at the point labeled (1) in Figure 5.

Now for $dt > 0$ if $d\theta_e/dt > 0$ we have $d\theta_e > 0$. That is, if $d\theta_e/dt > 0$ then the operating point moves to the right because θ_e must increase. Likewise, if $d\theta_e/dt < 0$ then the operating point moves to the left. Therefore, starting at point (1) we have $d\theta_e/dt = 2\pi K > 0$, so we move to the right to point (2). Point (2) is a stable operating point. If θ_e tries to decrease from (2), then $d\theta_e/dt > 0$, and so $d\theta_e > 0$, forcing the operating point back to (2). Likewise, if θ_e tries to increase, this results in $d\theta_e < 0$, again forcing the operating point back to (2).

Therefore, after a certain time interval the operating point is point (2) and it stays there. At point (2) we have a steady-state frequency error of $d\theta_e/dt = 0$, but we have a non-zero steady-state phase error, $\theta_{e_{ss}}$. It is easy to see that $\theta_{e_{ss}} = \arcsin(K/K_v K_d)$.

Note that we get frequncy lock ($d\theta_e/dt = 0$) only if the phase-plane plot crosses the $d\theta_e/dt = 0$ axis. Hence, to achieve frequency lock we must have $2\pi(K - K_v K_d) < 0$, or $K_v K_d > K$. For this reason $K_v K_d$ (in Hz) is called the *lock range* or *hold-in range*.[1]

Note also that for large loop gain $K_v K_d$ we get a small $\theta_{e_{ss}}$. To see this, look at the loop transfer function:

$$\frac{\Theta_0(s)}{\Theta_i(s)} = \frac{2\pi K_v K_d}{s + 2\pi K_v K_d} = \frac{1}{s/(2\pi K_v K_d) + 1} .$$

As $K_v K_d \to \infty$, $\Theta_0(s)/\Theta_i(s) \to 1$, so $\theta_0(t) = \theta_i(t)$. (This also follows from $\theta_{e_{ss}} = \arcsin(K/K_v K_d) \to 0$ as $K_v K_d \to \infty$.) Keep in mind that the first order loop behaves as a low-pass filter, so large loop gain implies large bandwidth.

Thus we have now seen that the phase error tends to drive the loop into lock, which justifies the linearity assumption $K_d \sin\theta_e(t) \approx K_d \theta_e(t)$. But we have also seen that the first order loop requires a large loop gain to work properly, which implies a large loop bandwidth. Furthermore, we have seen that the first order loop achieves frequency lock, but it has a steady-state phase error. In some applications the steady-state phase error does not present a problem, while it other applications it does. If we desire $\theta_{e_{ss}} = 0$ another type of loop filter must be used; we shall now consider a commonly used filter.

4.3 Second order loop

In the second order loop the loop filter is

$$H(s) = \frac{s + a}{s} .$$

Then the overall loop transfer function is second order:

$$\begin{aligned} G(s) &= \frac{2\pi K_v K_d H(s)}{s + 2\pi K_v K_d H(s)} = \frac{2\pi K_v K_d (s + a)}{s^2 + 2\pi K_v K_d s + 2\pi K_v K_d a} \\ &= \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} , \end{aligned}$$

[1] See also Equation (4-104) in [Couch]. The analysis we did was for the first order loop, $H(s) = 1$, and so the loop filter does not enter. In general, the lock range is $K_v K_d H(0)$, as shown in Couch's Equation (4-104).

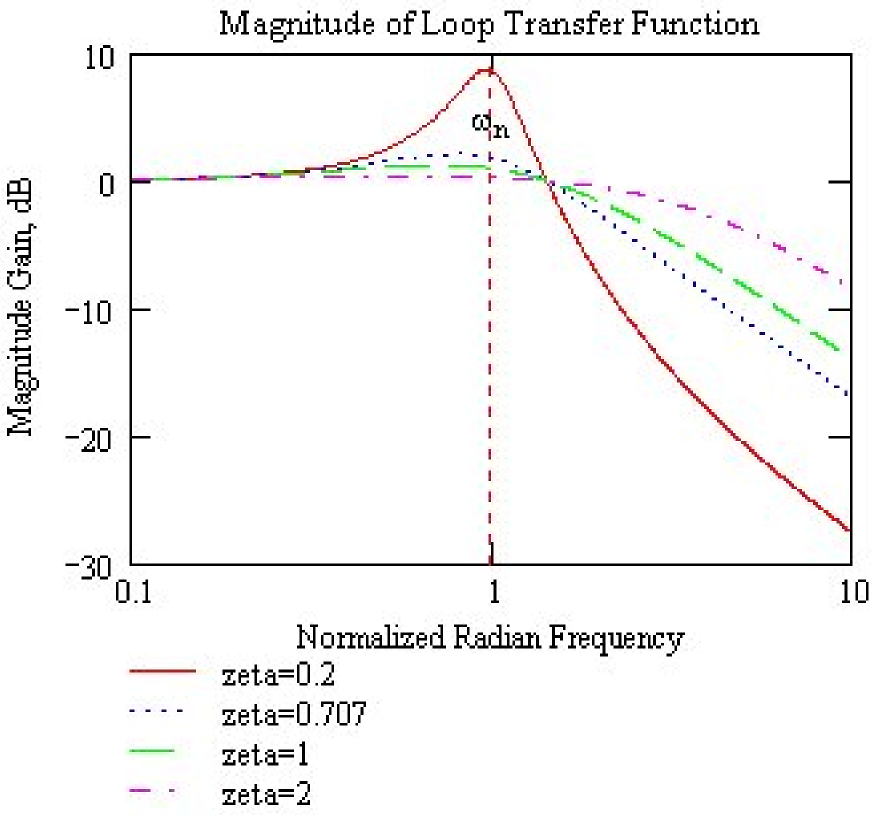


Figure 6: Frequency Response of Second Order Loop

where

$$\zeta = \frac{1}{2}\sqrt{\frac{2\pi K_v K_d}{a}} \qquad \text{is the damping ratio, and}$$
$$\omega_n = \sqrt{2\pi K_v K_d a} \qquad \text{is the natural frequency.}$$

This transfer function is that of a second order lowpass filter; see Figure 6. Again the loop acts as a low-pass filter with bandwidth

$$f_{\text{3dB}} = f_n \left(2\zeta^2 + 1 + \sqrt{(2\zeta^2+1)^2+1}\right)^{1/2} .$$

Also, as with the first order loop, a large loop gain implies a large bandwidth.

The major advantage of the second order loop is that the steady-state phase error is 0. To see this consider the transfer function from input phase to phase error from Equation 3:

$$\frac{\Theta_e(s)}{\Theta_i(s)} = \frac{s}{s + 2\pi K_v K_d H(s)} = \frac{s^2}{s^2 + 2\pi K_v K_d s + 2\pi K_v K_d a}$$
$$= \frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\,.$$

Suppose, as before, that $\theta_i(t) = (2\pi K t)u(t)$ so that $\Theta_i(s) = 2\pi K/s^2$. Then

$$\Theta_e(s) = \frac{2\pi K}{s^2 + 2\zeta\omega_n s + \omega_n^2}\,.$$

If $\zeta < 1$ then we have

$$\theta_e(t) = \frac{2\pi K}{\omega_n\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\!\big(\omega_n\sqrt{1-\zeta^2}\,t\big)\,,$$

and

$$\lim_{t\to\infty} \theta_e(t) = 0\,.$$

Hence $\theta_{e_{ss}} = 0$ for the second order loop.

5 The PLL as an FM Demodulator

Suppose that the input $v_i(t) = A_i\cos\!\big(2\pi f_c t + \theta_i(t)\big)$ is an FM signal. We shall show that the PLL can be used to demodulate this FM signal.

When using the PLL as an FM demodulator, we want $v_2(t) = m(t)$, so we need to know the transfer function $V_2(s)/M(s)$. For FM

$$\theta_i(t) = 2\pi f_\Delta \int_{-\infty}^{t} m(\xi)d\xi\,,$$

and so

$$\Theta_i(s) = \frac{2\pi f_\Delta}{s}M(s) \quad\text{or}\quad M(s) = \frac{s}{2\pi f_\Delta}\Theta_i(s).$$

But

$$\Theta_0(s) = \frac{2\pi K_v}{s}V_2(s) \quad\text{or}\quad V_2(s) = \frac{s}{2\pi K_v}\Theta_0(s).$$

Therefore,

$$\frac{V_2(s)}{M(s)} = \frac{f_\Delta}{K_v}\frac{\Theta_0(s)}{\Theta_i(s)} = \frac{f_\Delta}{K_v}G(s)\,.$$

Hence, if the VCO gain K_v is equal to the frequency deviation constant f_Δ of the FM signal,

$$\frac{V_2(s)}{M(s)} = G(s) = \frac{2\pi K_v K_d H(s)}{s + 2\pi K_v K_d H(s)}\,. \tag{6}$$

All of our preceding analysis shows us that in both the first and second order loops, a large loop gain results in $\theta_0(t) \approx \theta_i(t)$ which implies that $v_2(t) \approx m(t)$ when the input is an FM signal.

6 Concluding Remarks

- Using the PLL as an FM detector requires a large loop gain, which implies a large loop bandwidth. This is especially true for the first order loop. Too large a bandwidth is undesirable because it increases the output noise power which results in a decreased signal-to-noise ratio. Hence we always have to design with this trade-off in mind.
- Another drawback of the first order loop is the non-zero $\theta_{e_{ss}}$; this is eliminated in the second order loop. For FM demodulation, $\theta_{e_{ss}}$ is, however, of little concern.
- When using the PLL for carrier recovery a small $\theta_{e_{ss}}$ is required. Hence a second order loop would be preferred.
- The PLL can be used to demodulate PM by integrating the VCO output.
- The PLL can also be used for frequency generation; see Figure 4-25 in [Couch].
- See Section 5-4 in [Couch] or Section 7.3 in [Carlson] for a special PLL, called a Costas loop, for coherent demodulation of DSB.

References

[Carlson] A. Bruce Carlson, Paul B. Crilly, and Janet C. Rutledge, *Communication Systems: An Introduction to Signals & Noise in Electrical Communication*, 4th ed., McGraw-Hill (2002)

[Couch] Leon W. Couch, II, *Digital and Analog Communication Systems*, 6th ed., Prentice-Hall (2001)