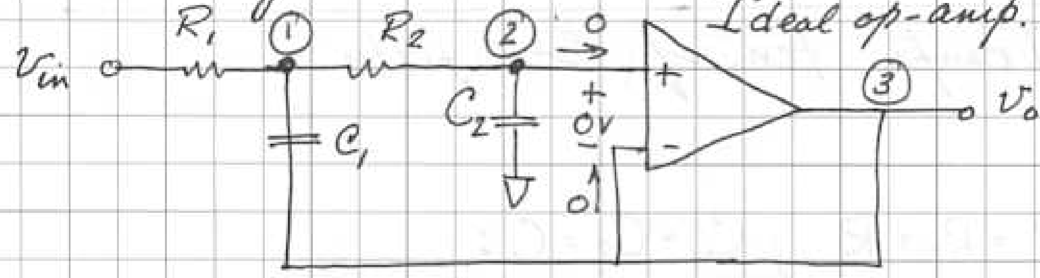
LAB 3: FREQUENCY RESPONSE OF SYSTEMS & DISTORTION

PRELAB

Sallen-Key



(1) $$\frac{V_1 - V_{in}}{R_1} + \frac{V_1 - V_2}{R_2} + \frac{V_1 - V_{out}}{1/C_1 s} = 0$$

(2) $$\frac{V_2 - V_1}{R_2} + \frac{V_2}{1/C_2 s} = 0$$

(3) $$V_2 = V_{out}$$

Eq. (2) $\Rightarrow$ $-\frac{1}{R_2} V_1 + \left(\frac{1}{R_2} + C_2 s\right) V_{out} = 0$

$\Rightarrow$ $(1 + R_2 C_2 s) V_{out} = V_1$

Eq. (1) $\Rightarrow$ $(R_2 + R_1 + R_1 R_2 C_1 s) V_1 - (R_1 + R_1 R_2 C_1 s) V_{out} = R_2 V_{in}$

$\Rightarrow (R_1 + R_2 + R_1 R_2 C_1 s)(1 + R_2 C_2 s) V_{out} - (R_1 + R_1 R_2 C_1 s) V_{out}$
$= R_2 V_{in}$

$\Rightarrow [\cancel{R_1} + R_2 + \cancel{R_1 R_2 C_1 s} + R_1 R_2 C_2 s + R_2^2 C_2 s + R_1 R_2^2 C_1 C_2 s^2$
$- \cancel{R_1} - \cancel{R_1 R_2 C_1 s}] V_{out} = R_2 V_{in}$

$$\Rightarrow \left[R_2 + (R_1R_2C_2 + R_2^2C_2)s + R_1R_2^2C_1C_2s^2\right]V_{out} = R_2V_{in}$$

$$\Rightarrow \boxed{\frac{V_{out}}{V_{in}} = \frac{1}{R_1R_2C_1C_2s^2 + (R_1 + R_2)C_2s + 1}}$$

Tranfer fcn. of S-K filter.

Put $R_1 = R_2 = R$, $C_1 = C_2 = C$:

$$H(s) = \frac{1}{(RC)^2s^2 + 2RCs + 1}$$

$$\text{also} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_ns + \omega_n^2}$$

$$\omega_n^2 = \frac{1}{(RC)^2}, \quad 2\zeta\omega_n = \frac{2}{RC} \Rightarrow \underline{\zeta = 1}$$

Always critically damped

$$H(\omega) = \frac{1}{1 - (RC)^2\omega^2 + j2RC\omega}$$

$$|H(\omega)| = \frac{1}{\sqrt{[1 - (RC)^2\omega^2]^2 + (2RC)^2\omega^2}}$$

$$H(0) = \underline{1} \quad (= 0\ dB).$$

6 dB break: $|H(\omega_b)| = 1/2$

$$\Rightarrow \frac{1}{2} = \frac{1}{\sqrt{[1 - (RC)^2\omega_b^2]^2 + (2RC)^2\omega_b^2}}$$

$$\Rightarrow \left[1-(RC)^2\omega_b^2\right]^2 + (2RC)^2\omega_b^2 - 4 = 0$$

$\Rightarrow$ ~~$(RC)^4\omega_b^4 + (2RC)^2\omega_b^2$~~

$$\Rightarrow (RC)^4\omega_b^4 + 2(RC)^2\omega_b^2 - 3 = 0$$

$$\Rightarrow \omega_b^2 = \frac{-2(RC)^2 \pm \sqrt{4(RC)^4 + 12(RC)^4}}{2(RC)^4} = \cancel{\frac{-3}{(RC)^2}},\ \frac{1}{(RC)^2}$$

$$\omega_b^2 = \frac{1}{(RC)^2} \Rightarrow \underline{\underline{\omega_b = \frac{1}{RC}}}$$

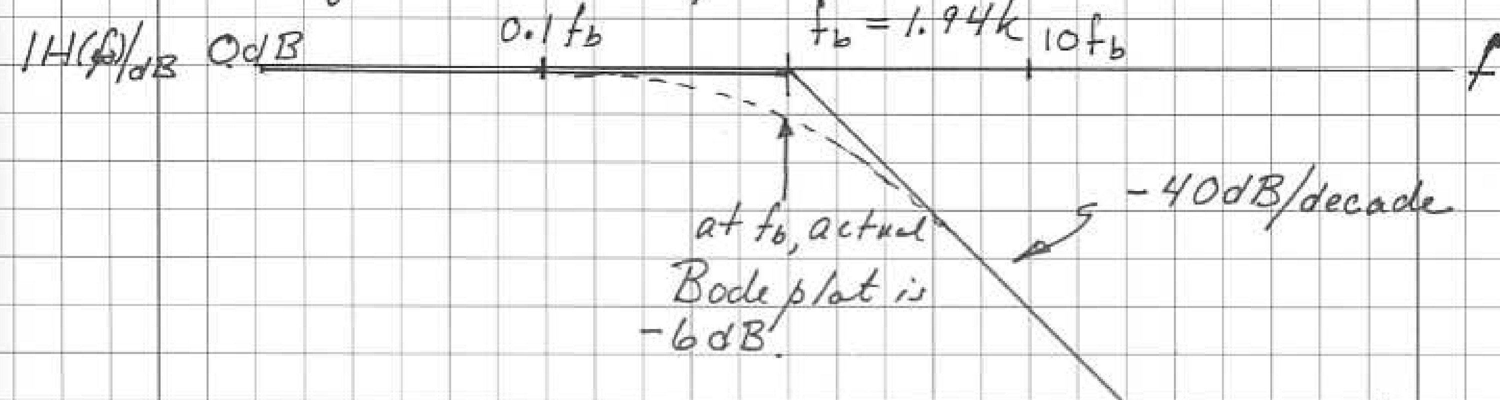
$$\underline{\underline{f_b = \frac{1}{2\pi RC}}}$$

For $R = 8.2\,k$, $C = 0.01\mu F$

$\omega_b = 12.20\,k\ rad/s$ $\quad \underline{\underline{f_b = 1.94\,kHz}}$

Also: $\omega_n = \frac{1}{RC} = \omega_b$, $\zeta = 1$

Straight line Bode plot:



Mathcad Bode plots next two pages

Lab 3 Prelab: Sallen-Key Filter

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$R := 8.2 \cdot 10^3 \qquad C := 0.01 \cdot 10^{-6}$

$f_{min} := 1 \qquad f_{max} := 10^5 \qquad N := 1000 \qquad m := 0 .. N-1$

$F_{min} := \log(f_{min}) \qquad F_{max} := \log(f_{max}) \qquad F_m := 10^{\left[F_{min} + m \cdot \frac{(F_{max} - F_{min})}{N-1}\right]}$

These parameters set up a logarithmic sequence of frequencies for the plot.

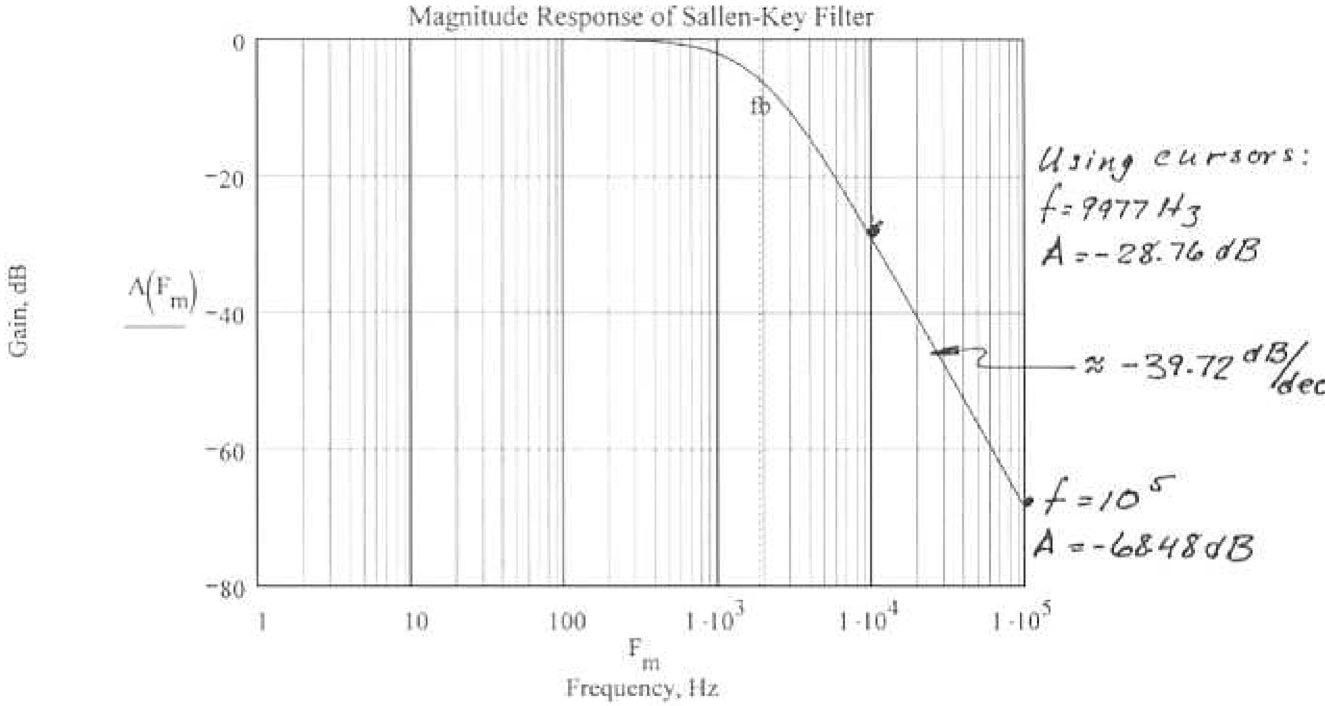
$$H(s) := \frac{1}{R^2 \cdot C^2 \cdot s^2 + 2 \cdot R \cdot C \cdot s + 1}$$ Sallen-Key transfer function

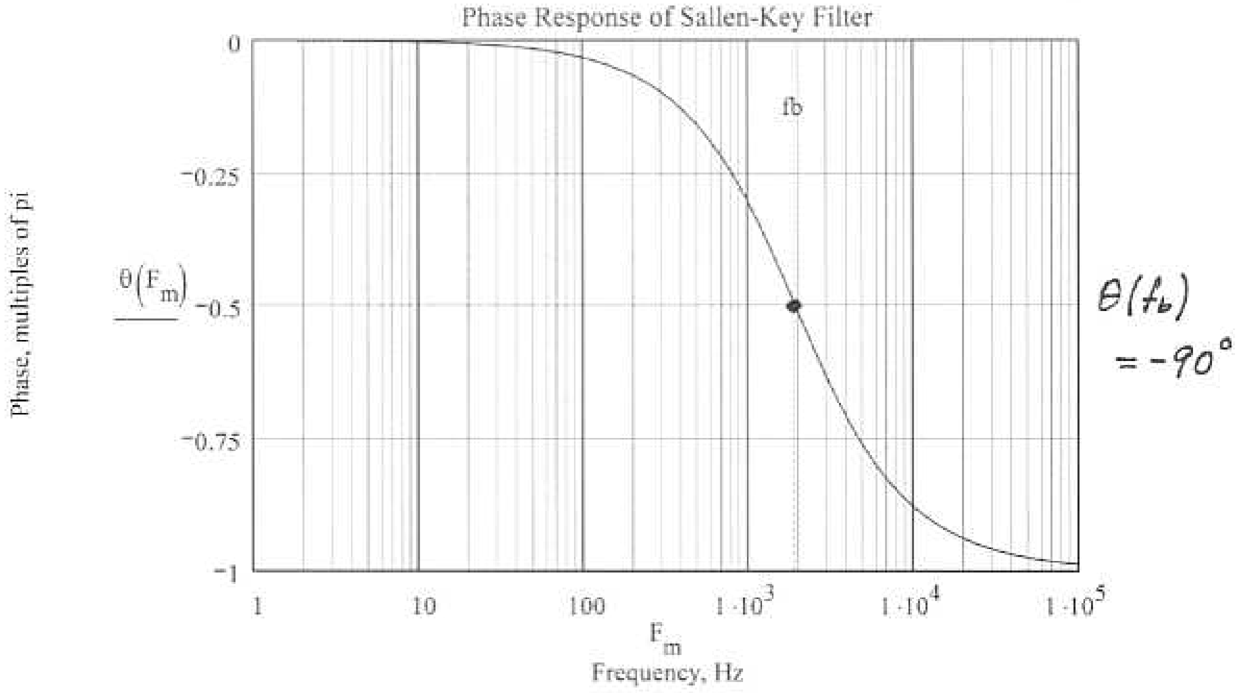
$$A(f) := 20 \cdot \log(|H(j \cdot 2 \cdot \pi \cdot f)|)$$ Magnitude response

$$\theta(f) := \frac{\arg(H(j \cdot 2 \cdot \pi \cdot f))}{\pi}$$ Phase response, in multiples of π

$$fb := \frac{1}{2 \cdot \pi \cdot R \cdot C} \qquad fb = 1.941 \times 10^3$$ 6dB break frequency $\quad A(fb) = -6.021$

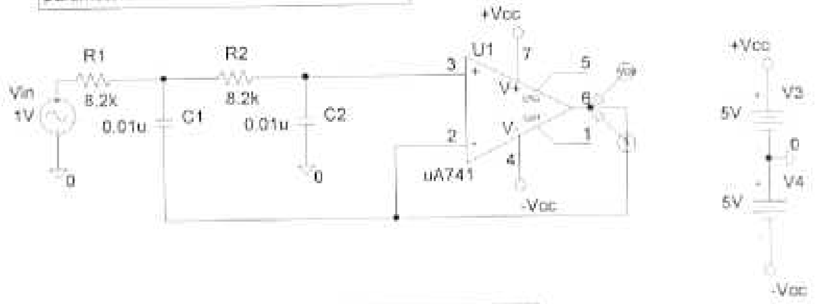
$\theta(fb) = -0.5$





PSpice simulations— following pages

Setup for frequency response of Sallen-Key circuit. Vin is a VAC part. In the Analysis Setup, check AC Sweep and enter parameters.



Note: set |Vin|=1V. Then output voltage amplitude is equal to gain.

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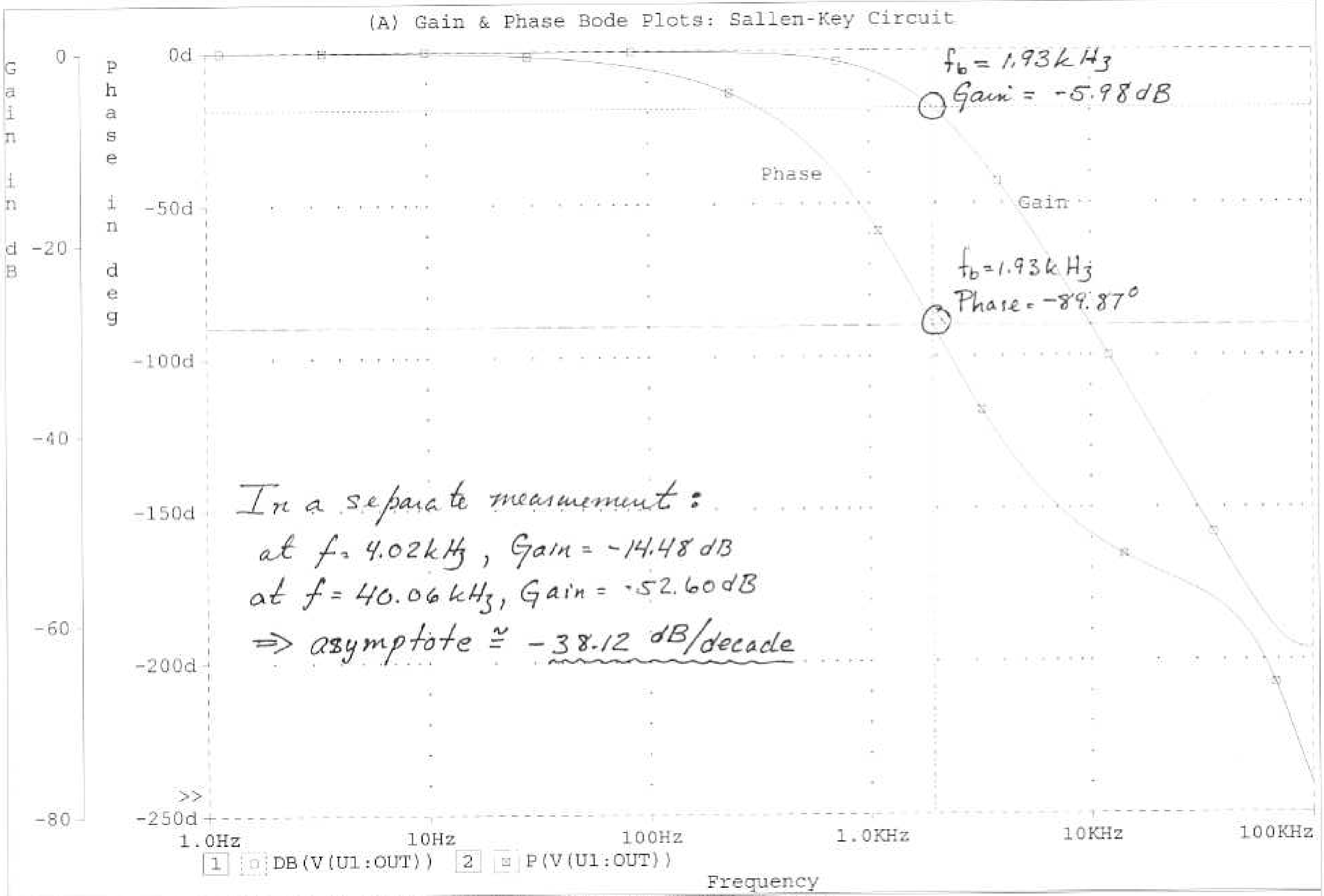
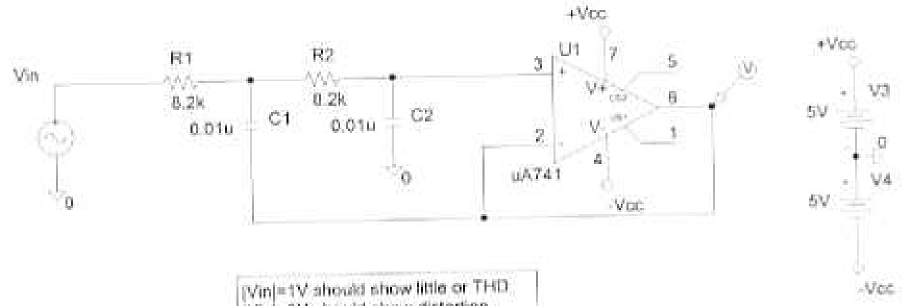
Temperature: 27.0



A1:(1.9307K,-5.9761) A2:(1.9307K,-89.865) DIFF(A):(0.000,83.889)

DISTORTION ANALYSIS

Measuring distortion in the Sallen-Key filter. For THD, Vin is a sinusoid (in this case 500Hz). Plot Vout and then use the FFT in Probe to view the spectrum of Vout.



|Vin|=1V should show little or THD
|Vin|=6V should show distortion.

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EEL 4514L: Prelab 3, Sallen-Key Filter

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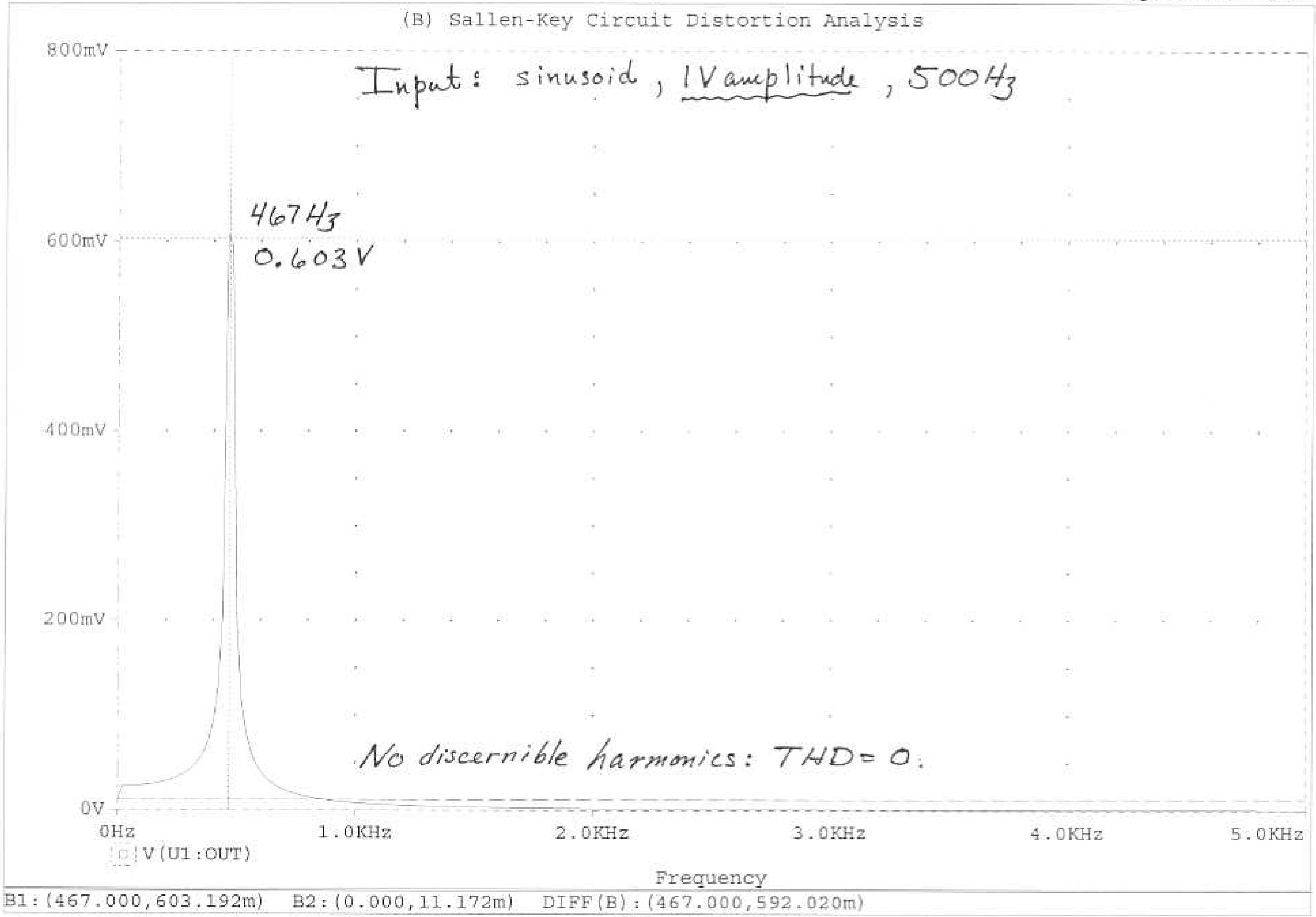
Temperature: 27.0



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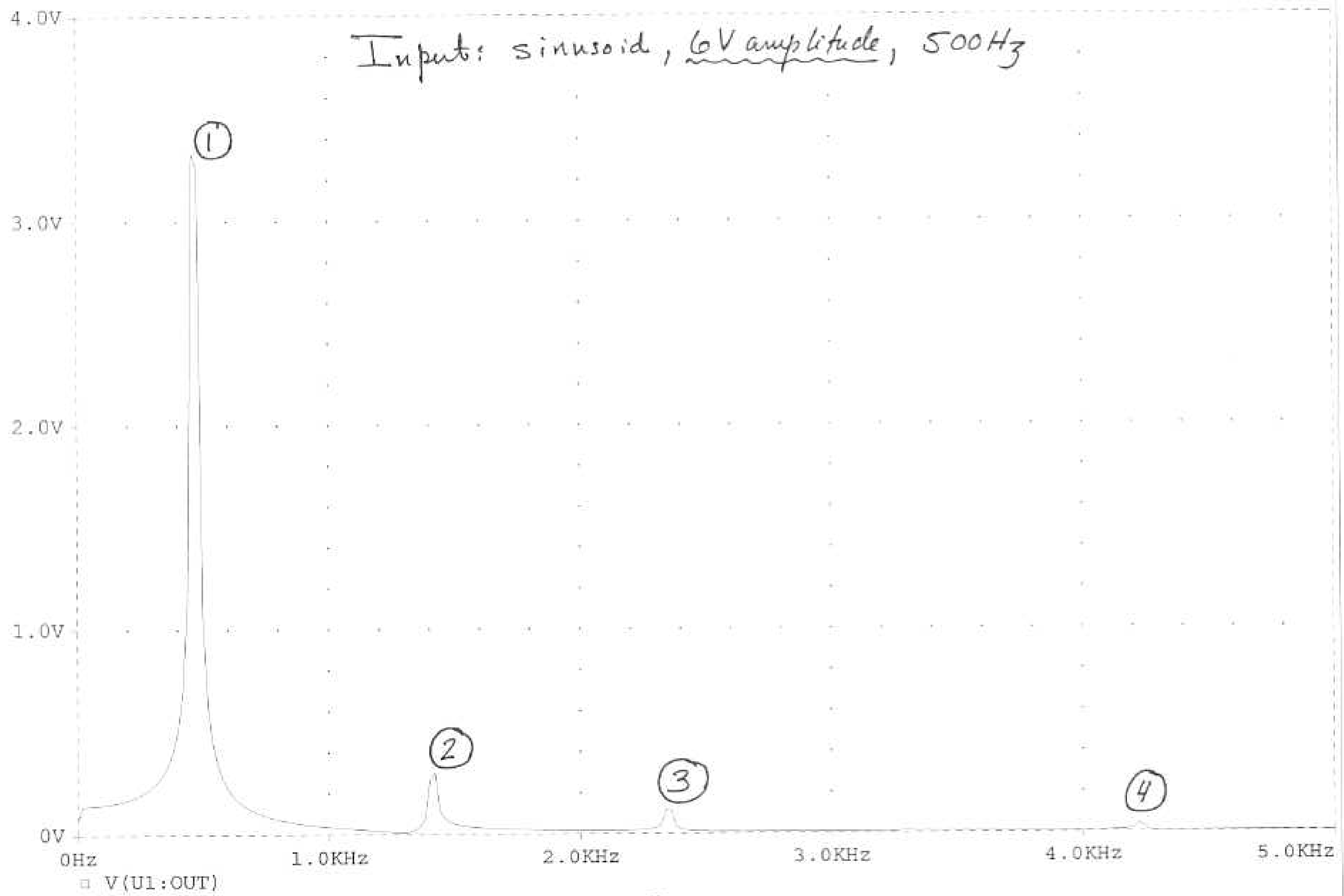
Temperature: 27.0

(B) Sallen-Key Circuit Distortion Analysis



With $|V_{in}| = 1V$, at 500 Hz, there is no discernible distortion. (p45)

With $|V_{in}| = 6V$:

	f, Hz	Line ampl., V	V_n/V_1
(1)	473.12	3.287 V	—
(2)	1.42 k	297.9 mV	0.0906
(3)	2.350 k	115.4 mV	0.0351
(4)	4.22 k	35.69 mV	0.0108

No lines discernible at higher frequencies.

Use Eq. (4-47) in Couch (also given in lecture):

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} V_n^2}}{V_1} \times 100\% = \sqrt{\sum_{n=2}^{\infty} \left(\frac{V_n}{V_1}\right)^2} \times 100\%$$

where V_n = amplitude of n^{th} harmonic. From table:

$$THD = \sqrt{(.0906)^2 + (.0351)^2 + (.0108)^2} \times 100\%$$

$$= \underline{\underline{9.78\%}}.$$