Lab 1 Prelab
EEL 4514L: Communication Lab
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Fourier series coefficeints for a square wave

The waveform is an odd square wave of amplitude A, dc value K, and duty cycle 0.5.

$N := 20$ Set the number of Fourier series coefficients to be calculated; N should be even. If you make N odd, then the program below will calculate N+1 coefficients.

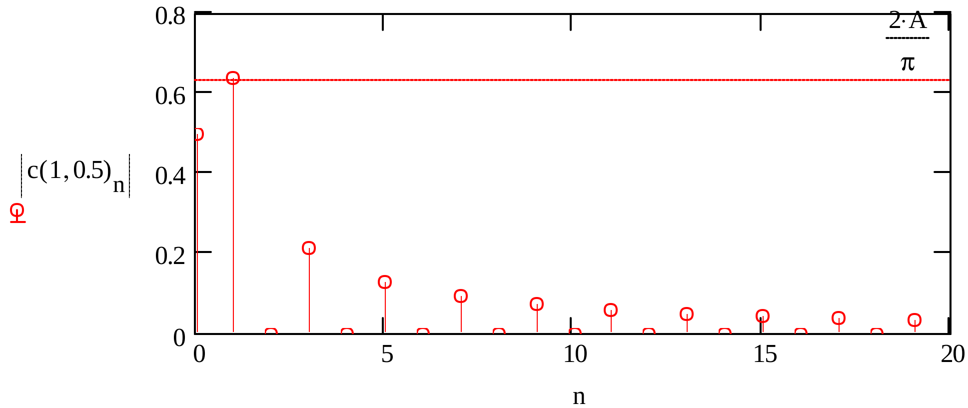
Recall that for a real function w(t), the F-series coefficients satisfy $c_{-n}=c_n^*$; i.e., the magnitude $|c_n|$ is an even function of n and the phase $\arg(c_n)$ is an odd function of n.

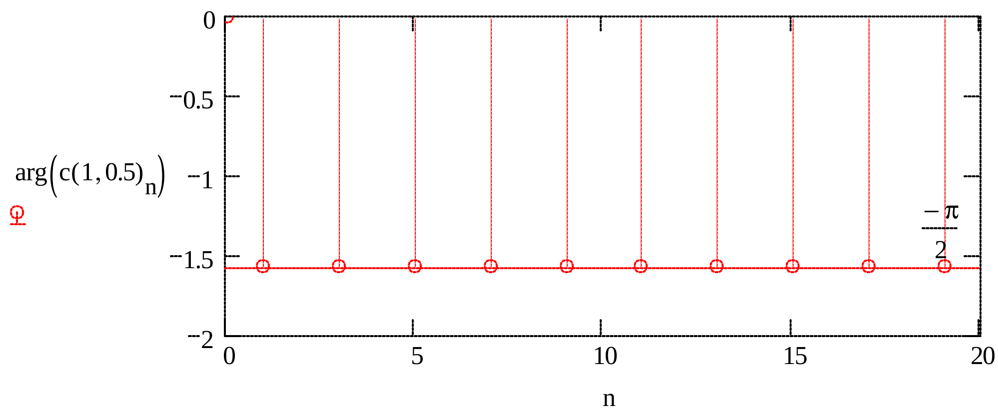
$$c(A,K) := \left| \begin{array}{l} a_0 \leftarrow K \\ \text{for } k \in 1,3..N \\ \quad a_k \leftarrow \dfrac{2 \cdot A}{j \cdot \pi \cdot k} \\ a \end{array} \right.$$

$c(1,0.5) =$

	0
0	0.5
1	-0.637i
2	0
3	-0.212i
4	0
5	-0.127i
6	0
7	-0.091i
8	0
9	-0.071i
10	0
11	-0.058i
12	0

$n := 0 .. N - 1$





<u>Output of an RC lowpass filter with a square input</u>

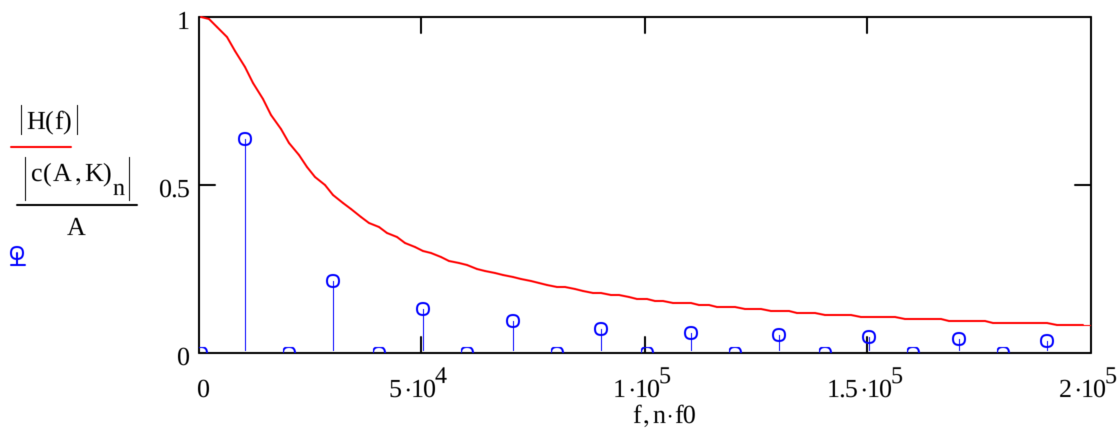
Pick a time constant: $\tau := 10 \cdot 10^{-6}$ (This is the value used in the lab.)

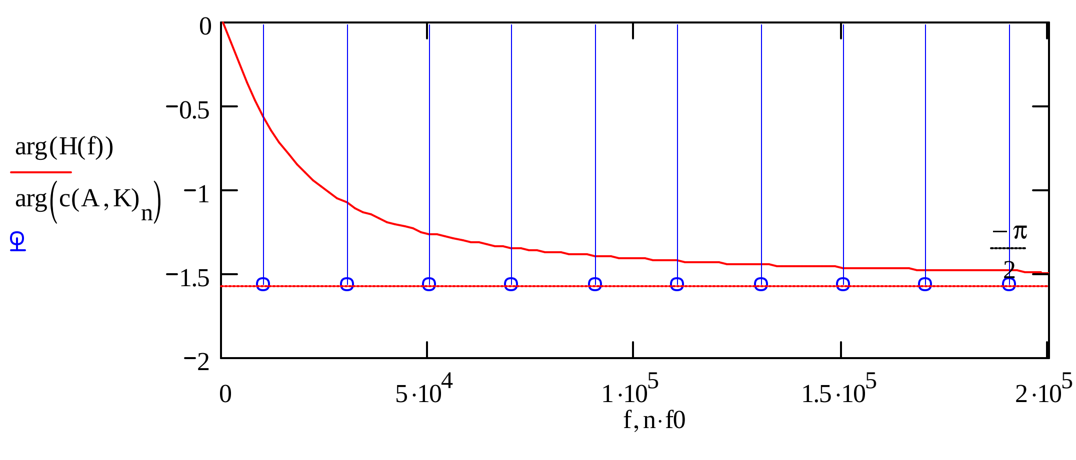
$$H(f) := \frac{1}{1 + j \cdot 2 \cdot \pi \cdot \tau \cdot f}$$

In lab a square wave with K=0, A=100mV, and f_0=10kHz will be used.

$f0 := 10 \cdot 10^3$ $\qquad$ $fmax := N \cdot f0$ $\qquad$ $f := 0, \frac{fmax}{100} .. fmax$

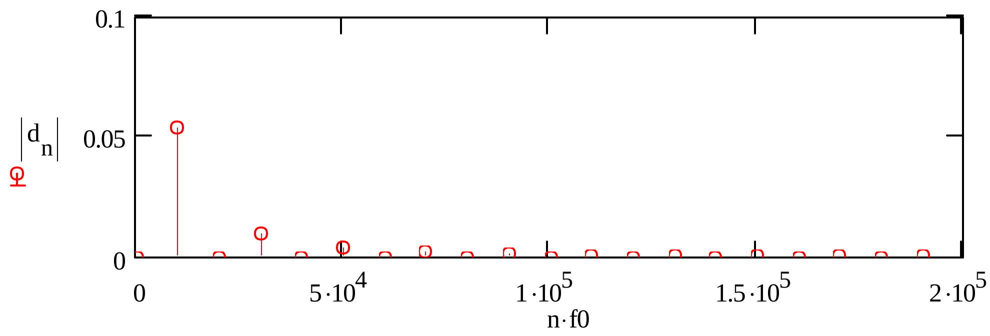
$A := 100 \cdot 10^{-3}$ $\qquad$ $K := 0$

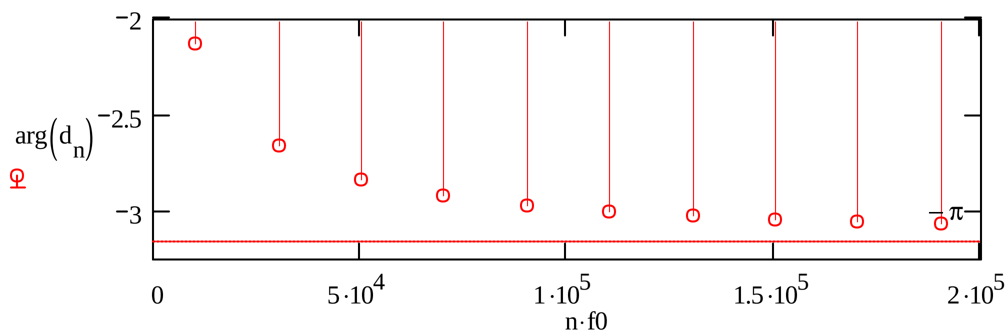




The Fourier series coefficients of the output are:

$$d_n := H(n \cdot f0) \cdot c(A, K)_n$$





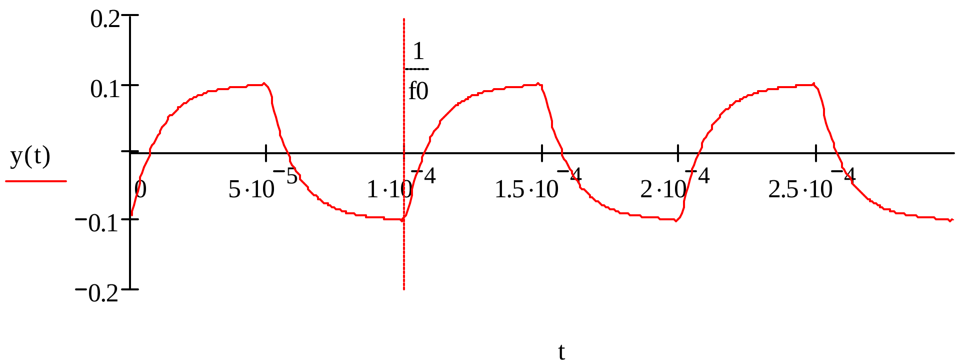
The output signal y(t) is represented by its Fourier series:

$$y(t) = \sum_n d_n \cdot \exp(j \cdot 2 \cdot \pi \cdot f0 \cdot t)$$

Of course we cannot calculate an infinite number of terms to obtain the plot; we must truncate to a finite number. Furthermore, we have only defined d_n for non-negative n; we obtain d_n for negative n from $d_{-n} = \overline{d_n}$:

$$y(t) := d_0 + \sum_{m=1}^{N-1} d_m \cdot \exp(j \cdot 2 \cdot \pi \cdot m \cdot f0 \cdot t) + \sum_{m=1}^{N-1} \overline{d_m} \cdot \exp(-j \cdot 2 \cdot \pi \cdot m \cdot f0 \cdot t)$$

$$tmax := \frac{3}{f0} \qquad t := 0, \frac{tmax}{500} .. tmax$$



Calculation of output from convolution:

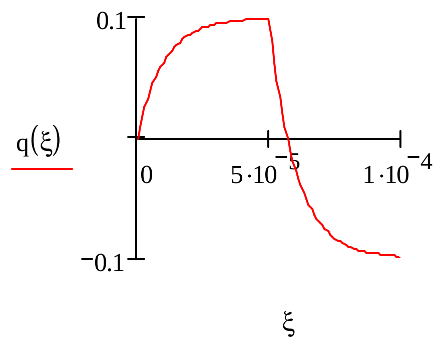
$$\frac{1}{1 + a \cdot s} \text{ invlaplace}, s \rightarrow \frac{\exp\left(\frac{-t}{a}\right)}{a} \qquad h(t) := \frac{1}{\tau} \cdot \exp\left(\frac{-t}{\tau}\right)$$

One period of the square wave: $$p(t) := A \cdot \left(\Phi(t) - 2 \cdot \Phi\left(t - \frac{1}{2 \cdot f0}\right) + \Phi\left(t - \frac{1}{f0}\right)\right)$$

One period of output:

$$q(\xi) := \int_0^{\xi} p(\lambda)\cdot h(\xi - \lambda)\, d\lambda$$

$$\xi := 0, \frac{1}{100\cdot f0} .. \frac{1}{f0}$$



$$y(t) := \sum_{m=0}^{2} q\left(t - m\cdot\frac{1}{f0}\right)$$