LAB 1

DIGITAL STORAGE OSCILLOSCOPE, FUNCTION GENERATOR, & MEASUREMENTS

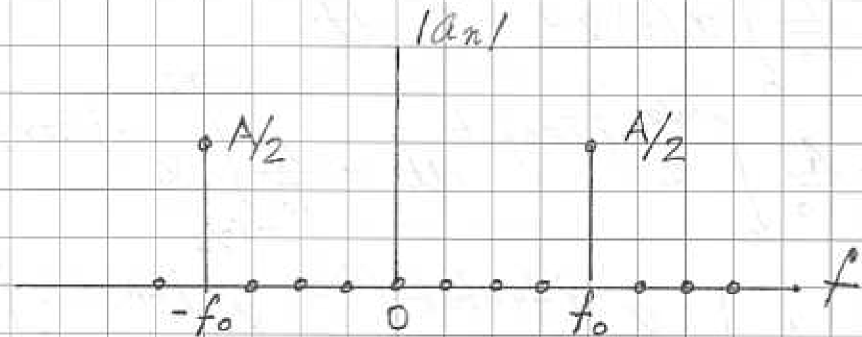
PRELAB

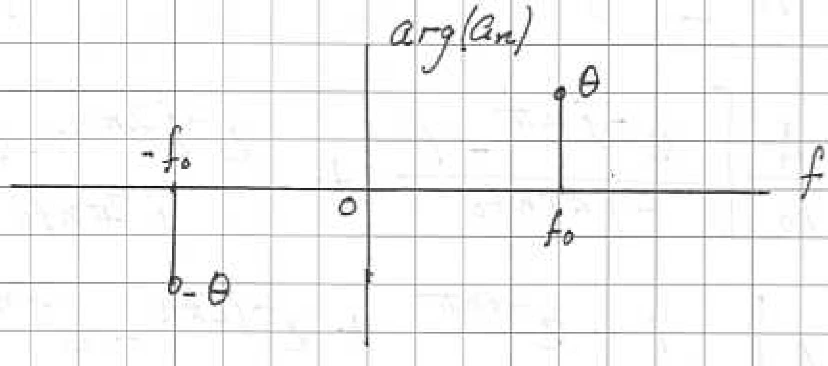
SPECTRUM OF SINUSOID: assume dc = 0 to start

$$x(t) = A\cos(2\pi f_0 t + \theta)$$
$$= \frac{A}{2} e^{j\theta} e^{j2\pi f_0 t} + \frac{A}{2} e^{-j\theta} e^{-j\omega_0 t}.$$

$$\Rightarrow \quad a_1 = \frac{A}{2} e^{j\theta}, \quad a_{-1} = \frac{A}{2} e^{-j\theta} = a_1^*$$

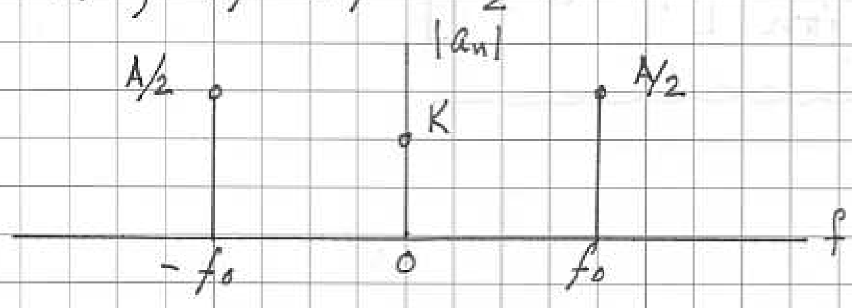
all other $a_n = 0$.

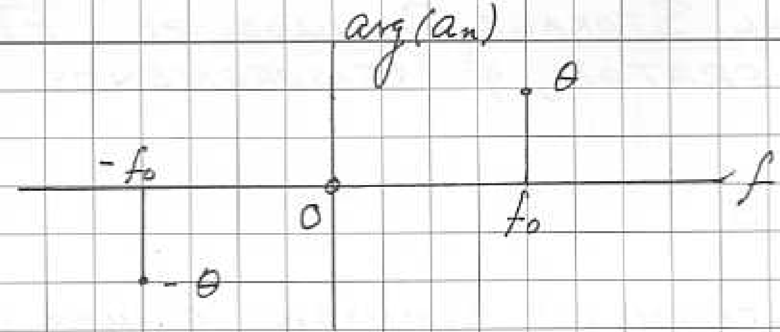




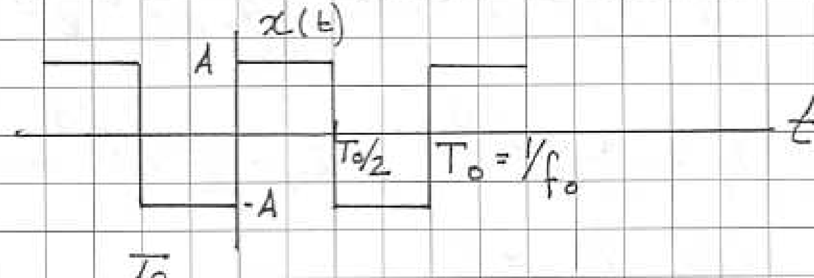
Now for non-zero dc: $x(t) = K + A\cos(2\pi f_0 t + \theta)$

$$a_0 = K, \quad a_1 = a_{-1}^* = \frac{A}{2} e^{j\theta}$$





SPECTRUM OF SQUARE WAVE: assume dc = 0 first.



$$a_n = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j2\pi n f_0 t}\, dt$$

$$= \frac{A}{T_0}\int_0^{T_0/2} e^{-j2\pi n f_0 t}\, dt - \frac{A}{T_0}\int_{T_0/2}^{T_0} e^{-j2\pi n f_0 t}\, dt$$

Provided $n \neq 0$

$$= \frac{A}{T_0}\left[\frac{e^{-j2\pi n f_0 t}}{-j2\pi n f_0}\Bigg|_0^{T_0/2} + \frac{e^{-j2\pi n f_0 t}}{j2\pi n f_0}\Bigg|_{T_0/2}^{T_0} \right]$$

$$= \frac{A}{T_0}\left[\frac{e^{-jn\pi} - 1}{-j2\pi n f_0} + \frac{e^{-j2\pi n} - e^{-jn\pi}}{j2\pi n f_0} \right]$$

$$= A\left[\frac{1 - e^{-jn\pi} + e^{-j2\pi n} - e^{-jn\pi}}{j2\pi n} \right]$$

$$= \frac{A}{j\pi n}\left[1 - e^{-jn\pi} \right]$$

For $n=0$: $a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt = \underline{\underline{0.}}$

OR

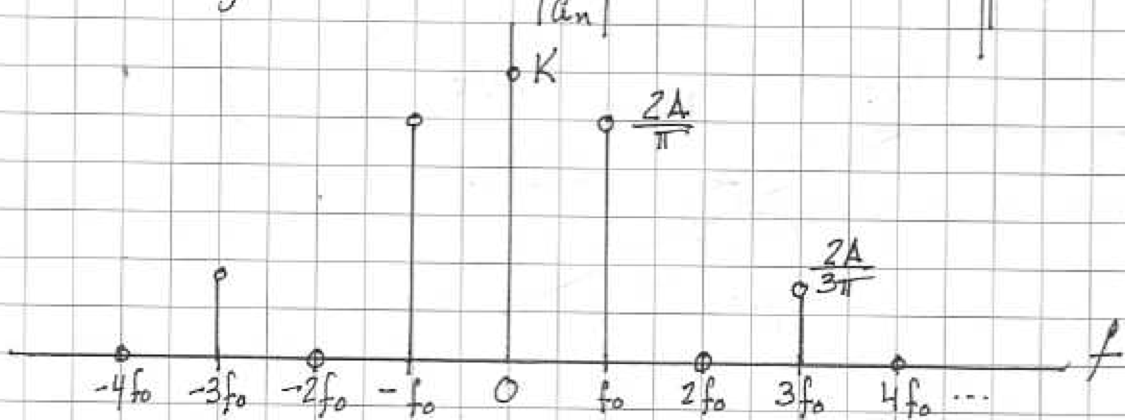
$$a_n = \begin{cases} 0 & (n=0) \\ \frac{A}{j\pi n}\left[1-e^{-jn\pi}\right] & (n\neq 0) \end{cases} = \begin{cases} 0, & n = 0 \text{ or even} \\ \frac{2A}{j\pi n}, & n \text{ odd} \end{cases}$$

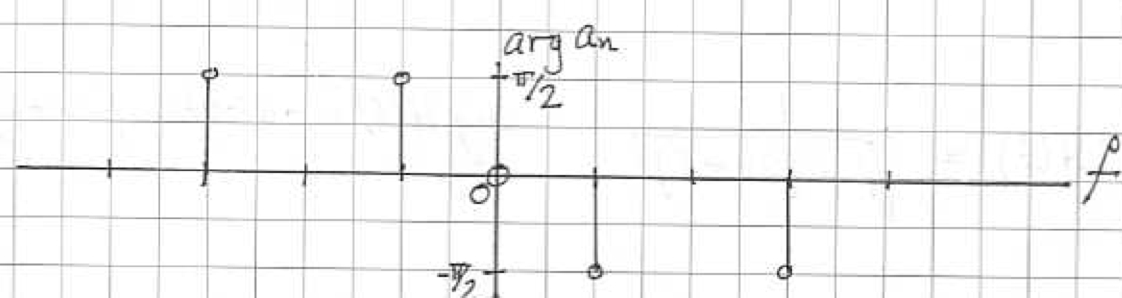
$n =$	0	1	2	3	4	-1	-2	-3	-4
$\lvert a_n\rvert =$	0	$\frac{2A}{\pi}$	0	$\frac{2A}{3\pi}$	0	$\frac{2A}{\pi}$	0	$\frac{2A}{3\pi}$	0
$\arg(a_n) =$	–	$-\pi/2$	–	$-\pi/2$	–	$\pi/2$	–	$\pi/2$	–

For a non-zero dc level K:

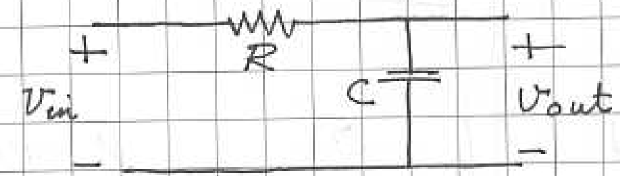
$$a_n = \begin{cases} K, & n = 0 \\ 0, & n \text{ even} \\ \frac{2A}{j\pi n}, & n \text{ odd} \end{cases}$$

F-coeff. for a square wave.





RC LOWPASS FILTER:



$\tau = RC$

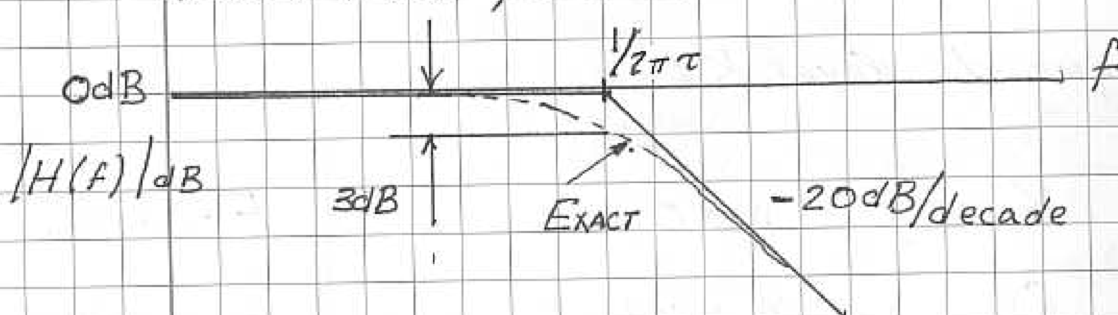
$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1/Cs}{R + 1/Cs} = \frac{1}{1+\tau s}, \; Re(s) =$$

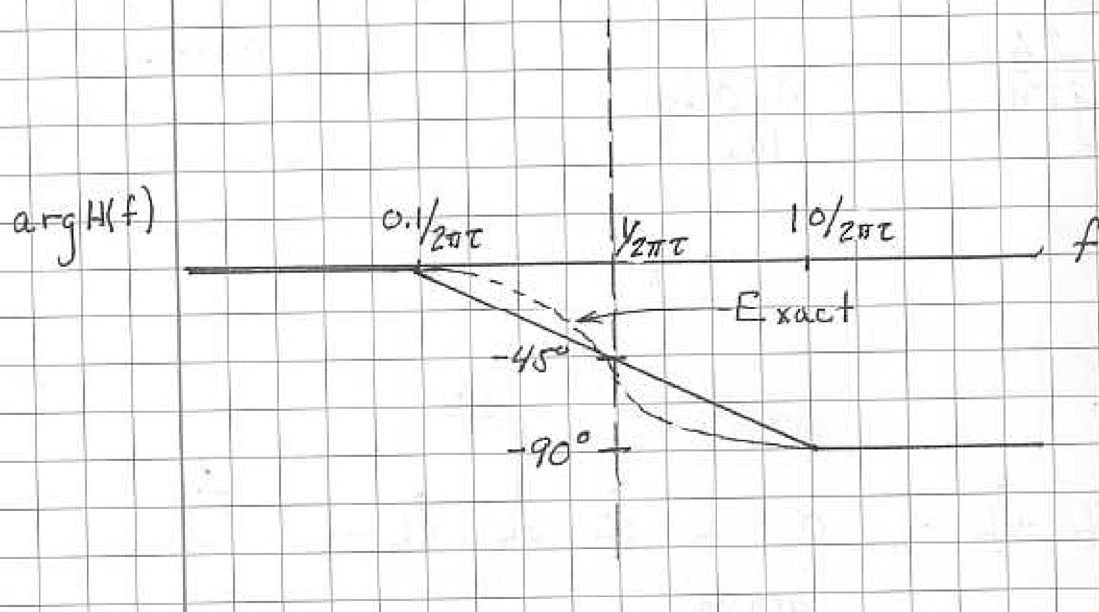
$$H(f) = \frac{1}{1 + j2\pi\tau f}$$

3dB break freq:

$$f_{3dB} = \frac{1}{2\pi\tau}$$

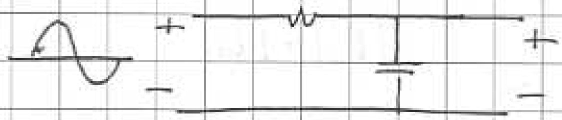
Bode Plots for $H(f)$:





$$|H(f)| = \frac{1}{\sqrt{1 + (2\pi\tau f)^2}} \qquad \arg H(f) = -\arctan(2\pi\tau f)$$

FILTER RESPONSE TO SINUSOIDAL INPUT:



$$v_{in}(t) = K + A\cos(2\pi f_0 t + \theta)$$

$$a_0 = K, \quad a_1 = \frac{A}{2}e^{j\theta}, \quad a_{-1} = \frac{A}{2}e^{-j\theta}.$$

$$H(f) = \frac{1}{1 + j2\pi\tau f}.$$

$$H(0) = 1, \quad H(f_0) = \frac{1}{1 + j2\pi\tau f_0}, \quad H(-f_0) = \frac{1}{1 - j2\pi\tau f_0}.$$

The output is sinusoidal with coefficients

$$b_n = a_n \cdot H(nf_0)$$

$$\Rightarrow \quad b_0 = a_0 = K.$$

$$b_1 = a_1 H(f_0) = \frac{A}{2}e^{j\theta}\frac{1}{1 + j2\pi\tau f_0}$$

$$= |b_1| e^{j\phi}$$

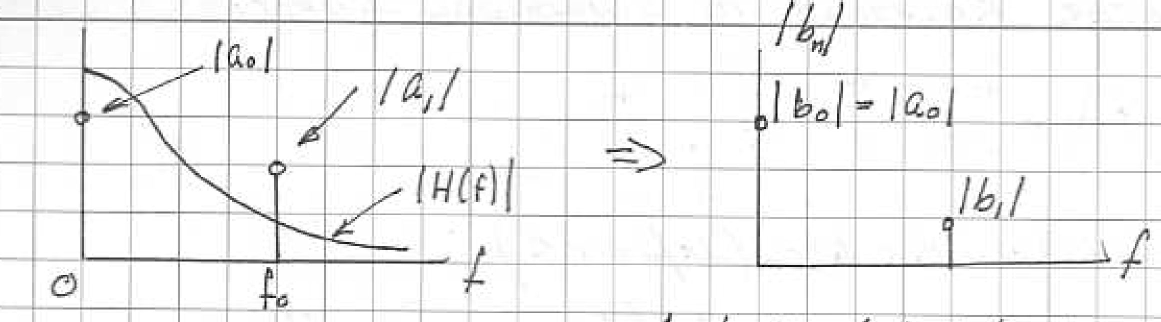
$$|b_1| = |a_1| \cdot |H(f_0)| = \frac{A}{2}\frac{1}{\sqrt{1 + (2\pi\tau f_0)^2}}$$

$$\phi = \arg a_1 + \arg H(f_0) = \theta - \arctan(2\pi\tau f_0)$$

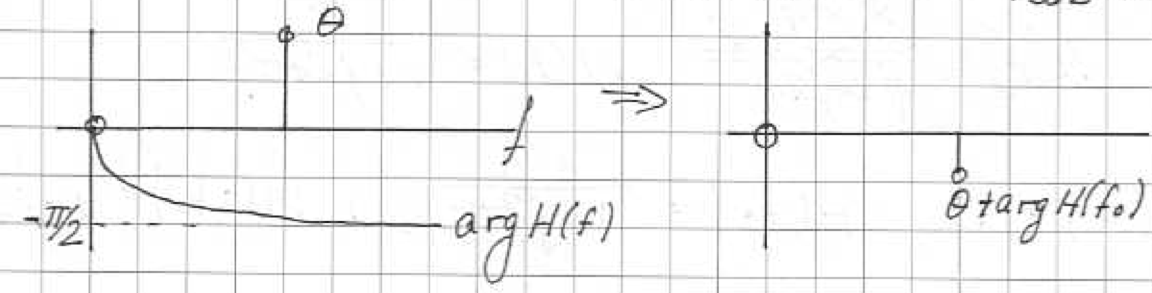
Likewise, $b_{-1} = b_1^*$

$$|b_{-1}| = |b_1|.$$

$$\arg b_{-1} = -\arg b_1$$



Actual plot depends on relation between f_{3dB} and f_0



Output signal:

$$v_{out}(t) = b_0 + b_1 e^{j2\pi f_0 t} + b_{-1} e^{-j2\pi f_0 t}$$

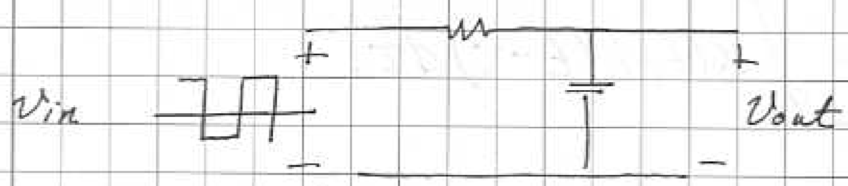
$$= b_0 + b_1 e^{j2\pi f_0 t} + \left(b_1 e^{j2\pi f_0 t}\right)^*$$

$$= b_0 + 2\mathrm{Re}\left(b_1 e^{j2\pi f_0 t}\right)$$

$$= b_0 + 2|b_1| \cos\left(2\pi f_0 t + \arg b_1\right)$$

$$= K + \frac{A}{\sqrt{1+(2\pi\tau f_0)^2}} \cos\left(2\pi f_0 t + \theta - \arctan 2\pi\tau f_0\right)$$

FILTER RESPONSE TO SQUARE WAVE INPUT



You could try to do the calculation in the frequency domain:

$$v_{in}(t) = \sum_{n=-\infty}^{\infty} a_n e^{j2\pi n f_0 t} \quad , \ a_n \text{ calculated on p. 19.}$$

Then $$v_{out}(t) = \sum_{n=-\infty}^{\infty} b_n e^{j2\pi n f_0 t}$$

$$b_n = a_n \cdot H(nf_0) = \begin{cases} K & , n = 0 \\ 0 & , n \text{ even} \\ \dfrac{2A}{j\pi n} \cdot \dfrac{1}{1 + j2\pi\tau n f_0} & , n \text{ odd} \end{cases}$$

Plot: $|a_n|$, arg a_n plot on p. 19, scaled by $|H(nf_0)|$ + phase shifted by arg $H(nf_0)$.

Then you must calculate $v_{out}(t)$ using

$$v_{out}(t) = \sum_{n=-N}^{N} b_n e^{j2\pi n f_0 t}$$

for large enough N. See $\boxed{\text{Prelab1.mcd}}$ for this method.

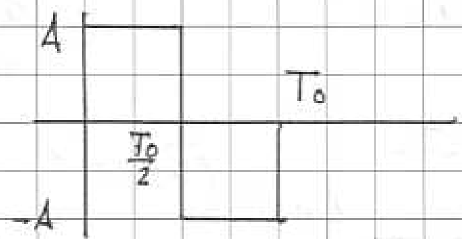
The output is easily calculated in the time domain by convolution: assume first $K = 0$.

$$y(t) = \int_{-\infty}^{\infty} x(t)\, h(t-\tau)\, d\tau.$$

The input in this case is periodic, and so the output also is periodic with same period.

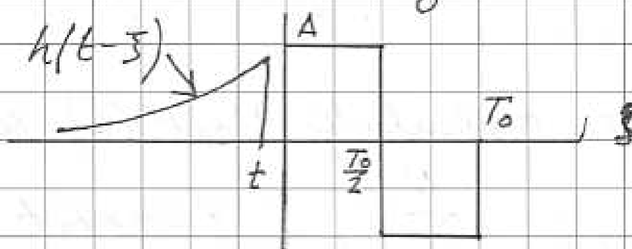
Calculate convolution of one period:

$v_{in}(t)$



$$h(t) = \frac{1}{\tau} e^{-t/\tau} u(t)$$

$$\text{One period of } V_{out}(t) = \int_0^t v_{in}(\xi)\, h(t-\xi)\, d\xi$$



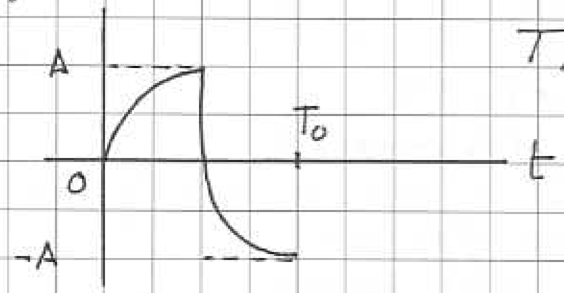
$\underline{t<0}$ $\quad V_{out}(t) = 0$

$\underline{0<t<T_0/2}$ $\quad V_{out}(t) = A\int_0^t \frac{1}{\tau} e^{-(t-\xi)/\tau} d\xi = A\left(1-e^{-t/\tau}\right)$

$\underline{T_0/2<t<T_0}$ $\quad V_{out}(t) = A\int_0^{T_0/2} \frac{1}{\tau} e^{-(t-\xi)/\tau} d\xi - A\int_{T_0/2}^{t} \frac{1}{\tau} e^{-(t-\xi)/\tau} d\xi$

$$= A\left(2e^{T_0/2\tau} - 1\right) e^{-t/\tau} - A$$

One period of $V_{out}(t)$:



This picture is for $T_0 \cong 5\tau$

For $K \neq 0$, we simply have a level shift by K in $V_{out}(t)$.

The convolution solution can also be done on the computer. See $\boxed{\text{Prelab1.mcd.}}$

<u>RC filter with $\tau = 10\mu s$</u> $(f_{3dB} = 15.92 kHz)$

$$\tau = RC = 10^{-5}$$

Must use standard R & C values; also must have R large enough so as not to load down the function generator.

Some possibilities:

R	C
$1k\Omega$	$10nF$
$10k\Omega$	$1nF$

Will need to adjust these in lab according to available R and C.