Cantilever Beam: Strain, Load, and Uncertainty

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Abstract—**A strain gage was mounted to a 6061 T6 aluminum cantilever beam and wired into a three-wire configuration Wheatstone quarter-bridge circuit that was connected to a strain gage amplifier. This instrumented cantilever beam setup was used to measure the weight of objects placed at the free end of the beam. The weight was calculated through a mechanics of materials approach and a calibration curve method. In the mechanics of materials, the weight is computed based on the measured strain and beam geometry. In the calibration curve method, bridge voltages readings for known calibration weights are used to produce a linear trendline. After comparison of the weight calculations and the propagation of uncertainty in both methods, it was determined that the mechanics of materials approach provided a smaller uncertainty. The mechanics of materials approach had a propagated uncertainty of 25.0 g and a statistical uncertainty of 1.6 g. The students average gulp size was found to be 26.8 g with a standard deviation of 4.4 g.**

Index Terms—**bending stress, calibration curve, cantilever beam, strain gage amplifier, three-wire quarter-bridge circuit**

I. INTRODUCTION

IN this report, a strain gage was mounted to an aluminum cantilever beam to approximate the weight of objects placed at the free end of the beam. The weight of the object was first determined using a mechanics of materials approach that related the strain experienced by the strain gage to the applied load at the free end of the beam. The weight of the object was also calculated using an experimentally determined linear trendline relating the amplified bridge voltage to the applied load [1].

This lab used a three-wire configuration Wheatstone quarter-bridge circuit with one active strain gage (seen in Fig. 1). In a three-wire configuration, the bridge voltage is measured through a lead wire with the same resistance, R_L, as the lead wires used to connect the gage to the Wheatstone bridge circuit [2].
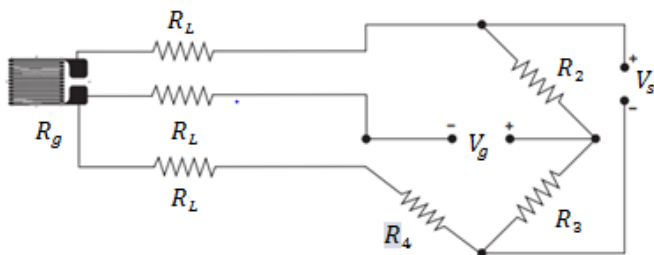

Fig. 1. This image was taken from [3]. It shows a three-wire configuration Wheatstone quarter-bridge circuit. The resistance of the lead wires is indicated by R_L. The resistance of the active strain gage is R_g. The differential voltage between the two voltage dividers is V_g. The source voltage, V_s, powers the Wheatstone bridge.

For the quarter-bridge circuit, the strain is related to the source voltage, V_s, the gage factor, G_f, and the change in bridge voltage ΔV_g, through the following relation [1]:

$$\epsilon = \frac{4\Delta V_g}{V_s G_f} \qquad (1).$$

The bridge voltage, V_g, is amplified with a gain of 220 using a Tacuna Systems Model EMBSGB200-M strain gage amplifier to give an amplified signal, V_{amp}. The amplified signal is also shifted by a trim potentiometer to 2.5V. The bridge voltage, V_g, is related to the measured V_{amp} by the following relation involving the gain, G [4, 5]:

$$V_g = \frac{V_{amp}}{G} \qquad (2).$$

The strain can be calculated more accurately using the amplifier because small deviations in bridge voltage, V_g, are converted into large deviations in V_{amp}. The data acquisition device (DAQ) can measure this amplified voltage with greater precision than the unamplified bridge voltage. Thus, the use of the strain gage amplifier reduces the uncertainty in strain measurements, despite the fact that the gain has its own uncertainty.

Applying topics from mechanics of materials, the applied force at the free end of a cantilever beam can related to the strain experienced by the gage. For a beam, the bending stress, σ, is related to the applied moment, M, the moment of inertia of the beam, I, and the distance from the neutral axis, c, through the equation:

$$\sigma = \frac{Mc}{I} \qquad (3).$$

In this lab, the moment, M, is equal to the applied load, W, multiplied by the moment arm from the point of application of the force to the strain gage, L. For the rectangular cross-section being considered, the distance to the centroid is half the height, h. In addition, the moment of inertia is given by the width, b, and the height, h, through the equation [6]:

$$I = \frac{bh^3}{12} \qquad (4).$$

Substituting these values into (3) gives:

$$W_{Mom} = k\sigma \frac{bh^2}{6L} \qquad (5).$$

The conversion factor $k = 101.97 \frac{g}{N}$ is introduced so that when all lengths are given in millimeters and the weight is given in grams, the stress is given in megapascals.

Now, applying Hooke's law to express stress, σ, as the product of strain, ϵ, and the modulus of elasticity, E:

$$\sigma = E\epsilon \qquad (6).$$

Plugging (6) into (5), the relation in (5) becomes [6]:

$$W_{MoM} = kE\epsilon \frac{bh^2}{6L} \qquad (7).$$

The conversion factor k from (5) is again used to give the weight in grams.

An alternative method, known as the calibration curve method, was also used to calculate the applied load at the free end of the cantilever beam. The calibration curve method relates the weight in grams, W, to the amplified voltage, V_{amp}, through the following linear function with experimentally determined constants, m and d:

$$W = mV_{amp} + d \qquad (8).$$

The relationship in (6) can be determined by placing known calibration weights at the end of the beam. Then, the measured value of V_{amp} was correlated to the value of the calibration weight. This method allows for weights to be measured using the cantilever beam without any knowledge of the mechanics of materials behind the problem. Because the value of the unknown applied load is computed differently than in the mechanics of materials approach, the calibration curve method has its own uncertainty in measurements [7].

II. PROCEDURE

Part 1: LabVIEW Program

A LabVIEW program (VI) was needed to calculate the weight of the applied load in grams given the beam geometry, the amplified bridge voltage, V_{amp}, and the source voltage, V_s. The VI took the measured V_{amp}, tared it to 2.5V, and divided it by the amplifier's gain of 220 to yield V_g. The VI then computed the strain using (1). This strain, along with the beam geometry, was plugged into (7) to calculate the weight in grams [1].

Part 2: Instrumented Cantilever Beam

A strain gage was applied to a 6061 T6 Aluminum bar according to the instructions in [8]. As seen in Fig. 2, the gage was centered along the width of the beam and places such that the center of the gage was approximately 8 inches from the free end [1].
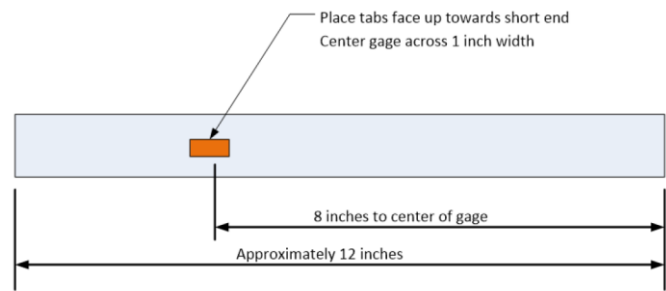

Fig. 2. This figure, taken from [1], shows the placement of the strain gage on the aluminum bar. The moment arm of the applied load is taken to be the distance from the center of the gage to the center of mass of object being weighed.

Marks were made on the aluminum beam to improve the repeatability of placement of the calibration weights and water bottle at the free end of the beam. The marks were made such that the water bottle is flush with the end of the beam. The calibration weights were centered on a line offset from the end of the beam by the radius of the water bottle (as seen in Fig. 3). The instrumented aluminum bar was then clamped to a fixture as a cantilever beam, with one fixed end and one free end [1].
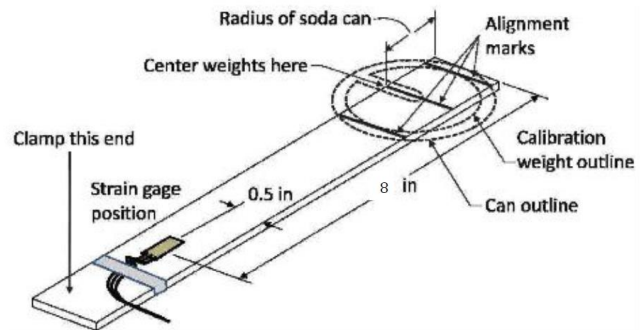

Fig. 3. The figure, taken from [1], shows the alignment marks used to ensure repeatable placement of the calibration weights and water bottle at the free end of the beam.

The strain gage was wired to the strain gage amplifier, within which is a quarter-bridge Wheatstone circuit. The DAQ was wired to the amplifier in order to acquire the voltage readings for V_s and V_{amp} on the windows -10V to 10V and -5V to 5V, respectively. The amplified voltage, V_{amp}, was set to approximately 2.5V using the trim potentiometer in the amplifier and then the VI was used to tare V_{amp} to 2.5V [1].

Part 3: Strain and Calibration Based Weights

A. *Weigh Calibration Weights*

In this part of the lab, calibration weights were used to determine the accuracy of the strain based weight readings and to acquire the experimental data needed to determine the constants in (6) for the calibration curve method.

The calibration weights were weighed on a commercial scale to determine their actual weight. Then, data was acquired as the 50g, 100g, 200g, 300g, and 350g calibration weights were placed on the free end of the instrumented cantilever beam [1].

B. Weigh Water Bottle

The weight of a water bottle was determined using the mechanics of materials strain based approached, as well as by applying the calibration curve method. First, the unopened water bottle was weighed 10 times on the cantilever beam. This weight data was used to provide a measure of the repeatability of placement of the water bottle on the free end of the beam and its effect on the weight measurements. To determine the accuracy of measurements, this weight was compared to the weight given by the commercial scale.

The strain measurement and calibration curve method weights were recorded for the water bottle after each gulp was drunk until the bottle was empty [1].

C. Weigh Object from Pocket

The beam was ultimately used to measure the weight of an object from the student's pocket. A Samsung Galaxy S3 cell phone was selected as the object to be weighed. The weights for this cell phone were recorded using the strain based calculation, the calibration curve method, and the commercial scale.

While the cell phone was on the beam, the student's thumb was placed on the strain gage. The variations in the data in response to this touching the gage were observed and recorded [1].

III. RESULTS

Cantilever Beam Geometry

The geometry of the cantilever beam was measured in order to calculated the strain-based weight using (7). The length from the center of the gage to the applied load, L, was measured with a ruler, the width, b, with a dial caliper, and the height, h, with a micrometer. The dimesions are shown in Table I.

TABLE I
CANTILEVER BEAM DIMENSIONS

Dimension	Measuring Device	Measurement	Converted (mm)
L	Ruler	17.29 cm	172.9
b	Dial caliper	1.003 in	25.476
h	Mircormeter	0.1275 in	3.2385

Calibration Weights

The values stamped on the calibration weights were confirmed with the commercial scale. Then, the average value of V_{amp} for each calibration weight was recorded in Table II. Using Microsoft Excel, the values for V_{amp} produced by the various calibration weights were used to produce a linear trendline of the form in (6). The trendline that relates V_{amp} to weight is given by:

$$W_{cal} \,[\text{g}] = (-3135 \,[\tfrac{\text{g}}{\text{V}}])V_{amp} + 7835 \,[\text{g}] \qquad (9).$$

Equation (9) makes up the calibration curve method. The calibration curve in (9) gives the calibration curve weight, W_{cal}, for any value of V_{amp}.

TABLE II
CALIBRATION WEIGHTS, AMPLIFIED VOLTAGE, AND STRAIN-BASED WEIGHT

Calibration Weight (g)	V_{amp} (V)	W_{cal} (g)	W_{MoM} (g)
50	2.483	50.8	48.8
100	2.468	97.8	92.0
200	2.434	204.4	191.9
300	2.404	298.5	278.8
350	2.388	348.6	324.4

Weigh Water Bottle

In this part of the lab, a water bottle was repeatedly placed on the free end of the cantilever beam. The measured weight of the water bottle was recorded for ten different instances. The average value of the calculated weight for each placement of the water bottle is shown in Table III.

TABLE III
STRAIN-BASED WEIGHT FOR REPEATED PLACEMENT OF WATER BOTTLE

Placement Number	W_{cal} (g)	W_{MoM} (g)
1	266.8	248.8
2	270.2	251.9
3	272.3	253.8
4	273.8	255.3
5	266.0	248.0
6	267.6	249.6
7	268.2	250.1
8	267.9	249.8
9	271.8	253.4
10	268.8	250.6

Weigh Water Bottle with Gulps

Next the weight of the water bottle was recorded after gulps were taken. As seen in Table IV, these weights were recorded using both the strain based method and the calibration curve method. The reported weight is the average weight for that instance of placing the bottle on the beam. This data was used to determine the average gulp size and the standard deviation of gulps sizes. The empty bottle, which is the reading for gulp number 10 in Table IV, weighed 10 g on the commercial scale.

TABLE IV
WEIGHT OF WATER BOTTLE WITH GULPS TAKEN

Number of Gulps	V_{amp} (V)	W_{cal} (g)	W_{MoM} (g)
1	2.413	269.1	250.9
2	2.422	240.6	224.6
3	2.434	205.2	192.0
4	2.443	176.1	165.0
5	2.451	150.2	141.1
6	2.461	118.5	111.7
7	2.471	88.0	83.6
8	2.482	53.5	51.8
9	2.491	26.6	26.9
10	2.497	8.4	10.0

Weigh Object From Pocket

The cantilever beam scale was finally used to weigh a cell phone. The cell phone weight was recorded in Table V using the strain based calculation, the calibration curve method, and the commercial scale (W_{scale}).

TABLE V
WEIGHT OF CELL PHONE

W_{scale} (g)	V_{amp} (V)	W_{cal} (g)	W_{MoM} (g)
184	2.442	180.8	169.3

While the cell phone was on the cantilever beam, the student placed his thumb on the strain gage. Placing his thumb on the strain gage caused a smaller weight reading for both the strain-based weight reading (shown in Fig. 4) and the calibration curve weight (shown in Fig. 5). The changes in the weight readings can be attributed to the change in V_{amp} seen in Fig. 6.
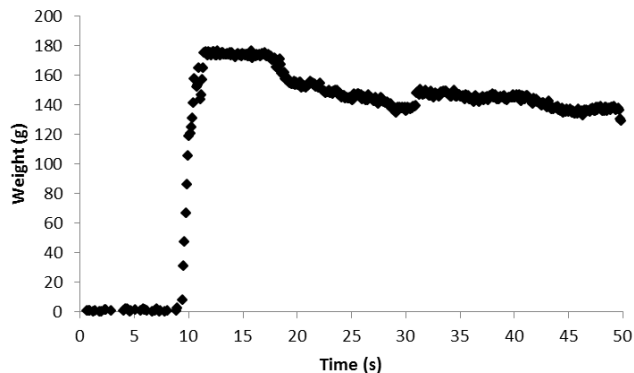

Fig. 4. The figure shows the strain-based weight, W_{MoM}, over time. The cell phone was placed on the cantilever beam at about 12 s. At around 17 s, the student's thumb was placed on the strain gage. The weight reading is seen to decrease to a lower value while the thumb was placed on the gage.
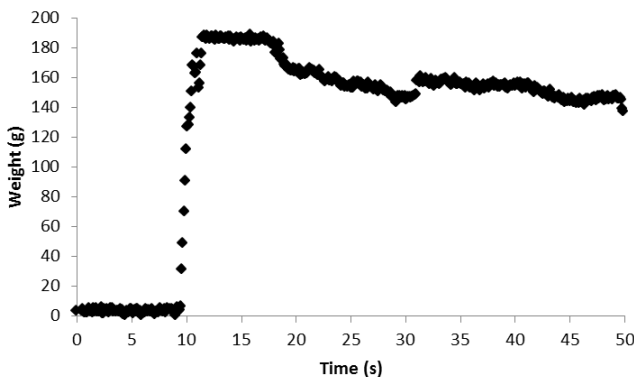

Fig. 5. The figure shows the calibration curve weight, W_{cal}, over time. The data in the graph was taken at the same time as the data in Fig. 4. Again, the weight reading is seen to decrease to a lower value while the thumb was placed on the gage.
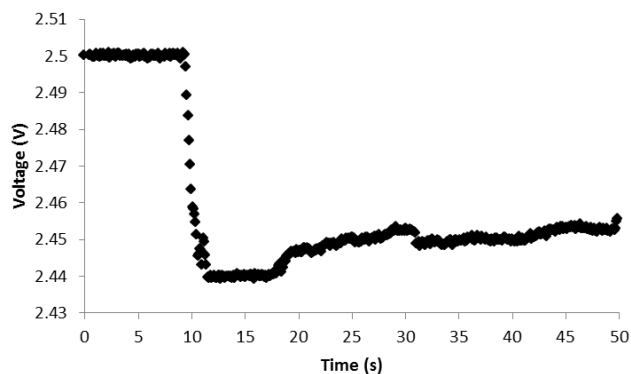

Fig. 6. The figure shows the amplified bridge voltage, V_{amp}, over time. The data in the graph was taken at the same time as the data in Fig. 4. The amplified bridge voltage was seen to increase when the thumb was placed on the gage.

IV. DISCUSSION

Maximum Weight Measureable with Beam

Knowing the yield stress for the material and the dimensions of the beam, the maximum weight that can be measured with the scale without plastically deforming the cantilever beam can be calculated using (5). The yield stress for 6061 T6 aluminum is taken as $\sigma_{yield} = 240$ MPa [6]. The beam dimensions (given in Table I) were measured to be $L = 172.9$ mm, $b = 25.48$ mm, and $h = 3.2385$ mm. Plugging these values into (5) yields:

$$W_{MoM} = (101.97)(240)\frac{(25.48)(3.2385)^2}{6(172.9)} = 6300 \text{ g} \qquad (10).$$

Thus, the maximum weight that can be measured is 6.3 kg.

Weight of Full Bottle

The full water bottle was weighed using both the mechanics of materials approach and the calibration curve method in the first reading in table IV. When weighed with the commercial scale, the full water bottle weighed 269 g. The weight of the water bottle using the mechanics of materials approach was recorded as 250.9 g. Using the calibration curve method, the full water bottle was found to weigh 269.1 g. The calibration curve method produced a more accurate weight reading in this instance, but an analysis of the uncertainty of both weight measurement methods must be conducted to determine which approach is consistently more precise.

Uncertainty in Weight Measurements

The detailed calculation for the uncertainty of both weight measurements can be found in the appendix. In short, the mechanics of materials approach uncertainty was determined using the Root Sum Squares (RSS) method for propagation of uncertainty through the steps in the calculation [9]. The uncertainty for the calibration curve method was determined using a Monte Carlo simulation combined with the RSS method. The Monte Carlo method was used to determine the uncertainty in the slope of the calibration curve linear trendline given the possible variation in the calibration

weights and the amplified voltage. Then the uncertainty in the slope was propagated through the linear relation in (9) in order to determine the uncertainty in the calibration curve method weight [10]. The uncertainty analysis in the appendix determined that the uncertainty in the mechanics of materials weight was, $U_{W_{MoM}} = 25.04$ g, and the uncertainty in the calibration curve weight was, $U_{W_{cal}} = 385.8$ g. Evidently, the mechanics of materials approach has a much smaller uncertainty than the calibration curve method. This large uncertainty for the calibration curve method is due to the large slope of the linear trendline. This large slope means that small variations due to uncertainty in V_{amp} result in large deviations in W_{cal}. In addition, the calibration curve method relies on a single measurement of V_{amp} to calculate the weight for a given reading. Conversely, the mechanics of materials approach utilizes an understanding of the physical set-up of the cantilever beam in order to relate the beam geometry, material properties, and the measured strain to the weight. From this point on, the mechanics of materials approach calculated weights will be used due to its advantage of having a smaller uncertainty.

Repeated Water Bottle Weight Measurements

The weight of the full water bottle was measured ten times and recorded in Table III using both the strain-based calculation and the calibration curve method. However, the strain-based mechanics of materials approach has since been determined to have the smallest accuracy of the two methods. Using the commercial scale, the water bottled was found to weigh 269 g. The mean value of the strain based weight, W_{MoM}, was 251.1 g with a standard deviation of 2.3 g.

The statistical uncertainty of the weight of the unopened water bottle for the data set in Table III can be determined using a t-distribution for a 95% confidence interval [9]. With ten samples ($N = 10$), then the t-value is given by $t = 2.201$. Then the random uncertainty, P_x, can be calculated as a function of the t-value, the sample standard deviation, S_x, and the sample size, N, as follows:

$$P_x = t \frac{S_x}{\sqrt{N}} = 2.201 \left(\frac{2.3}{\sqrt{10}} \right) = 1.601 \text{ g} \qquad (11).$$

Repeatability of placement of the can is not an issue. The standard deviation is small enough such that uncertainty due to other causes, such as the measurement of voltages, dominates the uncertainty. Using the guiding marks on the beam, the water bottle was able to be placed consistently enough such that the location of placement of the bottle does not play a significant role in the uncertainty of the overall measurements.

Mean and Standard Deviation for Gulp Size

The weight of the water bottle was recorded in Table IV after successive gulps were taken. The mechanics of materials approach weights will be used to determine the mean and standard deviation of the student's gulp size. The mean gulp size was 26.8 g with a standard deviation of 4.4 g.

Variation in Empty Water Bottle Weight

The empty water bottle was weighed using both the cantilever beam and the commercial scale. The empty water bottle weighed 10 g on the commercial scale. The empty water bottle was also recorded to weight 10 g using the mechanics of materials calculation on the cantilever beam.

The major variation in between the commercial scale and the beam scale is that the beam scale could not be effectively tared to 0 g. The beam scale always read some value of weight despite attempts to tare it to zero. This is a result of the random error in the beam scale measurement. Conversely, when the commercial scale was zeroed, it consistently read 0 g. The commercial scale, which had a printed sensitivity of 1 g, and hence an uncertainty of 0.5 g, has a much smaller uncertainty than the beam scale. All in all, the commercial scale gave much more stable weight readings than the beam scale, which showed fluctuations in the measured weight even with no weight applied.

Improvements to Reduce Uncertainty

There are a variety of sources of uncertainty for the mechanics of materials approach for calculating the weight. The uncertainty in weight is propagated from the uncertainty in voltage measurement, amplifier gain, youngs modulus, and beam dimensions. The uncertainty in measurement of bridge voltage V_g was mitigated through use of an amplifier. The amplifier amplified the bridge voltage to a level that can be easily read with the DAQ. Although this introduces an uncertainty in gain of the amplifier, the overall result was to reduce uncertainty.

The height of the cross-section of the beam contributed greatly to the uncertainty in the weight. This indicates the need for great accuracy when measuring the height, h. Although a micrometer was already used to reduce uncertainty in this measurement, a further reduction in uncertainty would reduce the overall uncertainty in weight. Another major contributor to uncertainty in the weight was the uncertainty in the elastic modulus of aluminum. If a more accurate value for the elastic modulus was used, uncertainty in the weight measurement would be reduced. Lastly, the use of a more elastic material would reduce the uncertainty of measurements. If the material whose strain was measured to calculate weight showed greater deformation under the applied load, the changes in measured voltage would be greater and the young's modulus would be smaller.

Effect of Weight of Beam on Calibration

When calculating the weight using the mechanics of materials approach, the weight of the beam was not taken into account. In reality, the cantilever beam will experience an applied load due to its own weight. This weight of the beam will produce a bending strain measurable by the strain gage. However, due to the taring of V_{amp}, this effect can be taken out of the calculation by setting the weight equal to zero when no mass is placed on the free end.

For the calibration curve method, the weight of the beam affects the data to which the linear trendline was fit. The

voltage readings were taken to correspond to an applied load equal to the value of the calibration weight placed on the free end. However, due to the weight of the beam itself, the actual applied load was due to the weight of the beam in addition to the calibration weight. The calibration curve method would experience an error due to not including the weight of the beam itself.

Effect of Placing Thumb on Strain Gage

As seen in Fig. 4 and Fig. 5, the calculated weight for both methods was reduced when the student's thumb was placed on the strain gage. This reduction in the weight reading is attributed to an increase in V_{amp}, as seen in Fig. 6, when the thumb was placed on the gage. Because (9) is a decreasing linear function, an increase in V_{amp} produces a decrease in W_{cal}. As seen in Fig. 6, the increase in V_{amp} brings the amplified voltage value closer to its initial, unstrained voltage. This means there is a decrease in ΔV_g, which, according to (1), translates into a decrease in strain. As shown by (7), this decrease in strain leads to a corresponding decrease in W_{MoM}. Thus, it is clear that the increase in V_{amp} when the student's thumb was placed on the gage results in a decrease in weight calculated by both methods. The increase in V_{amp} is due to the increase in temperature of the strain gage from the body heat transferred to the gage.

V. Conclusion

Instrumented Cantilever Beam

In this lab, an aluminum cantilever beam with a strain gage was used to measure the weight of objects placed at the free end. Measurements of the geometry of the beam were taken using a ruler, a dial caliper, and a micrometer. The micrometer, which has the smallest uncertainty of the three measuring devices listed, was used to measure the height, h. In this way, the micrometer provided the smallest uncertainty for the dimension that had the largest impact on the uncertainty of the weight measurement. The voltage readings from the Wheatstone bridge were amplified in order to reduce the uncertainty present in measuring the bridge voltage. Although this introduced an uncertainty in gain for the amplifier, the overall effect was a reduction of uncertainty compared to no amplifier.

Uncertainty of Both Weight Calculation Methods

The weight measured using the aluminum cantilever beam was calculated using two different methods: the mechanics of materials approach and the calibration curve method. The mechanics of materials approach utilizes an understanding of the mechanics of the cantilever beam to relate the strain to the applied load. The calibration curve method relies on fitting a linear trendline to the calibration data in order to relate V_{amp} to the applied load. This is done by taking readings of known calibration weights. The mechanics of materials approach was found to have a smaller uncertainty than the calibration curve method.

Mechanics of Materials Method Weight Measurements

Once it was determined that the mechanics of materials approach produced the smallest uncertainty, it was used to calculate the weights in the remainder of the lab. The repeatability of placement of the unknown weights was testing by placing the full water bottle on the free end of the beam ten times. Using a t-distribution with a 95% confidence interval, the statistical uncertainty of the weight measurement was determined to be about 1.6 g. Thus, the repeatability of placement the weights on the free end is not an issue. Uncertainty using this method was attributed to the uncertainty in voltage measurement, beam geometry, and aluminum elastic modulus.

Appendix

TABLE VI
Measurement Uncertainty

Quantity	Symbol	Value	Uncertainty
Gain	G	220	± 0.22
Amp Voltage	V_{amp}	2.5 V	$\pm 0.00383\ V$ [1]
Source Voltage	V_s	5 V	$\pm 0.00447\ V$ [1]
Width	b	25.476 mm	± 0.0127 mm [2]
Height	h	3.2385 mm	± 0.00127 mm [2]
Length	L	172.9 mm	± 0.5 mm [2]
Strain	ϵ	0.000149	Equation (20)
Stress	σ	11.17 MPa	Equation (24)
Elastic Modulus	E	75000 MPa	± 5000 MPa
Gage Factor	G_f	2.1	± 0.0105

[1] Obtained from [11]. [2] Obtained from device.

First, find the propagation of uncertainty in calculation of the bridge voltage, V_g. As seen in (2) the bridge voltage is a function of V_{amp} and G. Thus, applying the Root Sum Squares (RSS) method of uncertainty propagation, [9]:

$$U_{V_g} = \sqrt{\left(\frac{\partial V_g}{\partial V_{amp}} U_{V_{amp}}\right)^2 + \left(\frac{\partial V_g}{\partial G} U_G\right)^2} \qquad (12).$$

Where the values for $U_{V_{amp}}$ and U_G are taken from Table VI, and the partial derivatives are defined as follows:

$$\frac{\partial V_g}{\partial V_{amp}} = \frac{1}{G} = 0.0045 \qquad (13).$$

$$\frac{\partial V_g}{\partial G} = -\frac{V_{amp}}{G^2} = -5.165 \text{ x } 10^{-5} \qquad (14).$$

Plugging into (12), yields

$$U_{V_g} = 2.064 \text{ x } 10^{-5} \text{ V} \qquad (15).$$

Now, find the propagation of uncertainty in the calculation of the strain from (1). The strain, ϵ, is a function of V_g, V_s, and G_f. Thus, applying the RSS method of uncertainty propagation [2],

$$U_\epsilon = \sqrt{\left(\frac{\partial \epsilon}{\partial \Delta V_g} U_{\Delta V_g}\right)^2 + \left(\frac{\partial \epsilon}{\partial V_s} U_{V_s}\right)^2 + \left(\frac{\partial \epsilon}{\partial G_f} U_{G_f}\right)^2} \qquad (16).$$

Where the values for U_{V_s} and U_{G_f} are taken from Table V, $U_{\Delta V_g} = U_{V_g}$ is gotten from (15), and the partial derivatives are defined as follows,

$$\frac{\partial \epsilon}{\partial \Delta V_g} = \frac{4}{V_s G_f} = 0.38095 \qquad (17).$$

$$\frac{\partial \epsilon}{\partial V_s} = -\frac{4\Delta V_g}{V_s^2 G_f} = -3.8095 \text{ x } 10^{-5} \qquad (18).$$

$$\frac{\partial \epsilon}{\partial G_f} = -\frac{4\Delta V_g}{V_s G_f^2} = -9.0703 \text{ x } 10^{-5} \qquad (19).$$

Plugging into (16), yields

$$U_\epsilon = 7.922 \text{ x } 10^{-6} \qquad (20).$$

Next, find the propagation of uncertainty in the stress calculated in (6). The stress, σ, is a function of E, and ϵ. Thus, applying the RSS method of uncertainty propagation [2],

$$U_\sigma = \sqrt{\left(\frac{\partial \sigma}{\partial E} U_E\right)^2 + \left(\frac{\partial \sigma}{\partial \epsilon} U_\epsilon\right)^2} \qquad (21).$$

Where the values for U_E is taken from Table VI, and U_ϵ is given in (20). Then, the partial derivatives are defined as follows:

$$\frac{\partial \sigma}{\partial E} = \epsilon = 0.000149 \qquad (22).$$

$$\frac{\partial \sigma}{\partial \epsilon} = E = 75000 \qquad (23).$$

Plugging into (21), yields

$$U_\sigma = 0.9529 \text{ MPa} \qquad (24).$$

Now, find the propagation of uncertainty in the calculation of the weight from (5). The weight, W, is a function of σ, L, b, and h. Thus, applying the RSS method of uncertainty propagation [2],

$$U_W = \sqrt{\left(\frac{\partial \sigma}{\partial \sigma} U_\sigma\right)^2 + \left(\frac{\partial \sigma}{\partial L} U_L\right)^2 + \left(\frac{\partial \sigma}{\partial b} U_b\right)^2 + \left(\frac{\partial \sigma}{\partial h} U_h\right)^2} \qquad (25).$$

Where the value of U_σ is taken from (24), the values for U_L, U_b, and U_h are taken from Table VI, and the partial derivatives are computed as follows,

$$\frac{\partial W}{\partial \sigma} = k\frac{bh^2}{6L} = 26.263 \qquad (26).$$

$$\frac{\partial W}{\partial L} = -k\sigma\frac{bh^2}{6L^2} = -1.6975 \qquad (27).$$

$$\frac{\partial W}{\partial b} = k\sigma\frac{h^2}{6L} = 11.5202 \qquad (28).$$

$$\frac{\partial W}{\partial h} = k\sigma\frac{bh}{3L} = 181.2503 \qquad (29).$$

Plugging into (25), yields

$$U_{W_{MoM}} = 25.04 \text{ g} \qquad (30).$$

The above result is for the uncertainty in the strain-based, mechanics of materials approach weight calculation. What follows is the discussion of uncertainty in the calibration curve calculated weight, W_{cal}.

The uncertainty for the calibration curve method was derived using a Monte Carlo simulation. The spreadsheet provided in (10) was used to derive the uncertainty. In order to run this simulation, it was necessary to determine the values and uncertainty for the calibration weights and voltages, V_{amp}, used to generate the calibration curve.

The uncertainty in V_{amp} is given by the sum of the uncertainty of the amplifier and the uncertainty of the channel on the DAQ used to measure the voltage. Thus, the uncertainty is given by,

$$U_{V_{amp}} = U_{amp} + U_{DAQ} = \frac{0.22V_{amp}}{G} + U_{DAQ} \qquad (31).$$

The values for the uncertainty using (31) are given in Table VII. The value for U_{DAQ} is taken from the uncertainty data for channels on the DAQ taken in [11].

TABLE VII
CALIBRATION CURVE UNCERTAINTY

Calibration Weight	V_{amp}(V)	U_{amp} (V)	U_{DAQ} (V)	$U_{V_{amp}}$(V)
50	2.483	±0.002483	±0.00383	±0.006313
100	2.468	±0.002468	±0.00383	±0.006298
200	2.434	±0.002434	±0.00383	±0.006264
300	2.404	±0.002404	±0.00383	±0.006234
350	2.388	±0.002388	±0.00383	±0.006218

Using the above calculated values for $U_{V_{amp}}$, and the uncertainty of the commercial scale which was marked as ±0.5 g, the Monte Carlo simulation was run. In the simulation, the values of V_{amp} for each calibration weight

were allowed to vary randomly within the uncertainty range $U_{V_{amp}}$. Simultaneously, the calibration weights were allowed to randomly vary within their uncertainty range. In this way, random values for V_{amp} and each calibration weight were generating to provide 2000 simulated data runs. The standard deviation and the mean of the calibration curve trendlines for each of these simulations was used to determine the uncertainty. In this way, the uncertainty in the slope, m, was determined to be given as follows:

$$U_m = 141.77 \ \frac{\text{g}}{\text{V}} \qquad (32).$$

To determine the uncertainty in the mass predicted by the calibration curve method, the RSS method must be applied to (9) as follows:

$$U_W = \sqrt{\left(\frac{\partial W}{\partial m} U_m\right)^2 + \left(\frac{\partial W}{\partial V_{amp}} U_{V_{amp}}\right)^2} \qquad (33).$$

Where the values for $U_{V_{amp}}$ is the average of its values in Table VII, and U_m is given in (32). Then, the partial derivatives are defined as follows:

$$\frac{\partial W}{\partial m} = V_{amp} = 2.4354 \qquad (34).$$

$$\frac{\partial W}{\partial V_{amp}} = m = -3135 \qquad (35).$$

Plugging into (33), yields

$$U_{W_{cal}} = 385.8 \ \text{g} \qquad (36).$$

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