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A NUMERICAL STUDY OF UNCERTAINTY IN STABILITY AND SURFACE LOCATION ERROR IN HIGH-SPEED MILLING

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ABSTRACT

High-speed milling offers an efficient tool for developing cost effective manufacturing processes with acceptable dimensional accuracy. Realization of these benefits depends on an appropriate selection of preferred operating conditions. In a previous study, optimization was used to find these conditions for two objectives: material removal rate (*MRR*) and surface location error (*SLE*), with a Pareto front or tradeoff curve found for the two competing objectives. However, confidence in the optimization results depends on the uncertainty in the input parameters to the milling model (time finite element analysis was applied here for simultaneous prediction of stability and surface location error). In this paper the uncertainty of these input parameters such as cutting force coefficients, tool modal parameters, and cutting parameters is evaluated. The sensitivity of the maximum stable axial depth, b_{lim}, to each input parameter at each spindle speed is determined. This enables identification of parameters with high contribution to stability lobe uncertainty. Two methods are used to calculate uncertainty: 1) Monte Carlo simulation; and 2) numerical derivatives of the system eigenvalues. Once the uncertainty in axial depth is calculated, its effect is observed in the *MRR* and *SLE* uncertainties. This allows robust optimization that takes into consideration both performance and uncertainty.

INTRODUCTION

In manufacturing, devising cost-effective processes is a constant pursuit. High-speed milling can substantially reduce machining time; however, this can come at the sacrifice of product surface quality and dimensional accuracy. Milling models are used in predicting part quality (stable cutting process) and dimensional accuracy (surface location error) for a specific set of cutting parameters. Selecting optimum cutting parameters that maximize productivity and accuracy without sacrificing quality is highly desirable. In a previous study [1] optimization was used to find these conditions for two objectives: material removal rate (*MRR*) and surface location error [2-7] (*SLE*), with a Pareto front, or tradeoff curve, found for the two competing objectives. Although the milling model used in the optimization algorithm is deterministic (time finite element analysis), uncertainties in the input parameters to the model limit the confidence in these optimum predictions. These input parameters include cutting force coefficients (material and process dependent), tool modal parameters, and cutting parameters. By accounting for these uncertainties it is possible to arrive at a robust optimum operating condition.

In previous studies [8-10], uncertainty in milling process is handled from a control perspective. The cutting forces uncertainty is accommodated using a control system. The force controller is designed to compensate for known process effects and accounts for the force-feed nonlinearity inherent in metal cutting operations. In this study, the uncertainties in the milling model are estimated using sensitivity analysis and Monte Carlo simulation. This would enable selection of preferred design that accounts for inherent uncertainty in the model *a priori*.

The paper proceeds first with a description of the milling model considered and next a discussion of stability lobes and surface location error analysis with regard to their numerical accuracy. Sensitivity analysis is discussed in the next section. Then the next section presents case studies of numerical accuracy of sensitivity of b_{lim} and *SLE*. This will enable us to carry out the stability lobe and surface location error sensitivity analysis in the next two sections. Sensitivity is used to determine the effect of input parameters on the maximum stable axial depth, b_{lim}, and surface location error, *SLE*. This enables identification of parameters with high contribution to stability enhancement and *SLE* reduction. In the next section, the uncertainty in limiting axial depth, b_{lim}, and *SLE* predictions are calculated using two methods 1) Monte Carlo simulation; and 2) use of numerical derivatives of the system characteristic multipliers to determine sensitivities. The uncertainty in axial depth effects a reduction in the *MRR*, and the *SLE* uncertainty provides bounds on its expected value. This allows robust optimization that takes into consideration both performance and uncertainty.

NOMENCLATURE

b axial depth parameter
b_{lim} maximum stable axial depth
K_x modal stiffness in *x*-direction
M_x modal mass in *x*-direction
C_x modal damping in *x*-direction
K_y modal stiffness in *y*-direction
M_y modal mass in *y*-direction
C_y modal damping in *y*-direction
K_t tangential cutting force coefficient (N/m^2)
K_n normal cutting force coefficient (N/m^2)
K_{te} edge tangential cutting force coefficient (N/m)
K_{ne} edge normal cutting foce coefficient (N/m)
$\vec{\lambda}$ system characteristic multipliers
h step size used to estimate numerical derivative
X_i i input parameter
u_c combined standard uncertainty
Ω spindle speed (rpm)
ζ damping factor
ε absolute error limit

MILLING MODEL

A schematic of a two degree-of-freedom milling tool is shown in Figure 1. Tool/work-piece dynamics and cutting forces are used to formulate the governing delay differential equation for the system. Solution of the delay differential equation is found using time finite element analysis (TFEA) [12-14]. This method provides the means for predicting the milling process stability and quality (*SLE*). However, the uncertainty in the input parameters to the solution method places an uncertainty on the stability and *SLE* prediction. These parameters are divided into two groups; 1) uncertainty from lack of knowledge of the tool modal matrices, K, C and M, and the cutting force coefficients (mechanistic force model); and 2) uncertainty due to variability (aleatory uncertainty) in other machining parameters, such as spindle speed, chip load and radial depth. To estimate the parameters in the former, modal testing is used to measure the dynamic parameters while cutting tests are completed to estimate the cutting force coefficients. In the modal parameter estimation peak amplitude method is used to fit a theoretical transfer function to the milling model measured transfer function. In this method [11], the peak of the absolute value of the transfer function corresponds to the natural frequency. From which, the half power frequencies are used to estimate the damping ratio. Table 1 lists the mean modal values for 25.4 mm diameter endmill having a 12° helix angle with 114 mm overhang length and the corresponding cutting force coefficients for 6061 aluminum. These parameters will be used in the simulations. The milling process parameters are also listed in the table for a down milling case.

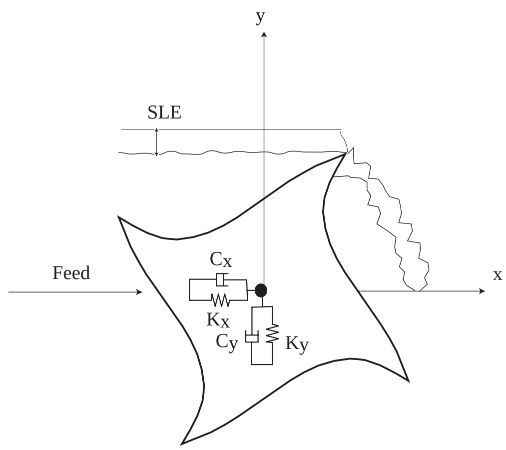

Figure 1 Schematic of 2-D milling model. The surface location error (SLE) due to phasing between tool force and displacement is also shown.

	M (kg)	K (N/m x10^6)	C (N.s/m)	ζ
x	0.44	4.45	83	0.030
y	0.44	3.55	90.9	0.036
K_t (N/m^2 x10^6)	K_n (N/m^2 x10^6)	K_{ne} (N/m x10^3)	K_{te} (N/m x10^3)	
600	180	6	12	
Tool diameter (mm)	radial depth,a (mm)	chip load, c (mm/tooth)	N	
25.4	0.508	0.1	1	

Table 1 Cutting force coefficients, modal parameters and cutting conditions of milling process.

STABILITY AND SURFACE LOCATION ERROR ANALYSIS

The stability lobes are used to represent the stable space of axial depth (b) and spindle speed (Ω) of a milling process. In TFEA [12-14], a discrete map is used to match the tool-free vibration while out of the cut with the tool vibration in the cut. The system characteristic multipliers ($\vec{\lambda}$) of the map provide the stable cutting zone where $\max|\vec{\lambda}|$ is less than one.

TFEA provides a field of $\vec{\lambda}$ in the design space of b and Ω. The limit of stability, b_{lim} can be found using root finding numerical techniques. Here we use the bi-section root finding method. The convergence criterion of the bi-section method should account for the amplification of numerical noise induced by sensitivity estimation. It should be noted that the number of elements affects the accuracy of the estimation.

For the case of calculation of *SLE* in TFEA, the numerical noise is only due to the number of elements. In this section we will discuss the effect of both the convergence criterion and the number of elements on the sensitivity estimation of b_{lim} and *SLE*.

Bi-section method convergence criterion

In order to find the axial depth limit, b_{lim}, at corresponding input parameters, the bi-section method is used in the TFEA algorithm to solve for b_{lim} at which the maximum characteristic multiplier is equal to one (stability limit). An absolute error is used as a criterion for convergence,

$$\frac{b_i - b_{i-1}}{b_i} \le \varepsilon \qquad (1)$$

where ε corresponds to the error tolerance and b_i is the root corresponding to $\max\left|\vec{\lambda}\right| = 1$ at iteration *i*. Although a relatively large value of ε can be adequate for the calculation of the stability lobes, a tighter limit is needed to calculate the sensitivities. This is attributed to amplification of numerical noise in the derivative calculation. This comparison is made in the case studies section.

Number of Elements

The accuracy of TFEA prediction of stability and *SLE* is highly dependent on the number of elements used. The effect of the number of elements is even more apparent when calculating the sensitivity of the prediction, where an even higher number of elements is needed to eliminate numerical noise from the sensitivity calculation.

SENSITIVITY ANALYSIS

The sensitivity of axial depth limit to input parameters $\left(\partial b_{\lim} / \partial X_i\right)$ is cumbersome to compute analytically using the TFEA method; therefore, a numerical derivative is used by implementing an infinitesimal perturbation.

Factors which affect accurate calculation of sensitivity to inputs include: 1) central difference truncation error; and 2) step size selection. Therefore, a balance needs to be used in calculating the sensitivity that does provide a stable estimation of the sensitivity while maintaining computational efficiency. In the following, we describe these factors and their consideration in the calculation of stability and *SLE* sensitivities.

Truncation Error

The central difference method is used in the sensitivity calculation. The formula for this method is,

$$\frac{\partial b}{\partial X_i} = \frac{b_1 - b_{-1}}{2h} + O\left(h^2\right) \qquad (2).$$

where h denotes the step size in input parameter X_i, $b_1 = b\left(X_i + h\right)$, $b_{-1} = b\left(X_i - h\right)$ and $O(h^2)$ is the 2[nd] order truncation error. A higher order formula with 4[th] order truncation error $O(h^4)$ can also be used. However, as shown in Eq. (3), it is two times more computationally expensive than Eq. (2),

$$\frac{\partial b}{\partial X_i} = \frac{-b_2 + 8b_1 - 8b_{-1} + b_{-2}}{12h} + O\left(h^4\right) \qquad (3)$$

In order to help decide whether the higher truncation error formula need to be applied (Eq. (4)), the sensitivity of b_{lim} with respect to modal stiffness K_x $\left(\partial b_{\lim} / \partial K_x\right)$ is calculated as a function of step size *h*. This comparison is made in the case studies section.

Step Size

The step size, *h*, in Eqs. ((2) and (3)) should be chosen carefully. This is especially important when there is numerical noise in the calculated b_{lim} due to the convergence criterion (Eq. (1)). The step size should be large enough to be out of the numerical noise range, however, not so large to incur large truncation errors. The case studies section will illustrate this idea.

CASE STUDIES

In this section numerical estimations of the sensitivity are made based on different variations of convergence criterion, number of elements, sensitivity analysis formula (Eq. (2) and Eq. (3)), and step size. The comparisons are made for a 10 (krpm) spindle speed, 10 elements and an $\varepsilon = 3\times10^{-4}$ unless noted otherwise. The logarithmic derivative can be used in making these comparisons. To evaluate percentage of change in an output (axial depth, *b*) due to a percentage change in the input, X_i. It is expressed as,

$$\frac{\partial \ln(b)}{\partial \ln(X_i)} = \frac{X_i}{b}\frac{\partial b}{\partial X_i} \qquad (4)$$

To illustrate the effect of convergence criterion, In Figure 2 the logarithmic derivative of b_{lim} with respect to M_x (the X direction modal mass) is calculated for two error limits as a function of step size percentage $\left(\%h = \Delta X_i / X_i \times 100\right)$. It can be seen that a tighter error limit nearly eliminates the numerical noise in the derivative calculation.

The effect of number of elements on *SLE* sensitivity is illustrated in Figure 3, where the *SLE* sensitivity with respect to K_x is calculated. A higher number of elements provide a larger stable region of sensitivity. Figure 4 compares the noise in the second and fourth order sensitivity calculation. A finite step size percentage $\left(\%h = \Delta K_x / K_x \times 100\right)$ is needed to reach a stable value of the derivative for both formulas. It can be seen that Eq. (3) gives a wider range of step sizes at which the sensitivity calculation is stable. However, the improved stability range, or reduction in numerical noise, is not significant to sacrifice computational efficiency for its usage.

The importance of step size selection is evident from Figure 5, which shows the logarithmic derivative of critical axial depth with respect to input parameters versus step size percentage. The figure also indicates the relative sensitivity of critical axial depth to each input parameter, spindle speed having the largest effect followed by modal mass and stiffness.

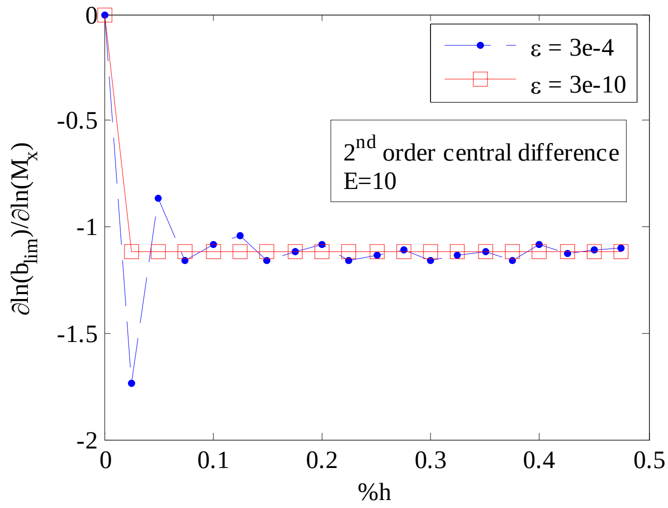

Figure 2 The effect of error limit in the bisection method on numerical noise in the sensitivity calculation.

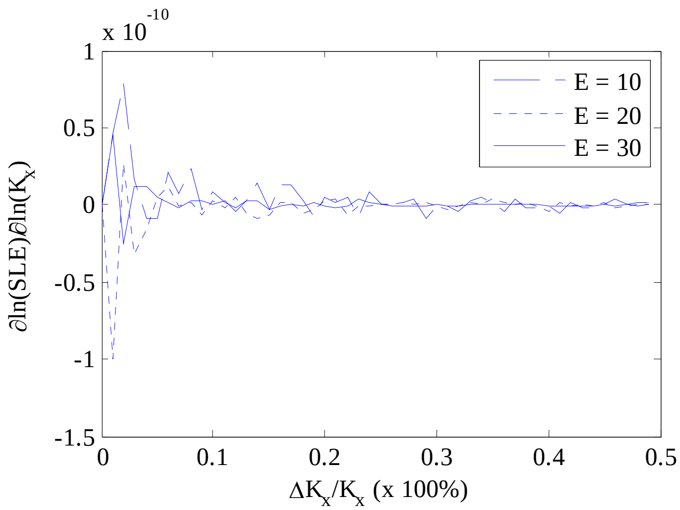

Figure 3 Sensitivity of *SLE* with respect to K_x. The higher number of elements, E, provides more stable sensitivity estimation. The second order finite difference formula is used here.

From Figures 4 and 5 it can be seen that an h=0.2% provides an accurate sensitivity estimation. To verify that a step size of 0.2%, convergence limit $\varepsilon = 3\times10^{-4}$, E=10, and the 2nd order finite difference approximation give correct calculation of sensitivity, the variations of b_{lim} to modal parameters and cutting coefficients are plotted in Figures 6 and 7 respectively. Also, the slope predicted using Eq. (2) is superimposed on the same plot .The figures verify that the sensitivity calculations approximate the slope accurately.

STABILITY SENSITIVITY ANALYSIS

In this section calculations of the sensitivity of axial depth limit b_{lim} to input parameters are provided. The parameters used

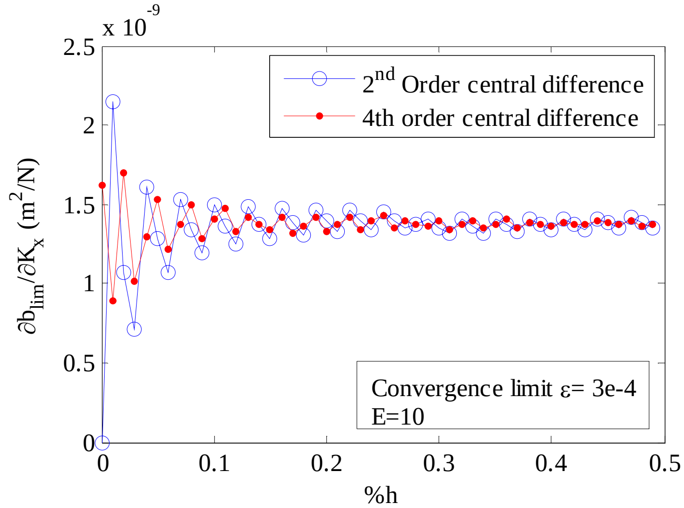

Figure 4 Comparison between 2nd and 4th order central difference formulas. The 4th order formula shows a wider stable region for step size, but higher computation time.

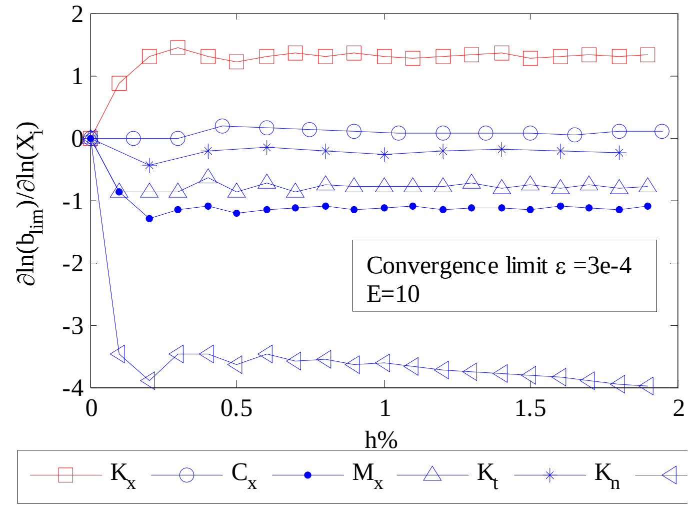

Figure 5 The logarithmic derivative of axial depth with respect to input parameters versus step size percentage.

in the sensitivity calculations are provided in Table 2. The axial depth limit calculated as a function of spindle speed is shown in Figure 8 where Eq. (1) is used to solve for b_{lim}. The variation of the characteristic multipliers ($\vec{\lambda}$) is also shown in the figure. Each calculated value of b_{lim} corresponds to $\max\left|\vec{\lambda}\right| = 1$. Any discontinuity in $\vec{\lambda}$ would affect the accurate sensitivity estimation of b_{lim} to input parameters. In Figure 9 the sensitivities of stiffness, K, and modal mass, M, are compared in the x (feed) and y-directions of the tool. As can be seen in the figure, the sensitivities in the x and y-directions are comparable in magnitude; however, the sensitivities are fairly large near discontinuities in the system characteristic multipliers (see Figure 8)

Table 2. Parameters used in sensitivity analysis.

h (%)	E	Central difference	ε
0.2	10	2nd order	3×10^{-4}

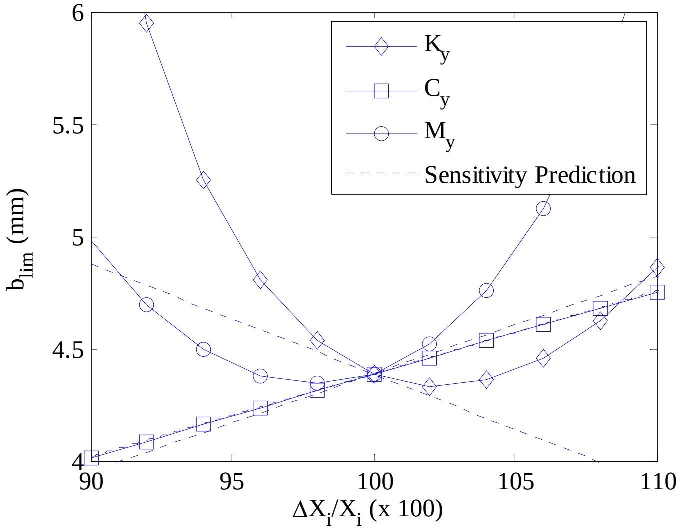

Figure 6 The variation of axial depth limit b_{lim} with a ± 10% change in nominal input parameters. The linear extrapolation based on sensitivity of b_{lim} with respect to each parameter is superimposed. Linearity and non-linearity of $b_{lim}(X_i)$ can be observed (see Table 2).

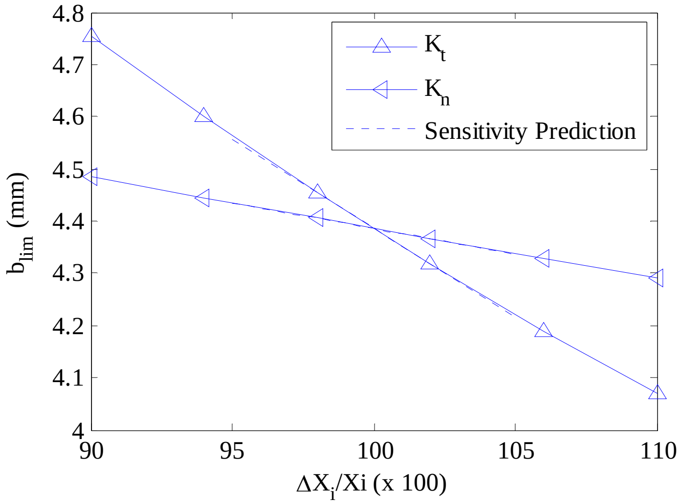

Figure 7 The variation of b_{lim} with a ± 10% change in K_t *and* K_n. The linear extrapolation based on sensitivity of b_{lim} with respect to each parameter is superimposed. Linearity of $b_{lim}(X_i)$ can be observed (see Table 2).

In Figure 10, the effect of damping on the stability is shown to be minimal compared to the modal stiffness and mass. This is a somewhat counter-intuitive result, but can be explained by regeneration, which is a primary physical phenomenon that causes instability. The modal mass and stiffness have a great effect on the system natural frequency, which has a significant effect on regeneration. This also explains the result shown in Figure 11, where the sensitivity of critical axial depth limit b_{lim} to a change in spindle speed Ω is significant and comparable to modal mass and stiffness. The effect of cutting force coefficients is shown in Figure 12, where the tangential cutting force coefficient, K_t, has larger effect on the axial depth limit than the normal direction coefficient, K_n.

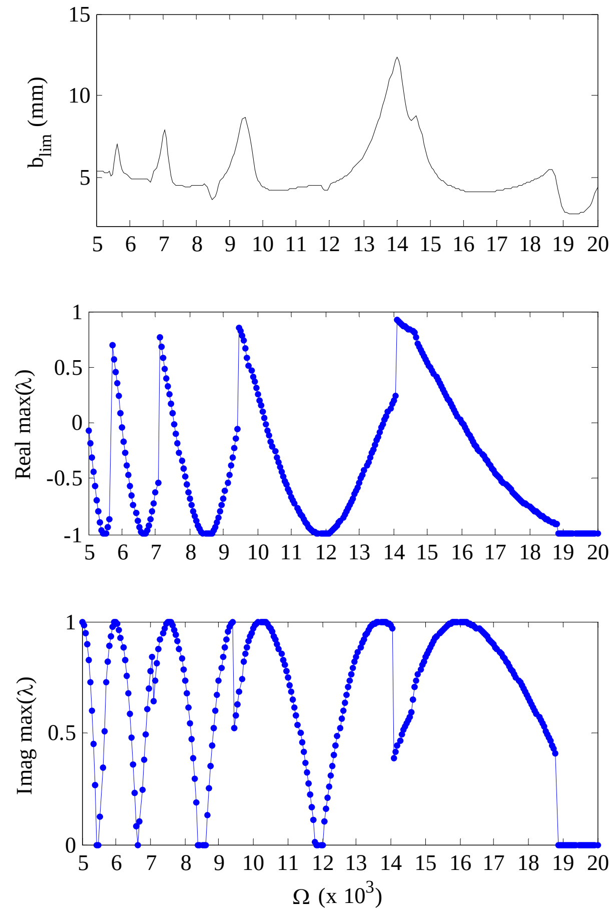

Figure 8 The axial depth limit as a function of spindle speed (stability boundary). Also shown is the variation of the system characteristic multipliers.

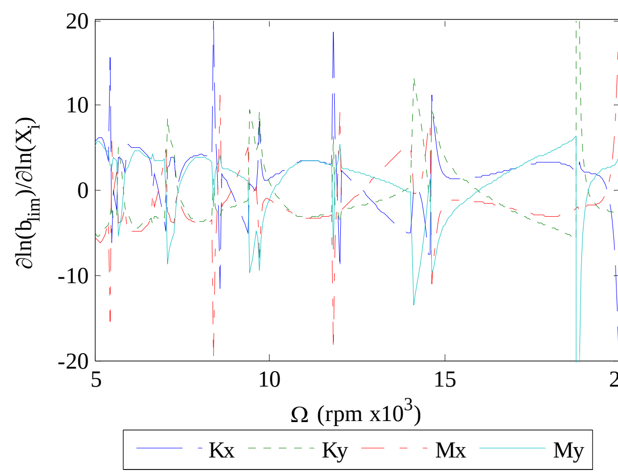

Figure 9 Sensitivity of critical axial depth limit b_{lim} to changes in modal mass *M* and modal stiffness *K* in the x and y-directions.

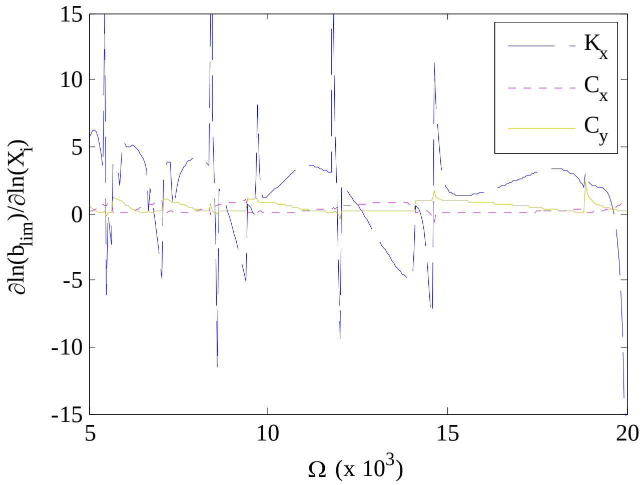

Figure 10 Sensitivity of critical axial depth limit b_{lim} to changes in modal damping *C* in the x and y-directions. It is compared to modal stiffness.

SURFACE LOCATION ERROR SENSITIVITY ANALYSIS

The sensitivity of surface location error, *SLE*, to changes in input parameters is examined here. The parameters used in the sensitivity calculations are provided in Table 2. In Figure 13, the sensitivity of *SLE* to changes in modal parameters in the y-direction is shown. Again, it can be seen that changes in K_y and M_y contribute more than the C_y to a change in *SLE*. In Figure 14, the effect of cutting force coefficients is observed, where it is observed that the highest contributors to *SLE* sensitivity are K_t and K_{te}. Also, in Figure 15, *SLE* sensitivity to spindle speed and radial depth is shown. Substantial sensitivity to spindle speed can be seen. This is due to the dependence of *SLE* on the relationship between the tool point frequency response and the selected spindle speed. As the spindle speed changes, it tracks different parts of the response.

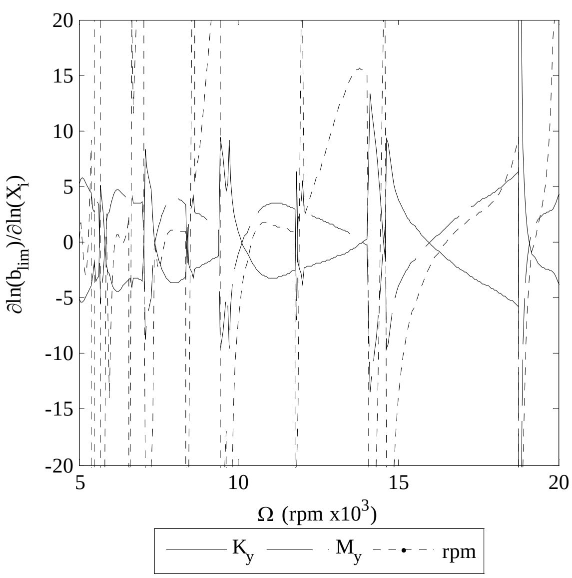

Figure 11 Sensitivity of critical axial depth limit b_{lim} to changes in spindle speed. It is compared here to the modal damping and stiffness in *y*-direction.

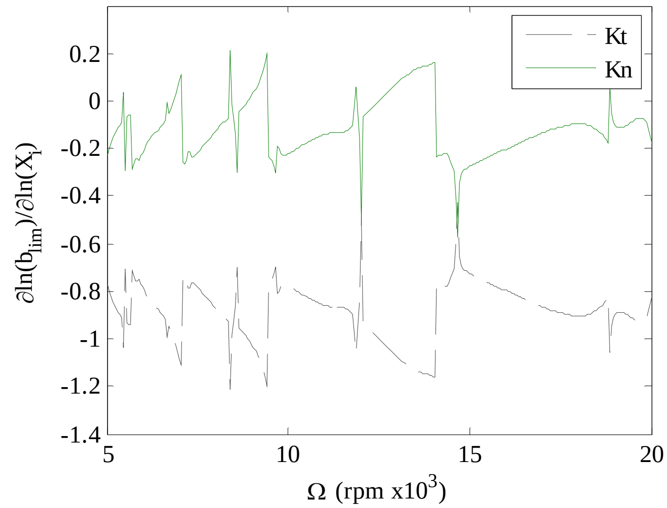

Figure 12 Sensitivity of critical axial depth limit b_{lim} to changes in force cutting coefficients in the tangential K_t and normal directions K_n. Higher sensitivity can be seen for K_t.

UNCERTAINTY OF STABILITY BOUNDARY AND SURFACE LOCATION ERROR

Monte Carlo Simulation

The combined standard uncertainty of stability boundary (b_{lim}) and surface location error (*SLE*) can be predicted using Monte Carlo simulation. In this method, a random sample of size *n* is selected from the population of each input parameters (such as K_t, K_x …). A normal distribution of the input parameters is assumed. In the sample *n*, the nominal value and standard deviation of each input parameter are used to generate

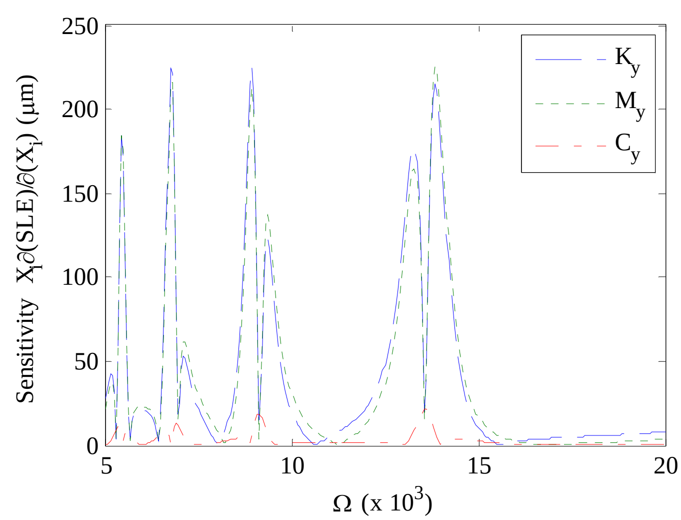

Figure 13 Sensitivity of surface location error *SLE* to changes in modal parameters in *y*-direction.

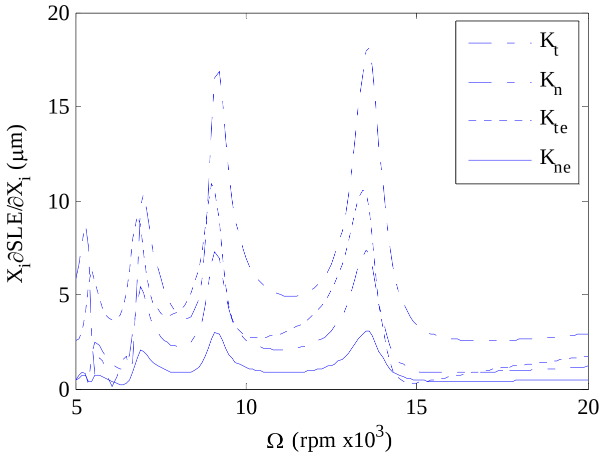

Figure 14 Sensitivity of *SLE* to cutting force coefficients.

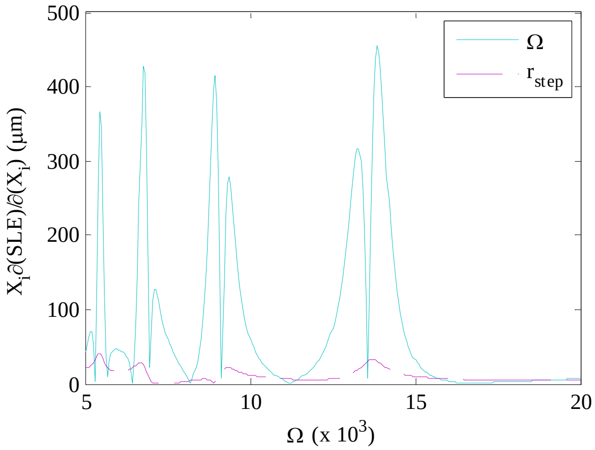

Figure 15 Sensitivity of *SLE* to spindle speed and radial step, or depth of cut.

the sample. The axial depth limit, b_{lim}, and surface location error, *SLE*, are then calculated using TFEA (the bi-section method is used to calculate b_{lim}) for each point in the sample. The standard deviation of the predicted b_{lim} and *SLE* is then calculated from sample output for the range of spindle speeds of interest. It should be noted here that in doing so, no correlation between the input parameters is assumed, which is the most conservative approach.

To illustrate the effect of uncertainty in the input parameters on stability boundary uncertainty, standard uncertainties of 5%, 0.5%, 0.0984% and 0.5% are assigned to nominal values of the cutting force coefficients, modal parameters, radial depth, and spindle speed, respectively. A sample size of 1000 is used. The stability boundary confidence level is found as shown in Figure 16 for a two standard deviation confidence interval.

Sensitivity Method

Uncertainty can also be found using sensitivities of output (b_{lim} or *SLE*) to input parameters. It is given as [15],

$$u_c\left(b_{\lim}\right) = \sqrt{\sum_{i=1}^{m}\left(\frac{\partial b_{\lim}}{\partial X_i}u\left(X_i\right)\right)^2}\,, \tag{5}$$

where $u_c(b_{lim})$ is the uncertainty in axial depth limit, $u(X_i)$ refers to the standard uncertainty in the input parameter X_i, and m is the number of input parameters. This relation assumes no correlation between input parameters. However, it should be

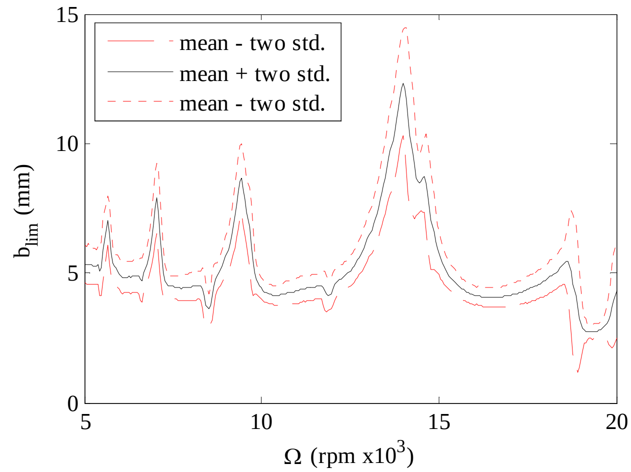

Figure 16 Confidence level in stability boundary due to input parameter uncertainties using Monte Carlo simulation.

noted that cutting force coefficients (K_t, K_n, K_{te}, K_{ne}) and modal parameters (K, C, M) may be correlated.

The same uncertainty is assumed in the input parameters as in Monte Carlo method and the uncertainty in axial depth limit $u_c(b_{lim})$ is calculated for $2u_c(b_{lim})$. Figure 17 shows the close agreement found using the two methods. However, it should be noted that the sensitivity method can be inaccurate near points where the function is C^1 or C^0 discontinuous. This is attributed to the characteristic multipliers ($\vec{\lambda}$) of the system. Figure 18 shows this direct correspondence between the inaccurate sensitivity and discontinuity in the $\vec{\lambda}$.

It should be noted here that predicting the uncertainty using Eq. (5) applies a linear approximation. If the uncertainties in the input parameters are large, then that linear approximation is no longer valid.

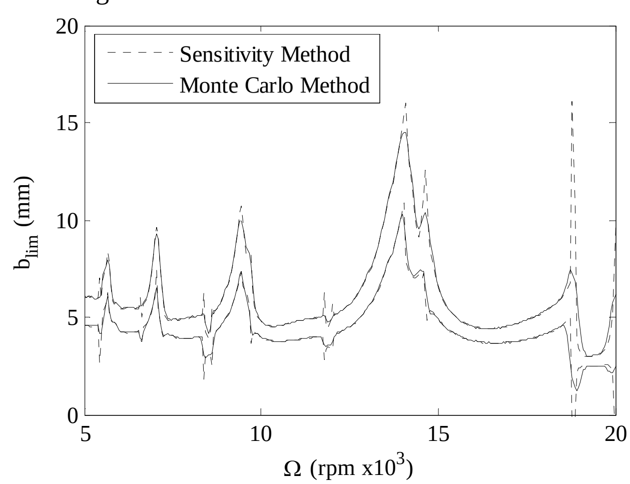

Figure 17 Comparison of 2-sigma confidence levels in stability boundary in axial depth limit using sensitivity method and Monte Carlo method.

The surface location error uncertainty is found similarly using both methods. However, as shown in Figures 13-15, the *SLE* sensitivities are accurate and do not depend on the characteristic multipliers continuity. This is due to the continuity of *SLE*. This explains the close prediction of uncertainty in *SLE* using sensitivity and Monte Carlo methods (Figure 19).

CONCLUSIONS

In this study, the sensitivities of axial depth limit and surface location error to model input uncertainties are studied. Numerical estimation of the sensitivities can be challenging, where several factors contribute to the accuracy of the estimation. The step size is one of the significant factors that affect the accuracy of the estimation.

The sensitivity analysis helps in identifying the relative contribution of the milling model input parameters to the sensitivity of either axial depth limit or surface location error. For the case of axial depth limit, the spindle speed, followed by modal stiffness and mass, is the most significant contributor. In the case of cutting force coefficients, the tangential cutting force coefficient is found to contribute more to the sensitivity than the normal cutting force coefficient. As for the surface location error sensitivity, the same trend can be observed. However, for the cutting force coefficients the edge tangential cutting force coefficient has significant contribution to *SLE*.

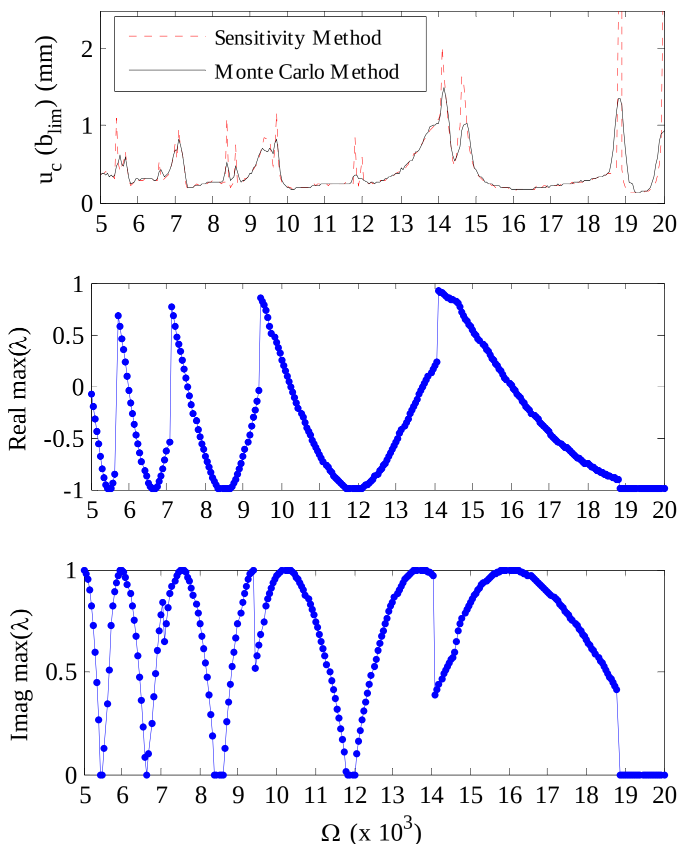

Figure 18 Uncertainty in axial depth using sensitivity and Monte Carlo methods. Inaccuracies in the sensitivity method can be seen near C^0 and C^1 discontinuity in the real and imaginary part of system characteristic multipliers.

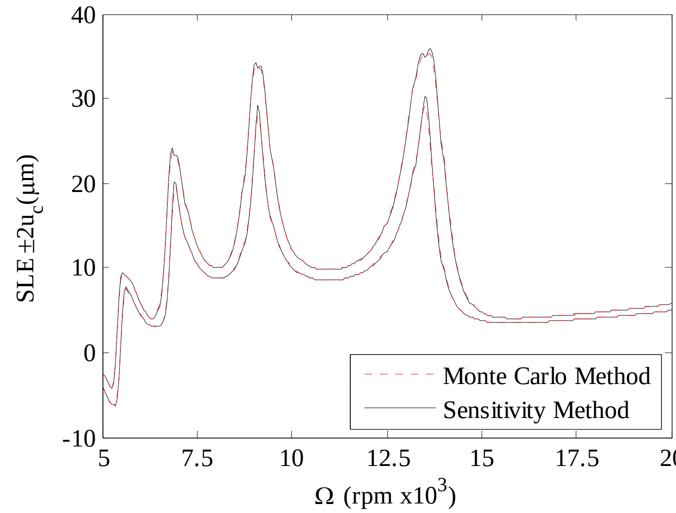

Figure 19 Surface location error with $2u_c$(SLE) confidence interval on the nominal SLE. Close prediction is observed.

The uncertainty in axial depth limit and surface location error is predicted using two methods: the sensitivity method and the Monte Carlo simulation approach. Comparable agreement is shown. However, the sensitivity method is more efficient computationally. For example, in the case of *SLE* uncertainty prediction, Monte Carlo simulation required 9.34 hours, while the sensitivity method needed only 0.26 hours (36 times more efficient). Noting that for $u_c(SLE)$ case, when the milling parameters are well into the stable region, the accuracy of sensitivity method is not sacrificed at the cost of efficiency as is the case for $u_c(b_{lim})$ at discontinuities in the characteristic multipliers.

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REFERENCES

1. Kurdi, M.H., Haftka, R.T., Schmitz, T.L., and Mann, B.P. Simultaneous Optimization of Removal Rate and Part Accuracy in High-Speed Milling in *Proceedings of American Society of Mechanical Engineers International Mechanical Engineering Congress and Exposition*, IMECE2004-60231, Anaheim, CA.
2. Kline, W., DeVor, R., and Shareef, I., 1982, The prediction of surface accuracy in end milling, *Journal of Engineering for Industry*, 104: 272-278.
3. Tlusty, J., 1985, Effect of end milling deflections on accuracy, R.I. King, Ed., *Handbook of High Speed Machining Technology*, Chapman and Hall, New York, pp. 140-153.
4. Sutherland, J. and DeVor, R., 1986, An improved method for cutting force and surface error prediction in flexible end milling systems, *Journal of Engineering for Industry*, 108: 269-279.

5. Montgomery, D., and Altintas, Y., 1991, Mechanism of cutting force and surface generation in dynamic milling, *Journal of Engineering for Industry*, 113/2: 160-168.
6. Smith, S. and Tlusty, J., 1991, An overview of modeling and simulation of the milling process, *Journal of Engineering for Industry*, 113/2: 169-175.
7. Schmitz, T. and Ziegert, J., Examination of Surface Location Error Due to Phasing of Cutter Vibrations. *Precision Engineering-Journal of the American Society for Precision Engineering*, 1999. **23**(1): 51.
8. Guerra, R.E.H., Schmitt-Braess, G., Haber, R.H., Alique, A., and Alique, J.R., Using Circle Criteria for Verifying Asymptotic Stability in Pl-Like Fuzzy Control Systems: Application to the Milling Process. *Iee Proceedings-Control Theory and Applications*, 2003. 150(6): p. 619.
9. Kim, S.I., Landers, R.G., and Ulsoy, A.G., Robust Machining Force Control with Process Compensation. *Journal of Manufacturing Science and Engineering-Transactions of the Asme*, 2003. 125(3): p. 423.
10. Rober, S.J., Shin, Y.C., and Nwokah, O.D.I., A Digital Robust Controller for Cutting Force Control in the End Milling Process. *Journal of Dynamic Systems Measurement and Control-Transactions of the Asme*, 1997. 119(2): p. 146.
11. Pandit, S.M., Modal and Spectrum Analysis: Data Dependent Systems in State Space. 1991: *John Wiley and Sons Inc*.
12. Mann, B.P., Bayly, P.V., Davies, M.A., and Halley, J.E., Limit Cycles, Bifurcations, and Accuracy of the Milling Process. *Journal of Sound and Vibration*, 2004, in press.
13. Insperger, T., Mann, B.P., Stepan, G., and Bayly, P.V., Stability of up-Milling and Down-Milling, Part1: Alternative Analytical Methods. *International Journal of Machine Tools and Manufacture*, 2003. **43**: 25.
14. Mann, B.P., *Dynamics of Milling Process*. 2003, Ph.D Dissertation, St. Louis, MO: Washington University.
15. International Organization for Standardization, *Guide to the Expression of Uncertainty in Measurement*. 1995.

