Performance Analysis and Simulation of MIMO Channels for Space-Time Coding

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***Abstract*—** For recent several years, wireless communication industries have been increased remarkably, and with the rapid growth of this area, many technologies also have been invented and used to improve communication performance and overcome the physical limitation in wireless communication environment, such as multiple propagation, time dispersion, and frequency dispersion. One of those technologies is the space-time coding (STC) which transmits and receives information at higher data rates with more reliability by using multiple antennas at both sides. Our project is aimed to simulate the space-time coding over the various channel models which are AWGN, Raleigh, and Ricean channel model and evaluate its performance in term of BER. As we know, space-time coding system also referred to as multiple-input multiple-output (MIMO) system. In this paper, we first introduce the MIMO system. Next, we are going to introduce space-time codes, trellis (convolutional) and block code, and give some cooperation based on the mathematical analysis. Finally, the simulation results will be used to analyze and compare their performance. And all realizations will be done by Matlab.

***Index Terms*— Simulation, Space-Time Coding, MIMO, Various Channels, BER, block code, trellis code**

INTRODUCTION-

I. MULTI-INPUT MULTI-OUTPUT

The multiple-input and multiple-output system (MIMO) is to use of multiple antennas at both the transmitter and receiver ends of a wireless communication system to improve communication performance. It is the extension of smart antennas system and can achieve capacity increases, under multi-environment, that vary linearly with the number of antenna used without increasing premium bandwidth or power [4]. It can also offer increased diversity advantages over traditional wireless systems [2].

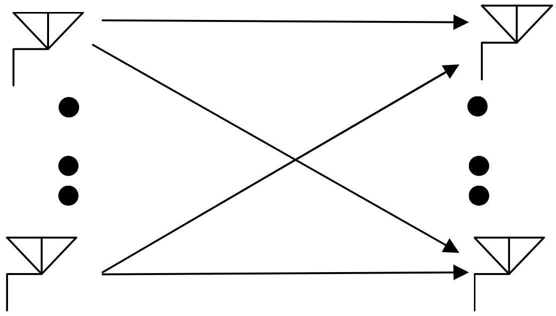

Figure 1. *Multiple antennas at source and destination*

Figure 1 shows the simple intuition of MIMO system. We define the number of transmit and receive antennas as N_T and N_R, respectively. So the system diversity order is given by $N_T \times N_R$ under sufficient spacing.

II. MIMO CHANNEL MODEL & CHANNEL CAPACITY

A. MIMO Channel Model

To design the MIMO system, we consider AWGN, Rayleigh and Ricean flat fading channel models. It is assumed here that the channel undergoes independent fading between the multiple transmit-receive antenna ends.

Here, we give the general picture about MIMO channel model.

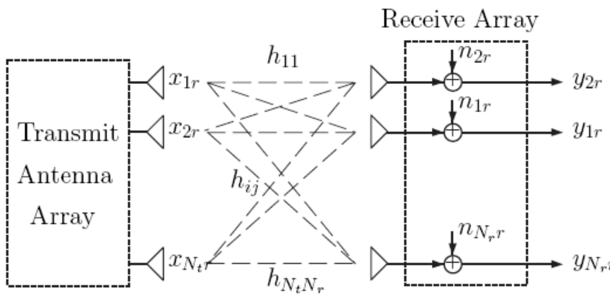

Figure 2. *Multiple antennas channel model*

This channel can be described by the following matrix equation:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \tag{1}$$

Here, $\boldsymbol{x}$ is the transmitted symbol vector. It can be real or complex signals. $\boldsymbol{n}$ is the real or complex additive Gaussian noise with variance N_0. And the $\boldsymbol{H}$ denotes $N_T \times N_R$ matrix, h_{ij} are gain coefficients modeling random phase shifts and channel gains. For more details, see [6,8].

For Rayleigh Channel, the h_{ij} are modeled as independent and identical distributed (i.i.d.) complex Gaussian random variables with variance ½ in each dimension.

For Ricean Channel model, we have both phase matrix and Rayleigh flat fading matrix. And we also have the K-factor for the Ricean distribution. It has been proved that Ricean model shows better BER performance than Rayleigh channel. See [6].

Comparison between Rayleigh channel and Ricean channel will be given in the remaining of this paper.

B. Channel Capacity

It was mentioned before that MIMO system can achieve capacity increases very linearly with the number of antennas. As we know, Shannon derived the following capacity formula for additive white Gaussian noise channel (AWGN):

$$C = W \log_2(1 + E/N_0) \quad (2)$$

Here, W is the bandwidth in Hz. E is the signal power. N_0 is the total noise power.

For single-input multiple-output (SIMO) system, the channel capacity can be written as:

$$C = W \log_2 \left(1 + \frac{E}{N_0} \cdot \boldsymbol{HH}^*\right) \quad (3)$$

Here, $\mathbf{H}$ is $1 \times N_R$ unit-power complex Gaussian amplitude of the channel.

For the MIMO system with N_T transmit and N_R receive antennas, the channel capacity is:

$$C = W \log_2[det\left(\boldsymbol{I}_{N_R} + \frac{E}{N_0 \cdot N_T} \cdot \boldsymbol{HH}^*\right)] \quad (4)$$

By the law of large numbers,

$$\begin{cases} \boldsymbol{HH}^* \xrightarrow{N_T \to \infty} N_T \boldsymbol{I_R} \Rightarrow C \approx W N_R \log_2(1 + \frac{E}{N_0}) \\ \boldsymbol{H}^*\boldsymbol{H} \xrightarrow{N_R \to \infty} N_R \boldsymbol{I_T} \Rightarrow C \approx W N_T \log_2(1 + \frac{N_R}{N_T}\frac{E}{N_0}) \end{cases} \quad (5)$$

From (5), we can see that the MIMO channel capacity increases linearly with the number of transmit or receive antennas. See the papers [6, 8].

At the result part, we will give the comparison about the channel capacity among different diversity.

III. SPACE-TIME CODING

Space-time coding is a theme which makes it possible to use the MIMO channel generated by multiple transmit and receive antennas.

Generally, there are two classes of Space-time coding, Space-Time Block Codes and Space-Time Trellis Codes. Figure 3 shows a simplified system diagram of Space-time coding. The rest of this part will introduce these two Space-time codes.

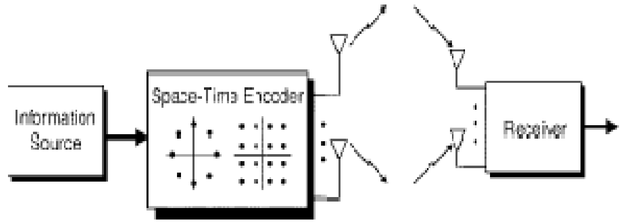

Figure 3. *The block diagram of space-time coding [3]*

A. Space-Time Block Codes

Space-time block codes (STBC) is one of the most widely used codes in MIMO system because of its low computation complexity. STBC was first mentioned by Alamouti [1, 7, 8]. Alamouti code is the case of two transmitter and N_R receiver antennas. While Tarokh et al. [5] improved Alamouti's code to adapt an arbitrary number of transmitter antennas based on the orthogonal design theory. Therefore, generally, we also call it *Space-Time Orthogonal Block Codes (STOBC)*.

As we know, MIMO channel model, see figure 2, can be described as Eq. (1). And we assume each antenna transmits a Double-sideband suppressed-carrier transmission (DSB-SC) signal. So we can modify (1) as:

$$\boldsymbol{y(t)} = \frac{\sqrt{E_s}}{N_T}\boldsymbol{Hx(t)} + \boldsymbol{n(t)} \quad (6)$$

Where $\boldsymbol{y} = (y_1, \cdots, y_{N_R})$ and $\boldsymbol{x} = (x_1, \cdots, x_{N_T})$. $\sqrt{E_s} =$ the signal energy per space $-$ time symbol,

STOBC is based on the theory of orthogonal designs [8]. We take matrix $\boldsymbol{X}$ in stand of $\boldsymbol{x}$. $\boldsymbol{X}$ is a space-time codeword of $N_T \times N_R$. So we have

$$\boldsymbol{Y} = \sqrt{\frac{E_s}{N_T}}\boldsymbol{HX} + \boldsymbol{N} \quad (7)$$

To achieve this equation, we use AWGN.m in MATLAB. At the receiver, we use ML detector to make a decision.

Since Alamouti codes generate two transmitters and N_R receivers, we further modify the codes to suit more than two transmitters. However, when using PSK or QAM modulations, it is impossible for us to generate fully orthogonal codes [3].

B. Space-Time Trellis Codes

Space-Time Trellis Codes (STTC) is convolutional codes that extend to the case of multiple transmit and receive antennas [1].

Details about Space-Time Trellis Codes are included in [2]. At the receiver side, the decoding of the STTC is performed by using soft outputs with the Viterbi algorithm (SOVA), which minimizes the metrics $m(r)$ to find the global minimum.

$$m(r) = \|\boldsymbol{y_r} - \boldsymbol{Hx_r}\|^2 \quad (7)$$

The metric is simply the squared Euclidean distance between hypothesis and received signal and needs channel estimation.

In [2], the STTC can maximize the diversity and coding gain. But, as is mentioned before, STBC is more wildly used than STTC; because STTC is computationally complicated, and will reduce the data rate of the transmission when maximize diversity. So there is a tradeoff. We have to maximize the data rate, and at the same time, get the high diversity. [2] gives the boundary equation.

We already mentioned the two Space-Time Codes. And STTC outperform s STBC [2]. On the other hand, STBC is computationally more attractive than STTC. STBC uses simple ML decoding algorithms which only use linear processing at the receiver.

COMPARISON AND RESULTS –

I. CAPACITY & DIVERSITY

It has been mentioned that using multiple antennas at both transmitter and receiver sides can mitigate the effects of fading over a communications link. Meanwhile, it can achieve a high channel capacity. We use equation (4) to plot figure 4.

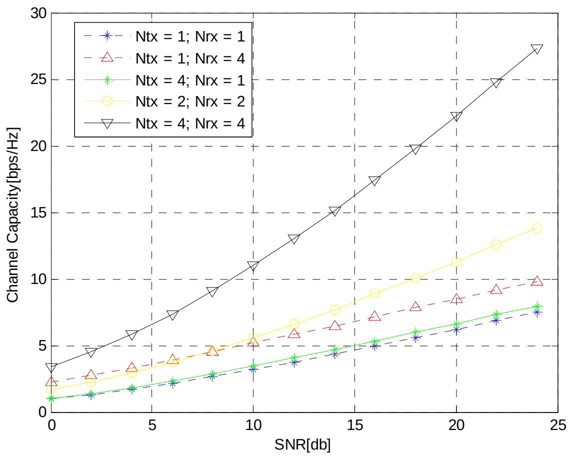

Figure 4. Capacity comparison of several multiple antenna systems.

This figure is plot through a Rayleigh channel with STBC. Upon the inspection of Figure 4, it is shown that MISO systems offer a smaller capacity gain than that of MIMO systems. We can see that when we provide multiple antennas with N_T=4 and N_R=1; it provides a smaller gain than SISO. While, when we change the number of antennas to N_T=1 and N_R=4 with keeping the diversity same, it is shown that there is an obvious gain. Hence, we can make the conclusion that

1. MIMO systems offer a higher channel capacity than SISO or SIMO systems.
2. Under the same diversity, more antennas at receiver side more capacity gain we will get.

Until now, we described the comparison of channel capacity between different diversities. Then, we plot Bit Error Rate (BER) over diversities to see whether it gives the similar conclusion or not. We expect the system to offer a diversity orders of 2and 4, and will compare it with various cases (1x2, 2x1 and 1x4, 4x1 and 2x2 systems, which have the same diversity order also).

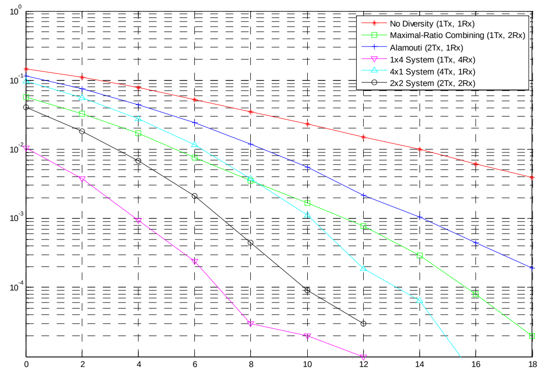

Figure 5. Transmit vs. Receive Diversity through STBC

Figure 5 shows BER of STBC via different diversities. As expected, the similar slopes of the BER curves for the 1x4, 4x1 and 2x2 systems indicate an identical diversity order. (Assuming that they are all under Rayleigh fading channel model).

Also observe that to when achieve the same BER, 1x4 system has a 3 dB advantage than 2x2 system; 2x2 system attributes 2dB better than 4x1 system.

So the conclusions about BER over different diversity through STBC are:

1. MIMO system does not offer the lowest BER under the same diversity. A SIMO system can offer the lowest BER.
2. Under the same diversity, more antennas at receiver side lower BER the system will gain. For example, 2x1 systems have a 3db disadvantage when compared to 1x2 systems.

Note here, the green curve and celeste curve come across each other. At first, 1x2 system outperforms 4x1 system, but when the SNR increase, 4x1 system outperforms 1x2 system. This can be explained by our conclusion. The receive antennas make main influence in the beginning, but when SNR increases, the bigger diversity order outperforms the smaller one.

II. COMPARISON OF RAYLEIGH AND RICEAN CHANNELS

We have given the basic ideas and general construction about the Rayleigh, Ricean channel models. And we already know that for the Rayleigh channel the fades are deeper than the Ricean channel case [9]. So, it should be true that there is a better BER performance for the Ricean channel case.

Below is the figure about the comparison of Rayleigh channel and Ricean channel.

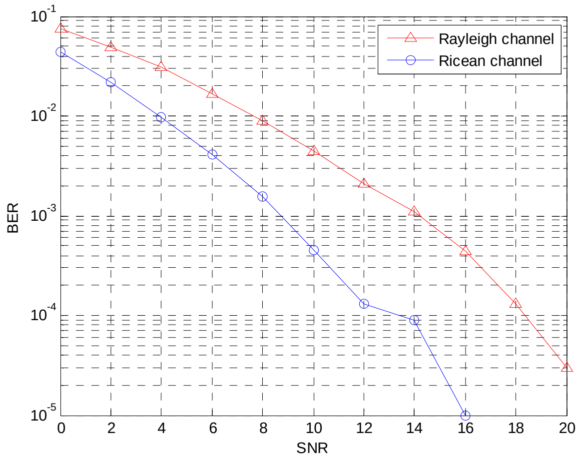

Figure 6. Rayleigh vs. Ricean comparison for 2x2 STBC, BPSK

As expected, the Ricean channel outperforms the Rayleigh channel. However, it is very important to see that this advantage in BER performance for Ricean channel is coming from cost of channel capacity. When we design the Ricean channel, we found that higher correlation in Ricean channel can make the BER performance good, but it can no longer achieve significant capacity gains.

III. SPACE-TIME BLOCK CODE VS TRELLIS CODE

Space-Time Block Code and Space-Time Trellis Code are the two Space-Time coding themes which we used to simulate the MIMO systems.

Figure 5 give the BER performance of space-time block codes for different number of transmit and receive antennas. For comparison, we will give the space-time trellis codes BER performance first, and then put these two codes together to make a comparison.

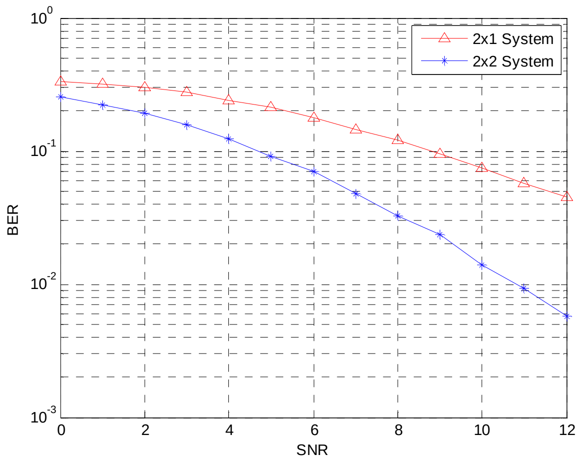

Figure 7. Performance of Space-time trellis code through Rayleigh channel, 4PSK

Different numbers of the trellis state will give different frame error rate (FER) performances.

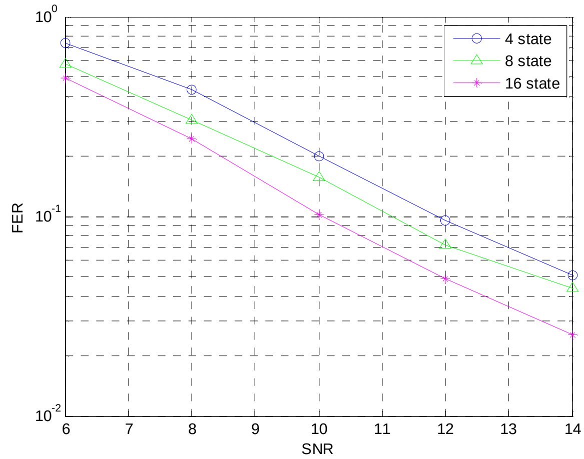

Figure 8. Performance of 2x1 Space-time trellis codes by different state numbers through Rayleigh channel, 4PSK

Figure 8 gives the comparison of STTC among different trellis states via a 2x1 system using 4PSK. It has been shown that the bigger trellis state number the better performance of FER. But as mentioned before, this gain is caused by the expense of increased complexity for the decoder at receiver. Of course, changing the number of transmit and receive antennas can also improve the performance.

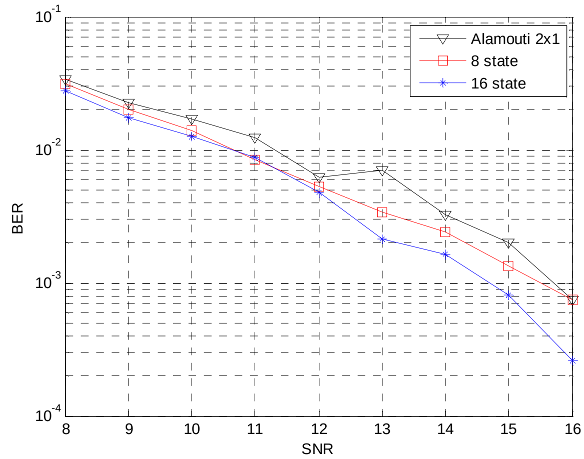

Figure 9. Comparison of Alamouti code and trellis codes with state number of 4 and 8, Rayleigh channel, 4PSK

As expected, upon the inspection of figure 9, trellis code has better BER performance than Alamouti code. However, the performance is not very good for small SNR. When the SNR increases, we can see trellis code outperforms Alamouti code. Here, state 4 and 8 trellis code seem to have similar performance at the low SNR. One way to avoid this problem is to increase the iteration time. But it will make the code more computational.

FUTURE WORK –

As for the limited time, we didn't do the comparison of Rayleigh and Ricean channel using Space-time trellis codes. Our future work will finish this comparison. But here we can safely get the conclusion that the Ricean channels always outperform the Rayleigh channels. So, using STTC, we will get the similar figure just like figure 6.

Due to the limitation of time, there will be some mistakes. But we learned a lot from the course project. We will modify these problems later and will also focus on the space multiplexing problem in the future.

CONCLUSION –

In this project, we do analyze and simulate MIMO systems for Space-Time Coding through Rayleigh and Ricean channel models. We also design the Space-Time Codes which is useful and efficient in MIMO systems. The space-time coding schemes described above were simulated in MATLAB.

One thing we have learned is how to divide the diversity into transmitters and receivers. Since the diversity order is the very important thing, we want to consider about it. First, we think if we divide the diversity order equally into the both sides, then we will get the highest channel capacity and the lowest bit error rate. But, in fact, it is shown that the equally divided the diversity order does not show the best result to achieve lowest BER. The maximal-ratio combined system can achieve the best way. This is because we modeled the total transmitted power to be the same in both cases (2x2, 1x4 systems). If we calibrate the transmitted power such that the received power for these two cases is the same, then the performance would be identical [10].

The BER performance between STBC and STTC are also analyzed.

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