Improved localization in autonomous vehicles through multi-vehicle cooperation

Joseph Knuth
Advisor: Dr. Prabir Barooah
University of Florida
Dept. Mechanical and Aerospace Engineering
knuth@ufl.edu

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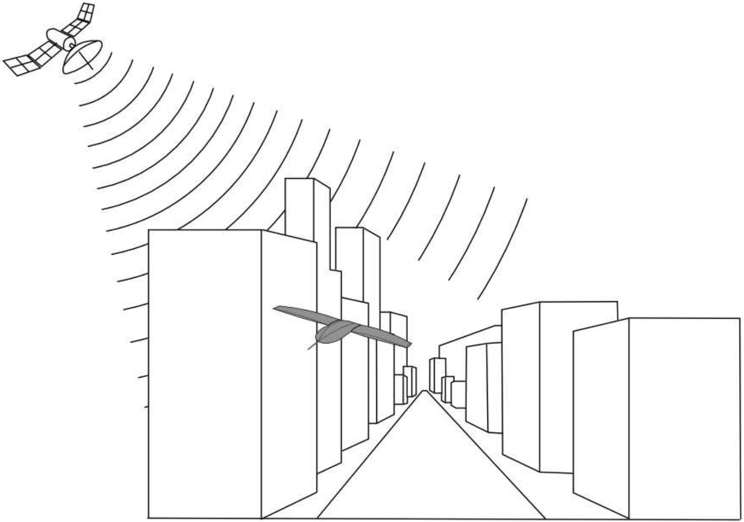
Limitations of GPS



Vision Based Systems

- Capture pictures in each time step
- Find features common in two consecutive pictures
- Determine movement of camera from movement of common features

Orientation Error in a Vision Based System

1-D system with error in position, but not in orientation.

Δx_i's are iid, $N(0, \sigma^2)$

$$p(n) = \Delta x_1 + \Delta x_2 + \cdots + \Delta x_n$$
$$\Rightarrow var(p(n)) = n\,\sigma^2$$

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2-D system with error in orientation

$$R_i = \begin{pmatrix} cos\,\theta_i & -sin\,\theta_i \\ sin\,\theta_i & cos\,\theta_i \end{pmatrix} \quad \theta_i\text{'s} \sim ?$$

$$p(n) = R_1\,x_1 + R_2\,x_2 + \cdots + R_n\,x_n$$
$$\Rightarrow var(p(n)) = ?$$

3-D Orientation Error and the VMF Distribution

In 3-D systems, the orientation can no longer be expressed as a scaler distribution.

Solution: Look at the distribution of the unit quaternion $q \in S(3)$
One example of such a distribution is the Von Mises - Fisher distribution

$$f(x) = \frac{k^{p/2-1}}{(2\pi)^{p/2}\, I_{p/2-1}(k)}\, exp(k\, \mu^T\, x) \ \text{ for } x \in S(3)$$

Points form a VMF Distribution on the 2-Sphere

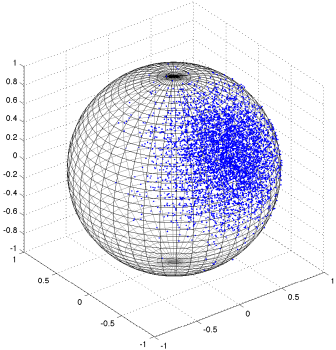
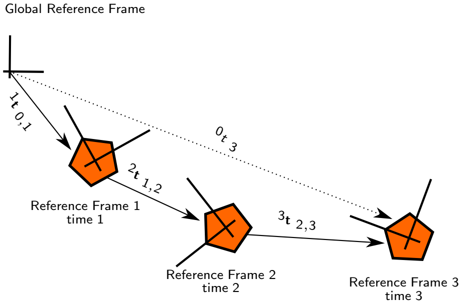


Figure: Data from a VMF distribution with p = 3, k = 10, and $\mu = [0, -0.8415, 0.5403]^T$

Position Error in a Vision Based System



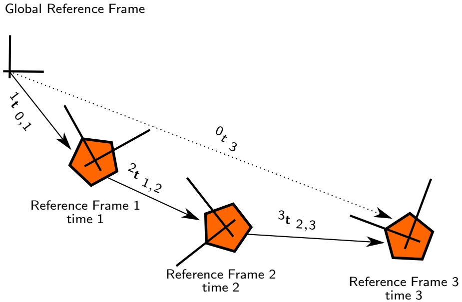
- Relative Measurements, Not Absolute
- Error Propagates

$$^0\hat{t}_1 = {}^0_1\hat{R}\ {}^1t_{0,1}$$

$$^0\hat{t}_2 = {}^0_1\hat{R}\ {}^1t_{0,1} + {}^0_1\hat{R}\ {}^1_2\hat{R}\ {}^2t_{1,2}$$

$$^0\hat{t}_3 = {}^0_1\hat{R}\ {}^1t_{0,1} + {}^0_1\hat{R}\ {}^1_2\hat{R}\ {}^2t_{1,2} + {}^0_1\hat{R}\ {}^1_2\hat{R}\ {}^2_3\hat{R}\ {}^3t_{2,3}$$

Position Error in a Vision Based System



- Relative Measurements, Not Absolute
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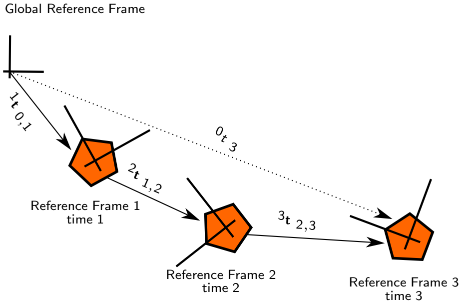
Uncertainty from: 1^{st} Rotation

$$^{0}\hat{t}_{1} = {}^{0}_{1}\hat{R}\ {}^{1}t_{0,1}$$

$$^{0}\hat{t}_{2} = {}^{0}_{1}\hat{R}\ {}^{1}t_{0,1} + {}^{0}_{1}\hat{R}\ {}^{1}_{2}\hat{R}\ {}^{2}t_{1,2}$$

$$^{0}\hat{t}_{3} = {}^{0}_{1}\hat{R}\ {}^{1}t_{0,1} + {}^{0}_{1}\hat{R}\ {}^{1}_{2}\hat{R}\ {}^{2}t_{1,2} + {}^{0}_{1}\hat{R}\ {}^{1}_{2}\hat{R}\ {}^{2}_{3}\hat{R}\ {}^{3}t_{2,3}$$

Position Error in a Vision Based System



- Relative Measurements, Not Absolute
- Error Propagates

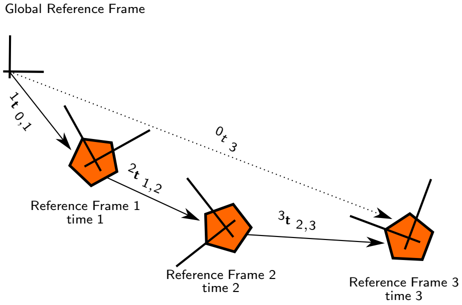
Uncertainty from: 1st Rotation 2nd Rotation

$$ {}^{0}\hat{t}_{1} = {}^{0}_{1}\hat{R}\;{}^{1}t_{0,1} $$

$$ {}^{0}\hat{t}_{2} = {}^{0}_{1}\hat{R}\;{}^{1}t_{0,1} + {}^{0}_{1}\hat{R}\;{}^{1}_{2}\hat{R}\;{}^{2}t_{1,2} $$

$$ {}^{0}\hat{t}_{3} = {}^{0}_{1}\hat{R}\;{}^{1}t_{0,1} + {}^{0}_{1}\hat{R}\;{}^{1}_{2}\hat{R}\;{}^{2}t_{1,2} + {}^{0}_{1}\hat{R}\;{}^{1}_{2}\hat{R}\;{}^{2}_{3}\hat{R}\;{}^{3}t_{2,3} $$

Position Error in a Vision Based System



- Relative Measurements, Not Absolute
- Error Propagates
- Complexity Increases

Uncertainty from: 1^{st} Rotation 2^{nd} Rotation 3^{rd} Rotation

$${}^{0}\hat{t}_{1} = {}^{0}_{1}\hat{R}\ {}^{1}t_{0,1}$$

$${}^{0}\hat{t}_{2} = {}^{0}_{1}\hat{R}\ {}^{1}t_{0,1} + {}^{0}_{1}\hat{R}\ {}^{1}_{2}\hat{R}\ {}^{2}t_{1,2}$$

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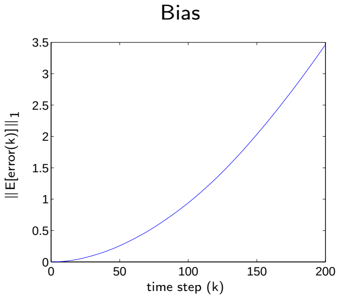
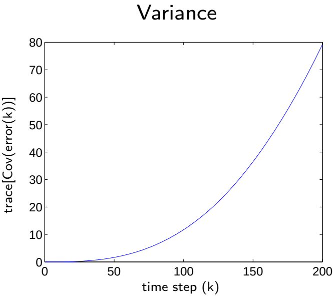
Monte-Carlo Simulation Results for Position Error

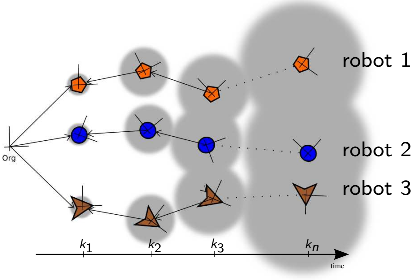




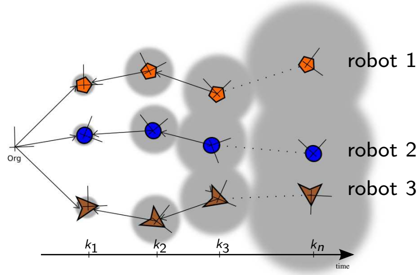
$$\text{error}(k) = X(k) - \hat{X}(k)$$

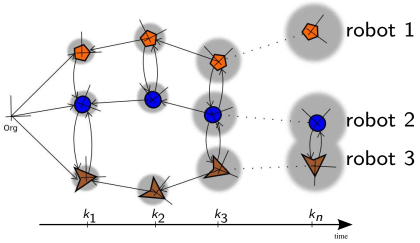
$$\text{Growth of Bias} \rightarrow \Theta(n)$$

$$\text{Growth of Variance} \rightarrow \Theta(n^2)$$



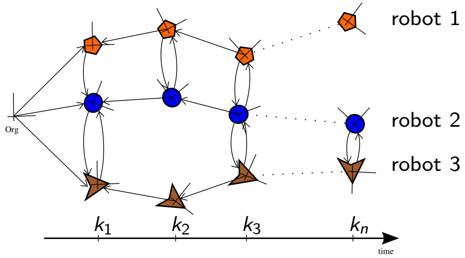
- When a group of vehicles work independently, the error in position grows quickly





- When a group of vehicles work independently, the error in position grows quickly
- Assume vehicles have the ability to find relative measurements between each other
- When the group cooperates and shares that data, the error increases more slowly

Multi-Vehicle System



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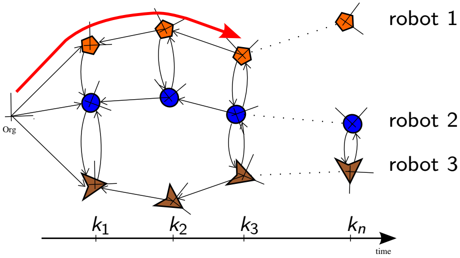

- Single agent gives a single estimate of $\hat{R}$

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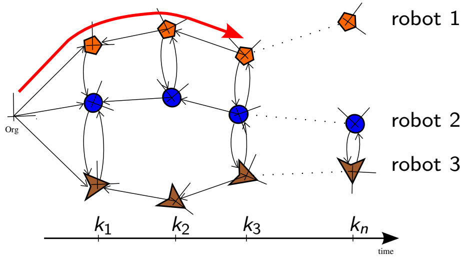
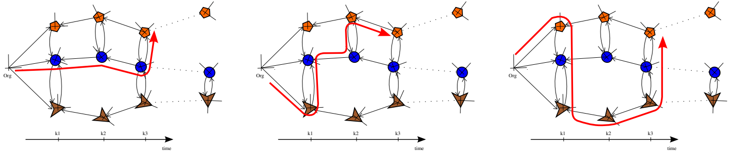
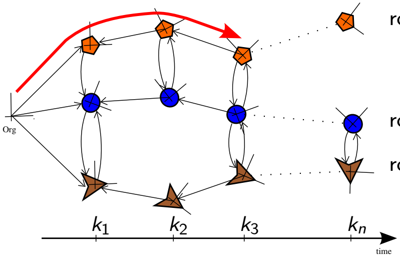


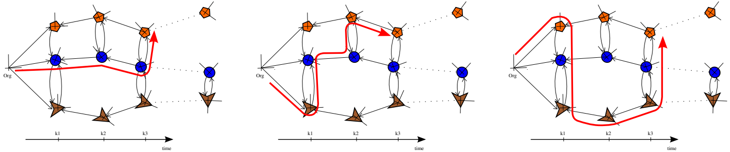
- Single agent gives a single estimate of $\hat{R}$
- Multiple agents in cooperation can give multiple estimates of $\hat{R}$



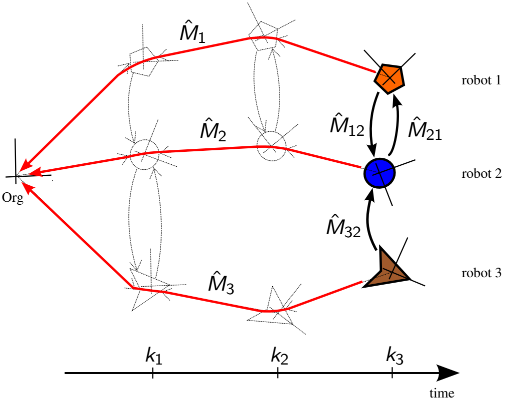
Multi-Vehicle System



- Single agent gives a single estimate of $\hat{R}$
- Multiple agents in cooperation can give multiple estimates of $\hat{R}$
- Averaging multiple measurements for $\hat{R}$ should give a better estimate
- Problem: $\frac{R_1+R_2}{2} \notin SO(3)$



Estimating POS in a Multi-Agent System

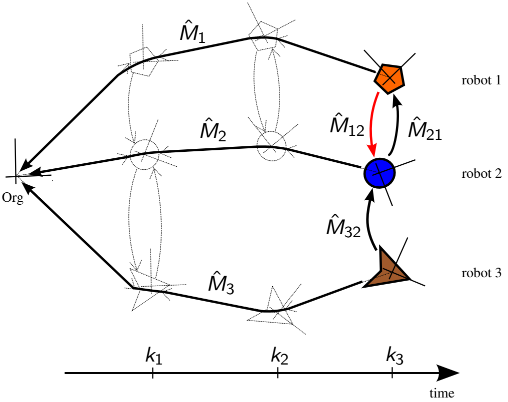


$M_i \longrightarrow$ Transformation from global reference frame to reference frame i

$M_{ij} \longrightarrow$ Transformation from reference frame i to reference frame j

$\hat{M} \longrightarrow$ Estimates of M

Estimating POS in a Multi-Agent System



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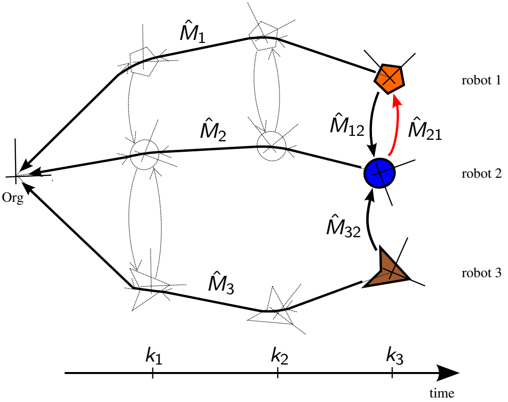

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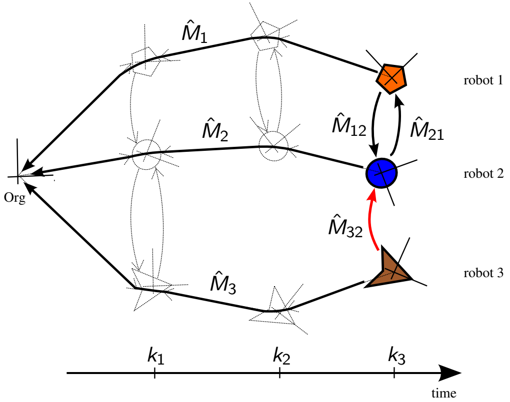

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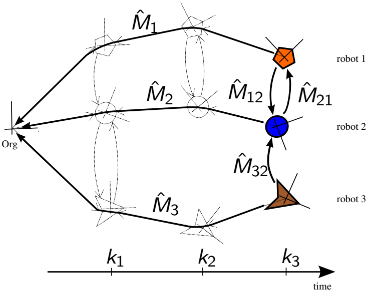
Combining Estimates

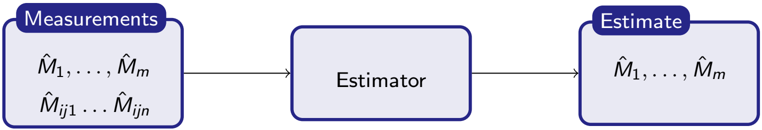


- Constraints
- Lie Algebra Equivalent Value
- First order Approximation

$$M_{ij} := M_j M_i^{-1}$$
$$\Rightarrow M_j^{-1} M_{ij} M_i = I$$
$$\text{Let } m := log(M)$$
$$m_{ij} = BCH(m_j, -m_i)$$
$$m_{ij} \approx m_j - m_i$$

$$\Delta \hat{M}_i := M_i \hat{M}_i^{-1}$$
$$\Delta \hat{M}_{ij} := \Delta \hat{M}_j \Delta \hat{M}_i^{-1}$$
$$\Delta \hat{M}_{ij} = \hat{M}_j^{-1} \hat{M}_{ij} \hat{M}_i$$
$$\Delta \hat{m}_{ij} \approx \Delta \hat{m}_j - \Delta \hat{m}_i$$

Estimating Absolute Position



Problem: Our desired estimates are not linearly dependent on our measurements

Solution: Use the linear equations from the first order approximation in the Lie Algebra

Linear System of Equations

$$\mathrm{D} \begin{bmatrix} \Delta\hat{m}_1 \\ \vdots \\ \Delta\hat{m}_m \end{bmatrix} \approx \begin{bmatrix} \Delta\hat{m}_{ij1} \\ \vdots \\ \Delta\hat{m}_{ijn} \end{bmatrix}$$

Algorithm for Intrinsic Averaging of Translations

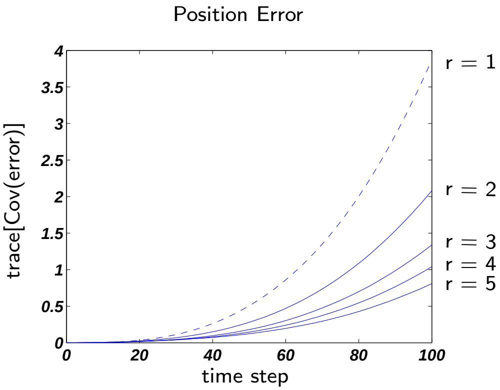
Least Squares Estimation

$$\begin{bmatrix} \Delta\hat{m}_1 \\ \vdots \\ \Delta\hat{m}_m \end{bmatrix} \approx \mathrm{D}^\dagger \begin{bmatrix} \Delta\hat{m}_{ij1} \\ \vdots \\ \Delta\hat{m}_{ijn} \end{bmatrix}$$

Algorithm

1. Use traditional methods to estimate the global position of each agent: $\left\{\hat{M}_i\right\}$
2. Find error in relative positions: $\left\{\Delta\hat{M}_{ij}\right\}$
3. Find lie algebra equivalent of error matrices: $\left\{\Delta\hat{m}_{ij}\right\}$
4. Use the least squares equation to find estimate of error in global positions: $\{\Delta\hat{m}_i\}$
5. Adjust the global position to compensate for error: $\hat{M}_i = \hat{M}_i\, exp(\Delta m_i)$
6. Repeat steps 2-5 until the global position errors become small: $\sum_{i=1}^{m} \|\Delta m_i\| < \varepsilon$

Centralized Method



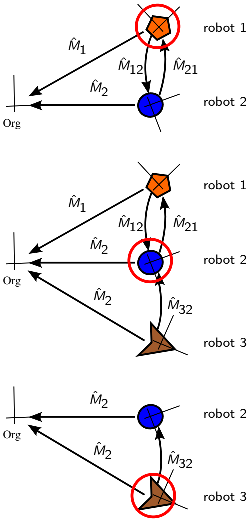
Advantages

- Reduces Error Growth

Disadvantages

- Single Vital Component
- Communication Limitations
- Problem Complexity Scales with Number of Agents

How to Distribute the Computation

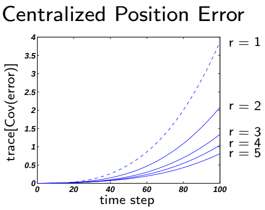


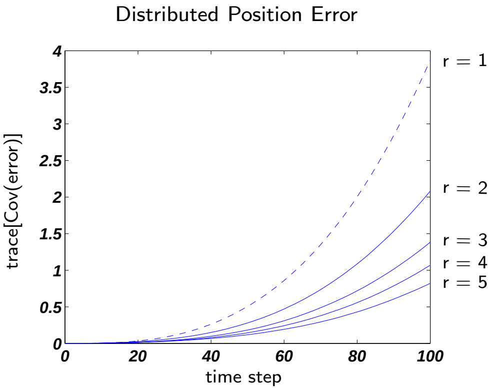
- Each agent sends an estimate of its global position and their relative position to any other agent it sees. Those other agents then return their global positions.
- Every agent runs the algorithm on its subset of the information
- Complexity no longer scales directly with number of agents
- Computational work divided among many

Distributed Results

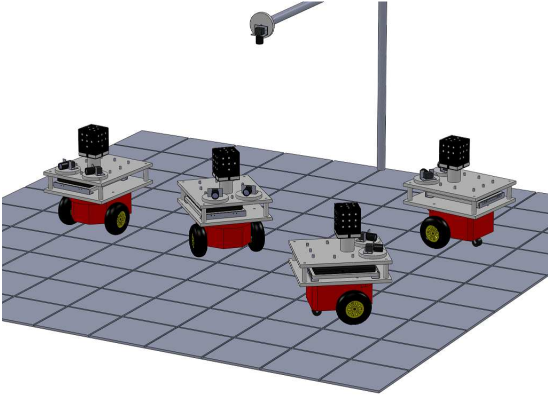
Results

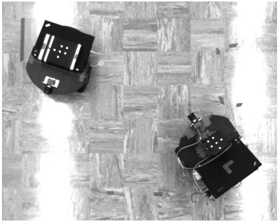
- Averaging algorithm can reduce complexity of error considerably
- Distributed just as good as Centralized

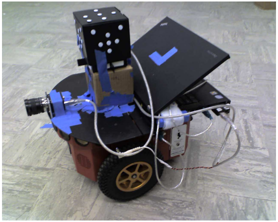




Future Work







Additional Slides

Baker-Campbell-Hausdorff

When A, B are two objects that do not commute:

$$e^A e^B = e^{BCH(A,B)}$$

Where:

$$BCH(A,B) = A + B + \frac{1}{2}[A,B] + \frac{1}{12}[A - B,[A,B]] + \mathcal{O}(|(A,B)|^4)$$

$$[A,B] = AB - BA \quad \leftarrow \text{Lie Bracket for Rotations and Translations}$$

Matrix Logarithm

$$log(A) = -\sum_{k=1}^{\infty} \frac{(I - A)^k}{k}$$