Jig-Shape Optimization of a Flapping Wing for a Micro Air Vehicle

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The purpose of this work is to develop a small ornithopter capable of slow flight in an indoor environment. Most of this work details the procedure for optimizing the wing structure such that the propulsive efficiency is maximized at the design point. A jig-shaped optimization is run in which an optimal wing shape as a function of time is determined and a wing structure is optimized such that single DOF actuated wing matches the desired deformation. An unsteady vortex-lattice method is used as the aerodynamic model and a nonlinear finite element shell method is invoked to model the structural deformations caused by both inertial and aerodynamic loads. The subsystems of a flight vehicle are considered and a motor is designed for maximum efficiency at the design point.

Additionally, such CFD techniques require adaptive grid generation at each time step which would be very computationally expensive and hard to obtain grid-independence. For these reasons, there has been a shift towards grid-free techniques for flapping wing applications. Unsteady Vortex Lattice Methods (UVLMs) have been successfully used to model flapping flight but have been unable to model wing-wake interaction such as those encountered in hover. There have been efforts to collapse the wake vortex rings into discrete vortons so that hovering cases can be modeled, but this method has produced mixed results.

The approach to the structural dynamics problem is more straightforward since Finite Element Analysis (FEA) techniques are well-founded in literature. However, high fidelity FEA techniques remain computationally expensive, especially for time-dependent problems. Shell models have successfully been used to model flapping wings but some thin shell elements fail under large strains or highly nonlinear problems.

The approach outlined in this research uses a decoupled solution of the fluid and structure sub-problems to generate a wing structure that, under aerodynamic and inertial loads, produces a deformed shape resulting in the minimization of an aerodynamic objective function. The aerodynamic model is based on a UVLM implementing vortex ring elements. A time-marching scheme is used for the nonlinear shell FEA model. The FEA code had been developed in a previous publication [1] and will only be briefly described in this work. This work will outline design point selection, description of the UVLM, and formulation of the Jig-shape optimization problem.

**Design Point Selection**

The intention of this MAV is to operate in an indoor environment for as long as possible. Since the MAV is designed for indoor flight, gust rejection is not a primary concern and a low flight speed is favorable in such scenarios. This design presented in this paper is the second iteration of the flight platform detailed in Moore [2]. The original design is used to estimate lift, drag, and power requirements of the new flight platform. These values dictate the design point which is used as the basis for a design optimization study.

**Introduction**

The study of flapping flight is a relatively new area of research and there are not many design tools available for the development of flapping vehicles. Conventional methods are generally meant for steady flow with negligible fluid-structure interaction. Therefore, a method is needed that is capable of resolving complex unsteady flows about structures with large deformations due to aerodynamic and inertial loads.

High fidelity Navier-Stokes (N-S) fluid solvers would be prohibitively expensive for any useful design study.

**Nomenclature**

\begin{align*}
\alpha &= \text{local angle of attack} \\
b &= \text{wing span} \\
c &= \text{mean aerodynamic chord} \\
C_D &= \text{coefficient of drag} \\
C_L &= \text{coefficient of lift} \\
C_P &= \text{coefficient of power} \\
C_T &= \text{coefficient of thrust} \\
[C_{\text{wing/wake}}] &= \text{influence matrix} \\
[C] &= \text{damping matrix} \\
\delta &= \text{wing bending parameter} \\
\Delta &= \text{incremental solution vector} \\
D &= \text{duty cycle} \\
e &= \text{induction} \\
F &= \text{vector of external forces} \\
\gamma &= \text{vorticity strength} \\
\Gamma &= \text{circulation} \\
g &= \text{gravitational acceleration} \\
G &= \text{objective function} \\
I &= \text{current} \\
k_v &= \text{speed constant} \\
k_o &= \text{torque constant} \\
[K] &= \text{stiffness matrix} \\
l &= \text{panel length} \\
[M] &= \text{mass matrix} \\
\eta &= \text{efficiency} \\
n &= \text{unit normal direction} \\
N &= \text{number of panels/elements} \\
\rho &= \text{fluid density} \\
P &= \text{vector of internal stresses} \\
R &= \text{resistance} \\
\theta &= \text{wing twisting parameter} \\
\mu &= \text{coefficient of friction} \\
\mathbf{u} &= \text{vector of FEA node displacements} \\
U &= \text{freestream velocity} \\
V &= \text{reference velocity for UVLM} \\
\omega &= \text{flapping angular velocity (2 \pi \cdot f)} \\
W &= \text{velocity influence matrix}
\end{align*}
The design problem is defined as follows:
Objective: Maximize flight time
Constraints
1. Span is 6 inches
2. Flight speed must be less than 2 m/s

Since the objective is to maximize flight time the weight of the avionics, structure, and input power should be minimized. Custom muscle wire actuators are chosen to drive the control surfaces due to their low weight and relatively high torque. Batteries are limited to commercially available cells which are currently available in sizes as small as 8.5 mAh capacity.

It is assumed that the drag coefficient of the design detailed in this work is similar to that of the original design since the construction techniques (which dictates parasitic drag) and tail design are similar. Water tunnel results of a flapping wing from [3] are used to relate the power coefficients to the reduced frequency. Although the results are only given for a wing with spanwise flexibility undergoing a plunging motion, the trend is assumed to be similar for roll-actuated wings with twisting.

\[ k = \frac{\omega \cdot c}{2U_\infty} \]  

(1)

Figure 2. Coefficients of Power and Thrust as a function of reduced frequency for plunging motion

The flapping frequency was selected to be 16 Hz since the 25 Hz used in the previous design required an excessively strong structure to handle the inertial loads. The root chord was chosen to be 2 inches and a Zimmerman planform was selected due to its similarity to avian planforms. This combination of flapping frequency and aspect ratio results in a reduced frequency of one, which has been shown in [3] to be ideal for maximizing propulsive efficiency. The power coefficient obtained by experiment in [3] is under-predicted when compared to that of the original platform by a factor of 0.67. For simplicity, it is assumed that this under-prediction applies over the entire range of reduced frequencies; this helps account for the differences in kinematics and structural damping between the experiment and the flight vehicle. It should be noted that this is a preliminary estimate and only affects the required coefficient of lift used as a constraint in the flapping kinematic optimization. A revised estimate for the coefficient of power is obtained from the UVLM analysis. From Figure 2, the power coefficient was determined to be 0.403 at the design-point reduced frequency of 1.00. This translates to a minimum power input of 12.0 mW. Since the shaft power is known, a preliminary motor can be designed for this power output using the method described in the following section. Now that the motor weight is known, the total weight of the MAV can now be determined. The lightest available lithium-polymer cell has a capacity of 8.5 mAh and a maximum discharge rate of 85 mA, much higher than the 12 mA needed by the motor and flight electronics. A 0.2 g 2.4 GHz receiver was chosen to control the MAV since it is currently the lightest commercially available 4-channel receiver. A muscle wire actuator is selected to drive the control surfaces due to its low weight and relatively high torque. The weight of the airframe was estimated based on the weight of the structure of the original design.

Table 1. Weight of components

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
<th>Weight (mg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor</td>
<td>Custom</td>
<td>127</td>
</tr>
<tr>
<td>Battery</td>
<td>GMB 8 mAh</td>
<td>380</td>
</tr>
<tr>
<td>Receiver</td>
<td>CORAL lite</td>
<td>200</td>
</tr>
<tr>
<td>MW Actuator</td>
<td>Custom</td>
<td>150</td>
</tr>
<tr>
<td>Airframe</td>
<td>Carbon/Mylar</td>
<td>300</td>
</tr>
<tr>
<td>Gears</td>
<td>62:1 Reduction</td>
<td>280</td>
</tr>
<tr>
<td>Total:</td>
<td></td>
<td>1.44 g</td>
</tr>
</tbody>
</table>

Table 2: Design point for 1st and 2nd iterations

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
<th>Weight (mg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flapping Freq.</td>
<td>25 Hz</td>
<td>16 Hz</td>
</tr>
<tr>
<td>Reduced Freq.</td>
<td>3.13</td>
<td>1.00</td>
</tr>
<tr>
<td>Shaft Power</td>
<td>2.04 W</td>
<td>12.0 mW</td>
</tr>
<tr>
<td>Span</td>
<td>10 in</td>
<td>6 in</td>
</tr>
<tr>
<td>Root Chord</td>
<td>4 in</td>
<td>2 in</td>
</tr>
<tr>
<td>Wing Area</td>
<td>31.4 in²</td>
<td>60.8 in²</td>
</tr>
<tr>
<td>Weight</td>
<td>20 g</td>
<td>1.437</td>
</tr>
<tr>
<td>Flight Speed</td>
<td>2 m/s</td>
<td>2 m/s</td>
</tr>
<tr>
<td>C_0</td>
<td>.302</td>
<td>.302</td>
</tr>
<tr>
<td>C_L</td>
<td>3.95</td>
<td>0.880</td>
</tr>
<tr>
<td>C_T</td>
<td>7.45</td>
<td>0.403</td>
</tr>
</tbody>
</table>

Table 2: Design point for 1st and 2nd iterations

The minimum required coefficient of lift and thrust listed in Figure 3 are used as nonlinear constraints in the optimization problem detailed later in this work.

Unsteady Vortex-Lattice Method

An UVLM is chosen as the aerodynamic model since it is a relatively computationally inexpensive technique and is grid-free. The UVLM is, however, very limited in the scope of problems it can solve. The continuous wing-wake ring assumption limits application to strictly forward flight which stipulates that the wake may never come into contact with the wing. Since it is an inviscid technique, only induced drag due to lift can be calculated. Therefore, viscous effects including flow separation and viscous drag are neglected although these generally play a significant role in thrust production.

Wing morphing is defined by sinusoidal deformations presented in [4] where the time-variant first and second bending and twisting modes are defined by the following functions:

\[
x' = -x_s - x_s \cdot \cos(\omega \cdot 2 \cdot \frac{y}{b}) + x_s \cdot \cos(\omega \cdot (-19 \cdot 2 \cdot \frac{y}{b}^4) + 224 \cdot (\frac{y}{b})^3 - 60 \cdot (\frac{y}{b})^5) \\
y' = y_s + f' \cdot (\frac{y}{b})^2
\]
\[ z' = z - x_\omega \cdot \sin(\theta_1) \cdot 2 \cdot y / b - x_\omega \cdot \sin(-192 \cdot (y / b)^4) + 224 \cdot (y / b)^4 - 60 \cdot (y / b)^5 + 4 \cdot (y / b)^7 \cdot (192 \cdot (y / b)^4 - 60 \cdot (y / b)^5) \] (2)

The influence of the wing vortex rings cancel out the freestream velocity and velocities due to wing motion. The contribution of each of the vortex rings is summed for each of the collocation point for each wing panel. This produces a size \( N \times N \) wing influence matrix, where \( N \) is the number of panels. The following linear system is solved for the circulation distribution, \( \Gamma_{\text{wing}} \), in order to obtain the initial circulation distribution [5].

\[ \left[ \Gamma_{\text{wing}}^i \right] \cdot \left[ \Gamma_{\text{wing}} \right] = \psi^i \] (3)

**Fig. 3 Velocity induced at point P due to vortex ring [4]**

where \( \psi \) is a vector of velocities due to freestream and wing kinematics acting at each collocation point. The line of vortex rings along the trailing edge of the wing is then shed into the wake and circulation for each of the shed ring elements is held fixed at the value at which the ring was shed in order to satisfy the force-free wake assumption [4]. The influence of the wing and the first row of vortex elements is then summed for each of the wake collocation points and the first row of wake rings is deformed. The induced velocity at the collocation points is determined from [5] as

\[ w = \left[ W_{\text{wing}}^i \right] \cdot \Gamma_{\text{wing}} + \left[ W_{\text{wake}}^i \right] \cdot \Gamma_{\text{wake}} \] (4)

For each of the subsequent time steps, a new wing circulation distribution is calculated and the next row of wake elements is shed. The influence of the wing and the newly-shed wake elements on each of the wake elements is then determined and the wake rings are translated by the sum of the freestream and induced velocities. Note that since the wing is deformable the wing influence matrix must be updated at each time step and the entire system must solved again, as opposed to simply updating the right hand side as is normally done with rigid geometries.

\[ \left[ \Gamma_{\text{wing}}^i \right] \cdot \Gamma_{\text{wing}} + \left[ \Gamma_{\text{wake}}^i \right] \cdot \Gamma_{\text{wake}} = \psi^i \] (5)

**Fig. 4 Wake shedding process using ring elements [4]**

The lift and induced drag force acting on each panel can be computed using the unsteady Bernoulli equation. The choice of \( \nu_{\text{ref}} \) and the local angle of attack vector are arbitrary and there is no standard for choosing these references. The choice of reference velocity given in [5] was used in this study and the change in lift and drag at each time step can be computed as

\[ \Delta L^i = \sum_{n=1}^{\text{wake}} \rho \cdot l_{\text{w},m,n} \left( \frac{\partial l_{\text{w},m,n}}{\partial t} \frac{v_{\text{ref}}}{\gamma_{\text{w},m,n}^i} \gamma_{\text{w},m,n}^i \right) \cdot \cos(\theta_{n,m}) \] (6)

\[ \Delta D^i = \sum_{n=1}^{\text{wake}} \rho \cdot l_{\text{w},m,n} \left( \frac{-w_{\text{ref}}}{\gamma_{\text{w},m,n}^i} \frac{\partial l_{\text{w},m,n}}{\partial t} \frac{v_{\text{ref}}}{\gamma_{\text{w},m,n}^i} \gamma_{\text{w},m,n}^i \right) \cdot \sin(\theta_{n,m}) \] (7)

where the local angle of attack \( \alpha_{n,m} \) is found by resolving the freestream velocity and velocity due to wing motions into a panel normal and chordwise component and calculating the angle of incidence at each panel. The velocities due to wing rotation and deformation are found using forward differences. The total force on a given time step is the contribution of each panel lift and drag coefficient summed over the entire wing. The pressure acting on each panel can be found resolving the individual quantities in (6) and (7) into the panel normal direction and dividing by the panel area.

The input power required to drive the wing is assumed to be dominated by aerodynamic forces and the inertial forces are neglected. This has been shown to be a good assumption for forward flight [6]. The power required at each time step can be found by using the following relation presented in [7].

\[ P^i = \sum_{m=1}^{\text{wing}} \sum_{n=1}^{\text{wake}} \rho \cdot (P_{\text{w},m,n}^i \cdot n_{m,n}^i) \cdot c_{m,n}^i \] (8)

Now that the forces are known at each time step the unsteady lift, drag, and power coefficients are calculated. The flapping efficiency can also be calculated by finding the cycle-averaged values of the thrust and power coefficients.

\[ \eta = \frac{C_t}{C_p} \] (9)
**Flapping Wing Optimization**

It is difficult to formulate a method to obtain analytical derivatives of an UVLM. It would be significantly easier for a steady scenario, but this is of no interest. Instead of focusing on a much faster analytic sensitivity analysis, the derivatives of the aerodynamic metric with respect to the wing mode design variables were computed from finite differences. This approach requires $(M_{vlm}+1)$ function calls where $M$ is the number design variables. Therefore, creating a fast code is of great importance.

Matlab's function `FMINCON` was invoked in all of the UVLM optimization studies. `FMINCON` uses a sequential quadratic programming algorithm for small to medium-scale problems, such as those presented in this work. The method relies heavily on the first derivative of the objective function with respect to the design variables and the Hessian is only approximated. Upper and lower bounds are set on the design variables in order to ensure a realistic geometry and to increase stability of the UVLM code. These bounds only apply to the control points of the cubic spline so it is possible for the wing twist and bending to violate the constraints. Two nonlinear inequality constraints are applied. As described in the previous section, the lift coefficient must exceed 0.88 and the thrust coefficient must be greater than 0.30. The work in [5] indicates that UVLMs over-predict the thrust coefficient by a factor of approximately two due to the inability to predict viscous drag and flow separation. The lower bound for the coefficient of drag was increased to 0.60 to account for this discrepancy.

Table 3. Design constraints for aerodynamic optimization

<table>
<thead>
<tr>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1^n$</td>
<td>-30°</td>
</tr>
<tr>
<td>$\theta_2^n$</td>
<td>-15°</td>
</tr>
<tr>
<td>$\delta_1^u$</td>
<td>-0.1·c</td>
</tr>
<tr>
<td>$\delta_2^u$</td>
<td>-0.08·c</td>
</tr>
<tr>
<td>$C_L$</td>
<td>0.88</td>
</tr>
<tr>
<td>$C_T$</td>
<td>0.6</td>
</tr>
</tbody>
</table>

The objective of the aerodynamic optimization is to maximize flapping efficiency (9) at the design point. Several initial geometries are investigated in order to determine the complexity of the design space. All initial geometries converge to identical optimal solutions suggesting that the design space in the region of interest is relatively smooth and not highly complex. When nonlinear constraints are introduced it is important that the vector of initial design variables produces a flapping motion that satisfies the nonlinear constraints. If the initial geometry violates a constraint the optimization algorithm will likely fail. It was determined that the baseline angle of attack must be at least 30° to satisfy the lift constraint. The thrust constraint is automatically satisfied with a rigid wing geometry. It is hypothesized that the second bending mode will produce a geometry that is unobtainable from passive aeroelastic effects. Therefore, the optimization is run twice, once for a purely twisting case and another allowing both bending and twisting.

Table 4. Summary of baseline and optimal wing performance

<table>
<thead>
<tr>
<th></th>
<th>$C_L$</th>
<th>$C_T$</th>
<th>$\overline{C}_P$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid</td>
<td>0.8892</td>
<td>0.6814</td>
<td>1.784</td>
<td>38.2%</td>
</tr>
<tr>
<td>Bending</td>
<td>1.045</td>
<td>1.637</td>
<td>2.301</td>
<td>71.1%</td>
</tr>
<tr>
<td>Bending &amp; Twisting</td>
<td>0.9779</td>
<td>1.550</td>
<td>2.061</td>
<td>75.2%</td>
</tr>
</tbody>
</table>

Fig. 5 Coefficient of lift and thrust through a flapping cycle for rigid, optimized twisting, and optimized bending/twisting geometries

Fig. 6 Coefficient of lift through the flapping cycle for rigid, optimized twisting, and optimized bending/twisting geometries
A rigid wing undergoing a sinusoidal rotation results in a roughly sinusoidal lift profile with positive lift being produced for most of the downstroke and negative lift for the upstroke, as would be expected. The maximum instantaneous lift is greater than the minimum value which is expected from a wing at a positive angle of attack. A rigid wing produces negative thrust (drag) during the first and last fifth of the flapping cycle, while providing thrust for the remainder of the cycle. Power input is greatest during midstroke and power required is negative at the end of both the upstroke and downstroke. Negative power indicates that the air loads are large enough to cause the prescribed motion.

If the wing morphing is restricted to purely twisting modes the coefficient of lift no longer has a sinusoidal form. Instead, the maximum lift is increased over the first quarter of the cycle. Lift appears to be generated by a vortex-shedding-like mechanism at the end of the down stroke. More negative lift is produced in the last third of the flapping cycle when compared to the rigid case. The cycle-averaged coefficient of lift is slightly higher than the baseline value of 0.889. The optimized deformation effectively eliminates any negative thrust produced; maximum thrust is produced in the middle of the upstroke. The majority of the lift is produced in the mid-stroke and, like the previous case, the power required is greatest during the mid-stroke. This may indicate that the aerodynamic power input is proportional to the angular velocity of the wing.

The optimization of both the bending and twisting modes results in a similar distribution of forces to that of the strictly bending case. The primary differences are that both the lift and thrust are less than the pure twisting case. The aerodynamic power required to generate this shape is less than the purely bending case, but larger than the rigid wing. Note that this is only true for the power required to overcome aerodynamic loads. The power lost due to structural damping for a bending wing may be significant.

Fig. 7 Optimal wing morphing coordinates for purely twisting and twisting/bending modes plotted against flapping angle.

Fig. 8 Wing deformation using bending and twisting modes and pressure distribution for a complete flapping cycle (L) and wake shed for 1.25 flapping cycles of wing optimized with bending and twisting modes (R)
The optimized deformations exhibit a negative first mode twisting angle through almost the entire downstroke before transitioning to a positive angle during the upstroke. This result is expected and matches trends exhibited in avian flapping motions. Interestingly, the second mode twisting remains positive for most of the flapping cycle, only becoming negative during the middle of the downstroke. The optimized wing bending coordinates show that the first bending mode traces the same path for both the upstroke and downstroke. This is an interesting result and differs from similar data presented in [5]. The second bending mode is almost constant throughout the cycle and has a value of approximately -20mm. At the beginning of the downstroke and the end of the upstroke the bending modes have an additive effect and produce a large downward tip deflection. The opposite is true for the end of the downstroke when the two modes help cancel one another. It is obvious that such a complex geometry cannot be produced in periodic form using only passive wing morphing. Therefore, the structural optimization detailed a following section will use the purely twisting case as the objective function of a jig-shaped optimization even though it as aerodynamically efficient as a coupled bending and twisting geometry.

Motor Design

There are very few commercially available motors that are light enough to power a 1.5 gram aircraft. The motors that are available are all coreless or multi-phase brushless and have poor efficiencies, ranging from 15-25%. Recently, several micro flight hobbyists have made small single-phase brushless motors for application in small fixed-wing models. Initially these were driven by a microcontroller which would be switched off for a brief period of time during which the back EMF would be read. The motor speed could then be found from the back EMF reading. However this method is not ideal since it sacrifices a portion of the duty cycle, reducing the available power output. Additionally, starting the motor from rest is difficult, if not impossible, using only a back EMF reading. Recently Allegro came out with a small integrated IC chip for powering single phase brushless motors. The IC incorporates a Hall Effect sensor for position feedback and has a thermal protection unit build in.

A simple model has been developed to design a micro brushless motor based on motor theory [8]. The model relies on experimental coefficients for various parameters including friction and induction. The motor torque and speed constant can be calculated as follows,

$$k_T = N_w \int (e \cdot n) dA$$

$$k_v = \alpha \cdot K_T$$

The induction normal to the coil surface is estimated to be 0.20 based on performance of similarly-designed single phase brushless motors. Similarly, the speed coefficient is approximated as 0.104. The static torque due to the weight of the motor shaft and the dynamic torque due to shaft rotation can be modeled as

$$T_{F, static} = \mu_s \cdot F_s \cdot r_{shaft}$$

$$T_{F, dynamic} = \beta \cdot (\omega_{shaft})^2 \cdot \frac{\Delta_{shaft}}{g} \cdot T_{F, static}$$

where $\beta$ has been experimentally determined to be 0.028 and $\Delta_{shaft}$ is the bearing air gap. Now, the voltage losses can be computed. The voltage drop due to the IC driver and the coil can be calculated from

$$\Delta V_{drive} = \frac{R_{drive}}{k_T} \cdot I$$

$$\Delta V_{out} = \frac{R_{out}}{D} \cdot I$$

The remaining voltage is the voltage due to speed and can be considered the useful voltage,

$$V_c = V_o - \Delta V_{drive} - \Delta V_{out}$$

The no-load current is found from the dynamic friction torque and the torque constant. The useful current is simply the difference between the design current and the no-load current. The shaft power is then found to be the product of the voltage due to speed and the useful current.

$$I_o = \frac{T_{F, dynamic}}{k_T}$$

Finally, the motor efficiency can be defined as

$$\eta = \frac{P_{shaft}}{V_o \cdot I_o}$$

As given earlier, the power coefficient predicted by the UVLM method is 2.301, which equates to a required shaft power of 68.6mW. This is significantly larger than the initial estimate of 0.403 and is likely a more accurate representation of the power required. The motor weight used in the selection of design point was extremely conservative and it is likely that the motor presented in this section will not be much heavier than the first estimate. The motor is designed based on a required shaft power of 80mW to account for errors in the inviscid UVLM code and variations in motor weight. A motor is designed using the model presented above based on the following operating point:

<table>
<thead>
<tr>
<th>Motor Design Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required Shaft Power</td>
</tr>
<tr>
<td>Nominal Voltage</td>
</tr>
<tr>
<td>Duty Cycle</td>
</tr>
<tr>
<td>Maximum Motor Weight</td>
</tr>
<tr>
<td>Minimum Efficiency</td>
</tr>
</tbody>
</table>

Table 5. Design constraints for single-phase brushless motor optimization

A 2.5mmODx2.5mmL N50 strength neodymium magnet magnetized across the diameter is chosen for application in the motor due to its small size. The magnet inner diameter is 1mm, therefore a 1mm OD brass tube is selected for the shaft bushing and magnet spacer. A motor is optimized for maximum efficiency subject to the constraints listed above.

<table>
<thead>
<tr>
<th>Motor Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire</td>
</tr>
<tr>
<td>Number of Turns</td>
</tr>
<tr>
<td>Magnet</td>
</tr>
<tr>
<td>No-load current</td>
</tr>
<tr>
<td>Shaft Power</td>
</tr>
<tr>
<td>Shaft Speed</td>
</tr>
<tr>
<td>Efficiency</td>
</tr>
<tr>
<td>Weight</td>
</tr>
</tbody>
</table>

Table 6. Optimal motor specifications
A motor is built according to the specifications given above. A manual mill is used to machine a mandrel on which a magnet- wire coil is wound. A hole must be drilled precisely through the center of the mandrel since the air gap assumed in the motor design is only 130μm. A brass tube is used for the shaft bushing as noted above since brass exhibits good wear characteristics compared to other metals. The shaft must only be slightly smaller than the inner diameter of the bushing. An initial motor design had poor performance due to only 25μm of play between the shaft and the bushing. Motor performance is significantly increased by drilling out the brass bushing with a 0.028" diameter drill. A drill blank of the same diameter is used as the motor shaft, resulting in a clearance of less than 5μm.

Fig. 9 Larger 810mg motor built to demonstrate design. Note the second coil under the brass rod mount which is used to measure motor speed

**Nonlinear Shell Model**

This section briefly details the computational framework for a nonlinear shell model presented in [1] in which the wing deformation response due to both aerodynamic and inertial loads is determined. An analytic sensitivity analysis implementing an adjoint method, also presented in [1], is used to obtain derivatives of the structural state which are then used in a gradient-based jig-shaped optimization. The derivation of analytic sensitivities is non-trivial and, although used in the optimization routine, will not be detailed in this work.

Several coordinate systems are needed to obtain the system response in a rigid body-attached reference frame. A global, inertial coordinate system, denoted by (X, Y, Z) and a rotating body-attached coordinate system denoted (x, y, z) are used to represent rigid-body motions. The wing deformations, accelerations, and forces are all written in the body-attached reference frame. The body-attached coordinate system (xe, ye, ze) is further discretized into element coordinate systems positioned at a corner of each triangular mesh element. An element deformation vector, de, is defined and another reference frame, (xd, yd, zd) is obtained by moving the element nodes by the deformation vector. Sinusoidal wing rotations about the roll axis are prescribed in the inertial coordinate system for each time step as

\[
\theta_i = (1 - e^{-2000\pi t}) \cdot A \cdot \sin(\omega t_i)
\]  

where A is the flapping amplitude. The exponential term is used to smoothly accelerate the wing over the first half cycle and helps avoid large transient forces at startup.

An updated Lagrangean procedure as described in [1] is used to obtain the system response within each time step. A wing node deformation vector u, written in the body-attached coordinate system, is defined at the beginning of each time step. This vector is used to generate an updated element coordinate system (xe’, ye’, ze’). The node displacements in each element due to deformations are described by a vector ud. Once the current and updated element coordinate systems have been computed an incremental vector is computed and used in a Newton-Raphson iteration scheme. Once the incremental vector is determined, it is added to the updated configuration resulting in an “iteration configuration”. The iteration configuration is then re-defined as the updated configuration and the iteration counter is incremented.

Fig 10. Coordinate systems used in the nonlinear shell model

An internal force vector is obtained by adding terms that represent in and out-of-plane stresses to form a linear stiffness matrix. A geometric stiffness matrix due to the elemental deformation is used to form a geometric stiffness matrix. The stiffness matrix and internal force vector can be generated from the aforementioned stiffness matrices, the incremental solution vector, the element deformation vector, and a transformation matrix between the body-attached reference frame and the updated element coordinate system. These terms are all written with respect to the body-attached reference frame and therefore must include inertial contributions. The element location within the inertial frame can be written as

\[
X = X_c + T \cdot (x_c + [R] \cdot (x_c + d_c))
\]  

The displacement within each triangular element can be written in terms of an interpolating matrix, H, composed of shape functions.

\[
d_c = [H] \cdot u_c
\]  

The acceleration of each wing node needed to calculate the inertial forces and virtual work can be determined by differentiating the position of the nodes in the body-attached coordinate system twice with respect to time. The derivative of the transformation matrix [T] is the product of [T] and a matrix [Ω] representing the angular velocity vector.
\[
\dot{X} = \dot{\bar{X}}_i + (\Omega + \Omega_i)[T]\left(x_i + [R_{ij}] \cdot x_j + [R_{ij}][N] \cdot u_i + 2[T][R_{ij}][N] u_j + [T][R_{ij}][N] u_j\right)
\]

The mass matrix \(M\), gyroscopic matrix \(C_{gpr}\) and dynamic stiffness matrix \(K_0\) are found from the formulation of virtual work given in [1]. The mass matrix is independent of time and only needs to be calculated once. The gyroscopic matrix is a function of time but the contribution to the total system damping is small. Therefore the gyroscopic term is neglected in order to improve code speed. The damping matrix is modeled as a scalar multiple of the mass matrix. The total stiffness matrix is the sum of the dynamic stiffness matrix and an elastic component. Similarly, the internal force vector is also the sum of its dynamic and elastic components. It should be noted that the stiffness matrix is the partial derivative of the internal force vector with respect to the vector of displacements. An internal force vector resulting from internal stress can be defined as

\[
P_i = [K_{ij}](u + \lambda)
\]

The total system response due to elastic and inertial contributions is written in the following form [7]:

\[
[M] \cdot \ddot{u} + [C] \cdot \dot{u} + [K] \cdot u + P = F
\]

The solution is integrated forward in time using an implicit method using a Newton-Raphson iteration loop within each time step. The solution at each iteration is the sum of the solution at the previous iteration and the current incremental solution vector

\[
\dot{\bar{u}}_{i+1} = \dot{\bar{u}}_{i} + \Delta t \cdot \ddot{\bar{u}}_{i}
\]

where \(i\) denotes the iteration counter and \(n\) denotes the time step. An updated Lagrangian method is implemented to compute the velocities and accelerations of the wing nodes at the beginning of each time step.

\[
\dot{\bar{u}}_{i+1} = \frac{1}{\beta} \cdot \dot{\bar{u}}_{i} + \frac{\lambda}{\beta} \cdot \Delta t \cdot \ddot{\bar{u}}_{i}
\]

The time derivatives of the incremental solution are defined in a similar manner. Derivatives of the solution at a given iteration are simply the sum of the derivative of the solution at the previous time step and the time-derivative of the current incremental vector:

\[
\dot{u}_{i+1}^{el} = \dot{u}_{i+1}^{el} + \lambda_{i+1}
\]

\[
\ddot{u}_{i+1}^{el} = \ddot{u}_{i+1}^{el} + \lambda_{i+1}
\]

Now that the deformation and its derivatives are all known, the iteration counter can be increased and the process repeated until the incremental solution vector is driven to a sufficiently small value.

**Wing Topology Optimization**

A structural topographical optimization usually requires a large number of design variables, limiting optimization algorithms to gradient-based techniques. It would be prohibitively expensive to obtain the sensitivities through finite differences for such a large number of variables and therefore analytical derivatives are desired. The formulation of a sensitivity analysis for a nonlinear shell model is presented in [1] and will not be discussed here. The approach detailed in [1] was used in this work to obtain analytic sensitivities of the objective function (optimal wing deformation) with respect to shell thicknesses. A steepest descent method is invoked in the optimization since such an algorithm is simple to implement when there are only linearly bounded constraints. It is found that the shell element model fails for small thicknesses, likely due to the inherent nonlinearity in the system. The minimum bound for element thickness is set to 60μm in order to avoid this region of the design space. The upper bound for element thickness is fixed at 2mm in order to prevent large disparities between element thicknesses. The first three nodes at the leading edge of the root chord are clamped and the wing rotation acts only through these three nodes. The objective function for the topology optimization is written as

\[
G = \int \sum_{i=1}^{N} \left(Z_i - Z_{i,jig-shape}\right)^{2} dt
\]

where the objective function \(G\) is the integral over the second flapping cycle of a least-squares fit between the current deformation in the Z-direction and the jig-shape deformation in the same direction. The initial transients are not included in the objective function since these are of no interest to a flight vehicle in cruise.

The thickness constraints chosen stipulate that a wing operating in air is dominated by inertial terms. Air loads acting on a wing of uniform thickness (250 μm) operating at 16Hz are too small to have any significant effect on the kinematics of the wing. It is found that it is not possible to obtain a reasonable result with purely inertial loads since a large moment applied to the wing due to aerodynamic forces near the trailing edge is needed to generate any significant amount of twisting. Therefore, the wing was optimized for application in water where the fluid loads would be much higher. This approach is far from ideal, but the trends should be similar to what would be seen for an air-operating design. A damping coefficient of 20 is used in all optimization runs so that the response becomes nearly periodic.

<table>
<thead>
<tr>
<th>Uniform Thickness</th>
<th>L.E. Baton</th>
</tr>
</thead>
<tbody>
<tr>
<td>Membrane Thickness</td>
<td>250 μm</td>
</tr>
<tr>
<td>Baton Thickness</td>
<td>--</td>
</tr>
<tr>
<td>Lower Thickness Bound</td>
<td>80 μm</td>
</tr>
<tr>
<td>Upper Thickness Bound</td>
<td>2mm</td>
</tr>
</tbody>
</table>

**Table 7. Topology optimization constraints**

The nonlinearity of the system and failure of the shell element under large strains makes the optimization process difficult. Even with conservative upper and lower thickness bounds the geometry will eventually be pushed to the point where the shell element fails. The optimization is carried out until failure of the shell model and it is likely that a local optima lies somewhere outside of the bounded design space. Various initial thickness distributions are investigated. It is found that the final topology is dependent on the initial thickness distribution. Thick (on the order of 1mm) uniform initial thicknesses do not produce adequate wing deformations. Thinner uniform profiles provide better results, likely due to the initial nonlinearity of the response. Several carbon-fiber baton configurations are also investigated. A leading edge baton produces a reasonable design but the shell elements fail relatively quickly. The carbon baton element thicknesses are fixed and not used as design variables in order to avoid unrealistic topologies.
The optimal geometries from the two cases presented in this work are unique. The optimal topology for the uniform initial shell reinforces the area near the clamped boundary condition, but creates a line of thin elements along a diagonal extending from mid-chord at the root to the leading edge. This effectively creates a hinge, allowing the wing trailing edge and tip to rotate about the segment of thin elements. The thickness near the center of the wing aft of the hinge is increased, increasing the moment due to inertial forces aft of the hinged section.

A similar type of topology is produced for the case of a baton-reinforced wing. Instead of the hinge being located along a diagonal it is now placed just behind the leading edge baton. The thickness of every element other than those that form the hinge segment is increased, again increasing the moment about the leading edge due to inertial forces. It should be noted that the average thickness of the topology resulting from the initial uniform shell is smaller than that of the baton-reinforced wing, resulting in a lighter wing. Figure 11 shows that the objective function evaluated for the initially uniform wing is smaller than the baton-reinforced wing after 17 optimization iterations. It should be noted that this does not indicate the diagonally-hinged wing is superior to the baton-reinforced topology since it is impossible to predict how the baton-reinforced geometry would evaluate past the 7th iteration where the FEA code failed.

Fig. 11. Objective function as a function of optimization iteration for two initial thickness distributions (L). Deformation at \( t/T = 0.5 \) during the first iteration of a uniform shell thickness, where the wireframe wing is the rigid body motion (R).
Conclusions

This work has detailed the design and optimization of a six-inch-span ornithopter for application in an indoor environment. Experimental data for a plunging wing has been used to estimate the power requirements for flapping flight and this estimate is used as the basis of selecting a motor and battery. A jig-shape optimization using an unsteady vortex-lattice method for the aerodynamic model and a nonlinear shell FEA method for the structural model is run at the design-point, with an objective of maximizing cycle-averaged flapping efficiency. The wing response is limited to passive mechanisms, namely inertial and aerodynamic loads. A revised power estimate is obtained from the UVLM analysis and is used to design a single-phase brushless motor which is intended to power the flight platform. The jig-shape obtained from the aerodynamic optimization indicates that twisting has a large effect on propulsive efficiency at high reduced frequencies, and bending modes are not as important in this operating regime.

It was thought that the design space for the structural optimization could be complex, therefore, several initial geometries were evaluated to investigate the design space. A uniform shell thickness and a leading-edge baton topology proved to be the most promising and were used in a gradient-based optimization procedure. The structural optimization was hindered by the instability of the nonlinear shell model. The design would be pushed into a region where the shell model would fail and therefore no optima were reached although significant reductions in the objective function were still realized. Since the jig-shape optimization resulted in large disparities between the obtained and desired response it is likely that the aerodynamic performance significantly deviates from the desired values. An iterative approach is needed in order to obtain an accurate estimate of the true performance for the proposed wing topologies.

There were several assumptions made in this work that may need to be re-considered. The first is that the design variables dictating the wing deformation will be pushed in the right direction when a UVLM is used as the aerodynamic model. The UVLM cannot predict massively separated flow which is sure to occur at these high reduced frequencies and the design gradients do not reflect sensitivities to such phenomenon. Secondly, it is assumed that the inertial power required to power the flapping wings is negligible. This may be an adequate assumption for wings operating in water, but this may not hold for an air-operating wing. Thirdly, it is assumed that the wing topology design gradients for a thin membrane are similar to that of a thicker membrane. This may not hold for very thin, billowing membranes.

This work provides a basis for future design of flapping wings using a jig-shaped approach which is much less computationally intensive than a fully coupled aeroelastic solution. Many of the problems associated with the nonlinear shell model may be solved by switching to a linear shell representation and would also significantly reduce computational cost. A Navier-Stokes solver could be used in place of the UVLM in order to accurately model vortices, although a three-dimensional unsteady N-S method would be extremely expensive. An alternative may be to use a coupled lifting-line theory and unsteady 2D N-S solver to predict flow separation while neglecting most 3D effects.

References


