CHAPTER 1
Introduction

Within this chapter, piezoelectricity is briefly reviewed along with some current technical applications of piezoelectric actuation. These examples provide the motivation for this research. The proposed work and its contribution to engineering are also described. Lastly, an overview of the remainder of the dissertation is presented.

1.1 Background

Since the discovery of piezoelectricity by Jacques and Pierre Curie over 100 years ago and the theoretical quantification of the relation between the piezoelectricity and crystal structure by Woldemar Voigt in 1894, piezoceramic materials have been widely used in lab instruments for micropositioning, sonar devices, communication devices, etc. Some of their advantages include: small size, high actuation energy density, relatively low power consumption, high resolution, and the capability of being used both as an actuator and sensor.

A piezoelectric ceramic is a material that demonstrates the piezoelectric effect and the inverse piezoelectric effect. When a mechanical force is applied across a piezoceramic element, electrical charges are generated. If electrodes are placed on both sides of the material, a voltage is generated. Therefore, mechanical energy is transformed into electrical energy. This is called the piezoelectric effect, or sensor effect. Conversely, if an electrical field is applied across the material, a mechanical deformation results. Thus, electrical energy is transformed into mechanical energy. This is called the inverse piezoelectric effect, or actuator effect. Therefore, piezoceramic materials can act as either a sensing element, or an actuation element, or both. These effects are described by Figure 1.1.
The piezoelectric effect occurs naturally in quartz crystals but with very low energy transferring efficiency. It can also be induced in other piezoelectric materials such as Lead Zirconate Titanate (PZT). Such materials usually have higher energy transferring efficiency, so they are used more often in practice. Because they are ceramics, they can be manufactured into virtually any wanted shape. To induce the piezoelectric effect of the material, the piezoceramic materials are first manufactured into the desired shape. Then electrodes are put on opposite sides of the materials and a strong DC electric field is applied to the materials so that the molecular dipoles in the ceramics are aligned along the direction of the electrical field. This process is called polarization. After the electrical field is removed, the materials exhibit piezoelectric properties. The material has the interesting property that the dipoles remain, to a large extent, aligned. The polarized material, whose polarization direction, aligned with the electrical field when being polarized, is called “polar”. Please refer to Appendix A for information about dipole moment and polarization. Figure 1.2 shows the polarization process in a microscopic view ([6] [141]).

To identify the directions in a piezoceramic element, according to the IEEE standard on piezoelectricity, three axes labeled as “1”, “2” and “3” are used, associated with the x, y, and z axes in a classical three dimensional orthogonal coordinate system. The polar direction is labeled as “3”, and is parallel to the direction of polarization within the ceramic.

The piezoelectric constants relating the mechanical strain and the applied electric field are termed the electromechanical coupling constants. By convention, they are
Material is put in a strong electrical field, the dipoles are aligned in the electrical direction. E: input electrical field

After the electrical field is removed, the material is a polar. The dipoles direction can vary away from the aligned direction, but the net effect is along the aligned direction.

Figure 1.2: Polarization process

represented by the “d” coefficients. This is an important coefficient in the positioning mechanism. For the piezoelectric effect, $d_{ij}$ is defined as the ratio of the electrical charge collected on the electrodes and the applied mechanical stress.

$$d_{ij} = \frac{\text{Short circuit charge density}}{\text{Applied mechanical stress}}$$

where, $i = 1, 2 \cdots 6$ and $j = 1, 2 \cdots 6$. Units are often expressed as $\frac{\text{Coulombs/Meter}^2}{\text{Newton/Meter}^2}$.

Conversely, in the inverse piezoelectric effect, $d_{ij}$ is defined as how much deformation the material will generate when an electrical field is applied.

$$d_{ij} = \frac{\text{Strain developed}}{\text{Applied electrical field}}$$

The units of this coefficient are often express as $\frac{\text{Meters/Meter}}{\text{Volts/Meter}}$. Large $d_{ij}$ constants indicate large mechanical displacements which are usually favorable in motion transducer devices. “$d_{33}$” and “$d_{31}$” are the two constants often used in a positioning mechanism in response to the input voltage. The “$d_{33}$” motion means when an input voltage is applied along the polar (or 3) axis, motion is produced in the polar direction. Thus we say that the actuator works in the “$d_{33}$ mode” or “longitudinal mode”. When an input voltage is applied in the polar direction and the motion we take advantage of
is in the “1” direction, we say the actuator works in the “d_{31} mode” or “transverse mode” or “in-plane mode”.

1.2 Constitutive Equation for Linear Piezoelectricity

The constitutive equations for linear piezoelectricity are available from many sources, and they can take many different forms. For example, two forms of such constitutive equations, which will be often referred to in this dissertation, from Richard Holland and E. P. EerNisse [5], are

\[\varepsilon_{ij} = S_{ijkl}^E \sigma_{kl} + d_{mij} E_m \]
\[D_n = d_{nkl} \sigma_{kl} + \epsilon^\sigma_{mn} E_m \]  

(1.1)

or

\[\sigma_{ij} = \epsilon_{ijkl}^E \varepsilon_{kl} - e_{mij} E_m \]
\[D_n = e_{nkl} \varepsilon_{kl} + \epsilon^\varepsilon_{nm} E_m \]  

(1.2)

The equation is written in the “matrix/vector” summation form. For an isotropic media, the tensors have the following relations:

\[\varepsilon_{ij} = \varepsilon_{ji}, \quad \sigma_{ij} = \sigma_{ji}, \quad d_{mij} = d_{mji} \]
\[S_{ijkl}^E = S_{klij}^E = S_{jikl}^E = S_{lkij}^E \]

The constitutive equation can be reexpressed as

\[\varepsilon_\nu = S_{\nu\mu}^E \sigma_\mu + d_{m\mu} E_m \]
\[D_n = d_{n\mu} \sigma_\mu + \epsilon^\sigma_{nm} E_m \]  

(1.3)

or

\[\sigma_\nu = \epsilon_{\mu\nu}^E \varepsilon_\mu - e_{m\nu} E_m \]
\[D_n = e_{n\mu} \varepsilon_\mu + \epsilon^\varepsilon_{nm} E_m \]  

(1.4)

where, \(\mu, \nu = 1,2\ldots6, \ m, \ n = 1,2,3\). The relation between \(\mu, \nu,\) and \(m, n\) are described in Table 1.1. The meanings of the symbols are described in Table 1.2.
\[ ij = ji \quad \mu, \nu \quad ij = ji \quad \mu, \nu \]

\begin{tabular}{lll}
11 & 1 & 12 = 21 & 4 \\
22 & 2 & 13 = 31 & 5 \\
33 & 3 & 23 = 32 & 6 \\
\end{tabular}

Table 1.1: Relation of subscripts

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Meanings</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon )</td>
<td>Strain</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Stress</td>
</tr>
<tr>
<td>( E )</td>
<td>Electrical field</td>
</tr>
<tr>
<td>( D )</td>
<td>Electric charge displacement</td>
</tr>
<tr>
<td>( S^E )</td>
<td>Adiabatic compliance at constant electric field</td>
</tr>
<tr>
<td>( d )</td>
<td>Piezoelectric coupling tensor</td>
</tr>
<tr>
<td>( \varepsilon^\sigma )</td>
<td>The permittivity at constant stress</td>
</tr>
<tr>
<td>( \varepsilon^\varepsilon )</td>
<td>The permittivity at constant strain</td>
</tr>
<tr>
<td>( c^E )</td>
<td>Elastic coefficient under constant electrical field</td>
</tr>
<tr>
<td>( e )</td>
<td>( e_{\mu\nu} = d_{\mu\nu}C_{\mu\nu} )</td>
</tr>
</tbody>
</table>

Table 1.2: Symbols and descriptions

1.3 Problem Statement and Motivation for Research

Since piezoceramic elements can generate a very small deformation under applied voltage, they can be applied to problems involving precision control and high-resolution positioning. For example, the actuators can provide a positioning resolution of up to several nanometers. Also, it can generate voltage in response to the applied mechanical force. This characteristic makes piezoelectric actuators a good candidate for vibration suppression of structures, machines and vehicles. The result of the research in this dissertation provides a theoretical foundation for several applications that include micropositioning, precision design and vibration suppression.

1.3.1 Application to Micropositioning

In the application of micropositioning and precision control, the piezoceramic transducers are typically used to position a mechanical component. At lower drive
levels, the piezoceramic materials exhibit nearly linear dynamics and negligible hysteresis. Thus, a linear model is sufficient to study the dynamic behavior. However, the materials are usually driven at moderate to high levels. At these drive levels, due to their ferroelectric nature, piezoceramic materials show a non-linear hysteretic behavior. Most smart materials exhibit hysteresis in the relationship between the input electrical field and the output response, such as velocity and displacement. Also it has been shown that the hysteresis response of the materials is frequency-dependent. To provide a framework amenable to characterize these relationships, one topic of current research in this dissertation is related to modeling the rate-dependent hysteresis of a piezoelectric (PZT) stacked actuator. A literature review of hysteresis modeling is presented in Section 2.1 of Chapter 2.

1.3.2 Application in Vibration Suppression

When a PZT actuator is attached to a vibrating structure, the PZT is strained because of the structural vibration, thus a voltage is generated. The generated electrical energy can be dissipated through a shunt circuit and thus the vibration of the structure is suppressed. Many methodologies have been proposed for the use of piezoelectric devices in motion control. A recently introduced methodology relies on switching strategies to achieve vibration control. This thesis provides a theoretical foundation for some of these techniques. The shunt circuits, control strategies and literature review relating to vibration absorption will be presented in detail in Section 2.2 of Chapter 2.

1.4 Proposed Work and Contributions

The research in this dissertation is related to the two distinct topics that were introduced in Section 1.3. This dissertation presents the necessary theoretical foundations and derivations, numerical simulations and experimental results. Thus, this work provides a contribution to the field of engineering in two primary ways.
1. A frequency dependent hysteresis model is created to predict the displacement of a PZT actuator. The model can be used to improve the control performance of these actuators.

2. An averaging method is applied to a discontinuous state switched system to obtain the effective response of some state-switched piezostructural systems such as shunt capacitance system and shunt resistance system.

1.4.1 Frequency Dependent Hysteresis Modeling

Contribution of this portion of the research is related to the applications of the PZT materials in micropositioning and precision control. One example is the atomic force microscope (AFM) positioning system. The response of the PZT materials with respect to the input exhibits a frequency dependent hysteretic behavior, which considerably affects the resolution of the positioning. Recently, some linear models or static hysteresis models to characterize the behavior of the piezoceramic materials have been derived. However, the frequency dependency in hysteresis in piezoelectric actuators has not been studied extensively within the controls literature. The objective of this research is to develop a frequency dependent hysteresis model based on the elementary hysteresis operators. The creation of such a model will allow us to predict the hysteresis displacement response for the PZT actuators over a wider range of frequencies than currently exists. To accomplish this objective, the history dependent and rate-independent KP kernels are employed as the elementary hysteresis operator, as expressed in Equation 1.5.

\[
P_\mu[u, \xi, \theta](t) = \int_{\mathcal{S}} [k_s(v, \xi(s))] (t) d\mu_f(s)
\]  

(1.5)

where, \( P \) is a quantity which has hysteretic behavior, \( k_s \) stands for the kernel, and \( \mu_f \) is a frequency-dependent probability measure. The subscript \( f \) indicates that the probability measure \( \mu \) is frequency dependent.
For modeling the frequency dependent hysteresis model, the following tasks are performed:

1. A rheological model is derived based on an elementary hysteresis operator. The electrical field in the governing equation is expanded as superposition of weighted Preisach KP kernels, where the weights are frequency dependent. This is expressed in Equation 1.5.

2. An approximation methodology is derived. This includes:
   a. Creating a finite element model of the governing equation, including conventional elastic effects and hysteresis effects.
   b. Expressing the control influence operator $B(\mu)$ as
      \[
      B(u) = \int K(\bar{s}, u(\xi, t)) \mu_f(ds)
      \]
      where the probability measure is discreted as $\mu_f(\cdot) = \sum w_k \delta_k(\cdot)$, and the subscript $f$ indicates that the probability measure is frequency dependent.

3. The weights are identified via the least square method from experimental data.
4. An experiment is created to obtain data in order to identify the displacement characteristics of the actuator. Within the experiment, the responses of a PZT stacked actuator is measured for several frequencies within a range of input voltage.
5. A mathematical formulation of how each weight changes with frequency is created.
6. The performance of the model is validated by using the empirical data.

1.4.2 Averaging Method

Contribution in this portion of the research is directed to reducing vibration within a structure through dissipation using the PZT materials. The switching strategies to achieve vibration control introduce discontinuities in the state space model of the system. To study the dynamic properties of the system, it is favorable to make
the system continuous. In this dissertation, the averaging method is applied to the state space model to find a continuous approximation to the original system.

Specifically, the following tasks are performed:

1. The governing equation of motion of a 2-DOF vibrating system is derived. The system dissipates energy through a PZT patch attached to the structure mass. The state space model for the system is derived.

2. The state space model of the system has a discontinuous coefficient matrix. An averaging method is applied to the state space equation in order to find an approximation to the governing equation. The resulting system is a continuous one.

3. Two cases of state switch circuits are studied — the shunt capacitor and the shunt resistor.

4. Circuit realization of the continuous system is presented.

1.5 Dissertation Outline

Within Chapter 2 a comprehensive literature review pertaining to hysteresis modeling and tunable vibration absorption is presented. Specifically, the hysteresis generating mechanisms and previous work performed on the hysteresis modeling within the controls literature are reviewed. Models for tunable vibration absorbers and new techniques in vibration absorption utilizing piezoceramic materials are investigated. General analysis about modeling and averaging on state space models for periodic switching control signals are also discussed.

The theoretical foundation about the Preisach method is reviewed in Chapter 3. This chapter establishes the nonlinear constitutive equation of PZT materials, and presents the theoretical foundation for the current research. The Preisach method to quantify the nonlinearity of the piezoceramic materials via an effective electrical field is described.
Chapter 4 provides the theoretical derivation of the motion of the PZT stacked actuator. The strong form and weak form of the governing equation are derived. Analytical and numerical solutions are discussed. A finite element approach and reduced order approximation will be applied to solve the governing equation.

Chapter 5 explains the experimental objectives. It also shows the experimental set up and results.

In Chapter 6, the frequency dependent hysteresis model is developed based on the theoretical derivation presented in Chapters 2, 3 and 4 along with the experimental results that are provided in Chapter 5. Within this chapter, the model is validated.

The governing equation of a 2-DOF vibration system, which employs a PZT patch to suppress the vibration with a shunted switched circuit is derived in Chapter 7. The averaging method is applied on the system and a circuit simulation of the governing equation is given. The performance of the systems before and after being averaged is compared.

Finally, within Chapter 8 a summary of the current research and recommendations for future work is presented.