Homework #11

(Due December 5 at the beginning of the class.)

Numerical Method Series #6:

Engineering applications of Newton-Raphson Method to solving systems of nonlinear equations

Goals:

1. Understanding the solution procedure of a modified Newton-Raphson Method: Problems 1 and 2.
2. Understanding engineering application of Newton-Raphson Method and convergence of numerical approximation: Problem 3.
3. Understanding the application of the Newton-Raphson method to solving a system of nonlinear equations: Problem 4.

Problem 1 (20 points)

Develop a Mathcad function that performs root finding using the *modified* Newton-Raphson method.

Instruction for Problem 1**:**

1. You **must** follow the homework template format as posted at the web page (www.ce.ufl.edu/~jchun/CGN3421/downloads.html)
2. You **must** validate your program.

Problem 2 (20 points)

(a) Using both the Mathcad functions of the standard and modified Newton-Raphson methods, evaluate a double root of

$$f(x) = x^3 - 5x^2 + 7x - 3 = (x-3)(x-1)(x-1)$$

with an initial guess of x=0. **Compare** the convergence rate between the standard and modified Newton-Raphson methods using a maximum number of iterations equal to 6.

(b) Using both the Mathcad functions of the standard and modified Newton-Raphson methods, evaluate a single root at x=3 with an initial guess of x=4.

Compare the convergence rate between the standard and modified Newton-Raphson methods using a tolerance of 0.003.

Instruction for Problem 2**:**

1. You **must** follow the homework template format as posted at the web page (www.ce.ufl.edu/~jchun/CGN3421/downloads.html)
2. You **must** provide your logical reasoning in the comparison.

Note:
You may solve either Problem 3-1 or Problem 3-2 at preference for the credit of Problem3 (40 points)

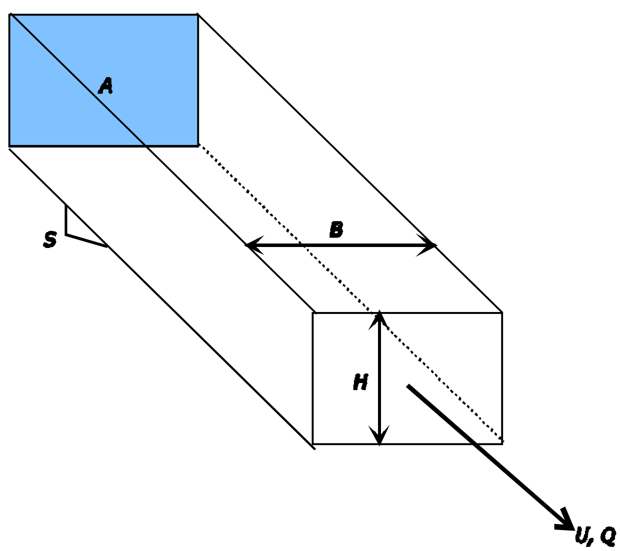
Problem 3-1 (40 points)

Civil engineering is a broad field that includes such diverse areas as structural, geotechnical, transportation, construction management, environmental, and water-resources engineering. The last two specialties deal with both water pollution and water supply. Thus, fluid mechanics are extensively used. One of general problem relates to the flow of water in open channels such as rivers and canals. The flow rate, which is routinely measured in most major rivers and streams, is defined as the volume of water passing a particular point in a channel per unit time ($m^3 / \sec$).
Although the flow rate is a useful measurement, a further question relates to a specific flow rate in a sloping channel. The water will reach a specific depth H (*m*) and move at a specific velocity U ($m/\sec$). Therefore,

$$Q = UA \qquad \text{Eqn. (1)}$$

where A represents the cross sectional area of the channel (m^2). If it is a rectangular channel (See the figure below), then

$$Q = UBH \qquad \text{Eqn. (2)}$$



However, although B (the width of the rectangular channel) is specified, we have one equation and two unknowns (U and H). We need additional equation to solve the unknowns.

For uniform flow, using the semi empirical equation called Manning equation, the flow rate through a rectangular channel can be evaluated as:

$$Q = \frac{S^{1/2}}{n} \frac{(BH)^{5/3}}{(B+2H)^{2/3}} \qquad \text{Eqn. (3)}$$

where n= the Manning roughness coefficient (a dimensionless number representing the channel friction), S=the channel slope.

If $Q = 5\, m^3/\sec$, $B = 20\, m$, $n = 0.03$, and $S = 0.0002$, **then what would be the specific velocity *U* of the channel?**

Your tasks are given as follows:

1. Write the flow rate equation, i.e., Eqn. (3), as a function of *H*. Set the resulting equation equal to zero. This equation should be solved for the value of *H* (5 points).
2. Plot the flow rate equation as a function of the depth *H* in a range of 0 *m* to 2 *m* (5 points)
3. Using the program developed in Homework #10, find the root of the flow rate equation, which represents a specific depth of the flow that does not vary spatially and temporally, i.e., uniform flow (10 points)
4. Find out the specific velocity of the flow *U* (10 points)
5. Use your own judgment to terminate your iterations when the value of *U* is sufficiently accurate (10 points)

Instruction for Problem 3-1:

1.You **must** follow the homework template format as posted at the web page (www.ce.ufl.edu/~jchun/CGN3421/downloads.html)
2.You **must use** a MathCAD graphical tool to make an initial estimate of the root.
3. You must validate your answer.

If any of the above instructions is not followed, your solution for Problem 3 will be returned with **zero** grades.

Problem 3-2 (40 points)

Two fluids at different temperatures enter a mixer and come out at the same temperature, The heat capacity of fluid A at constant pressure is given :

$$C_p = 3.381 + 1.804\times10^{-2} T - 4.300\times10^{-6} T^2 \qquad \text{Eqn. (4)}$$

and the heat capacity of fluid B at constant pressure is given :

$$C_p = 8.592 + 1.290\times10^{-1} T - 4.078\times10^{-5} T^2 \qquad \text{Eqn. (5)}$$

where C_p is in units of $cal/gram\ K$ and T is in units of K (Kelvin).

Heat is always transferred from the object at the higher temperature to the object with the lower temperature. Note that

$$\Delta H = \int_{T_1}^{T_2} C_p\, dT \qquad \text{Eqn. (6)}$$

which represents that the increase of specific enthalpy in a fluid is equal to the energy gained if the temperature of fluid rises (or the decrease of specific enthalpy in a fluid is equal to the energy lost if the temperature of fluid falls) for a constant pressure process.

For your information, the heat capacity is a constant that tells how much heat is added per unit temperature rise. The value of the constant is different for different materials. When scientists began to study the coldest possible temperature, they determined an **absolute zero** at which molecular kinetic energy is a minimum (but not strictly zero!). They found this value to be at -273.16 degrees of Celsius ^{O}C. Using this point as the new zero point we can define another temperature scale called the **absolute temperature**. If we keep the size of a single degree to be the same as the Celsius scale, we get a temperature scale that has been named after Lord Kelvin and designated with a *K*:

$$K = {}^{O}C + 273.16$$

Fluid A enters the mixer at 400 ^{O}C and Fluid B enters the mixer at 700 ^{O}C. There is twice as much as Fluid A as there is Fluid B entering into the mixer. **At what temperature do the two fluids exit in the mixer at thermodynamic equilibrium?** Use the Mathcad function that finds a root using the Newton-Raphson method, which was developed in Homework #10, to determine your answer.

Your tasks are given as follows:

a. Write the enthalpy balance equation, i.e., Eqn (6), such that the enthalpy gained in Fluid A is equal to the enthalpy lost in Fluid B. Set the resulting equation equal to zero. This equation should be solved for the value of *T* at a thermodynamic equilibrium (5 points).
b. Plot the enthalpy balance equation as a function of the temperature *T* over a range of 600 Kelvin to 1200 Kelvin (10 points)
c. Using the program developed in Homework #10, find the root of the enthalpy balance equation, which represents a temperature at equilibrium (15 points)

d. Use your own judgment to terminate your iterations when the value of T (in the unit of Kelvin) is sufficiently accurate (10 points)

Instruction for Problem 3-2:

1.You **must** follow the homework template format as posted at the web page (www.ce.ufl.edu/~jchun/CGN3421/downloads.html)
2.You **must use** a MathCAD graphical tool to make an initial estimate of the root.
3. You must validate your answer.

If any of the above instructions is not followed, your solution for Problem 3 will be returned with **zero** grades.

Problem 4 (20 points)

A system of two nonlinear equations is given;

$$f(x,y) = x^2 + xy - 10 \text{ and } g(x,y) = y + 3xy^2 - 57$$

(a) (15 points) **Solve** for **x** and **y** (i.e., f(x,y)=0 and g(x,y)=0) by hand using both the Newton-Raphson and Gauss Elimination methods:

$$\begin{bmatrix} \frac{\partial f_i}{\partial x} & \frac{\partial f_i}{\partial y} \\ \frac{\partial g_i}{\partial x} & \frac{\partial g_i}{\partial y} \end{bmatrix} \begin{Bmatrix} x_{i+1} \\ y_{i+1} \end{Bmatrix} = -\begin{Bmatrix} f_i \\ g_i \end{Bmatrix} + \begin{bmatrix} \frac{\partial f_i}{\partial x} & \frac{\partial f_i}{\partial y} \\ \frac{\partial g_i}{\partial x} & \frac{\partial g_i}{\partial y} \end{bmatrix} \begin{Bmatrix} x_i \\ y_i \end{Bmatrix}$$

where $f_i = f(x_i, y_i)$ and $g_i = g(x_i, y_i)$. Use initial guesses as x=1.5 and y=3.5. Perform only 2 iterations of the algorithm**.** You can differentiate *f(x,y) and g(x,y)* using the Mathcad symbolic solver.

(b) (5 points) **Describe** what a Jacobian matrix physically represents.

Instruction for Problem 4**:**

1. Your work **must** be written on engineering paper.
2. Your work **must** illustrate solution iteration process in details.

If any of the above instructions is not followed, your solution for Problem 4 will be returned with **zero** grades.