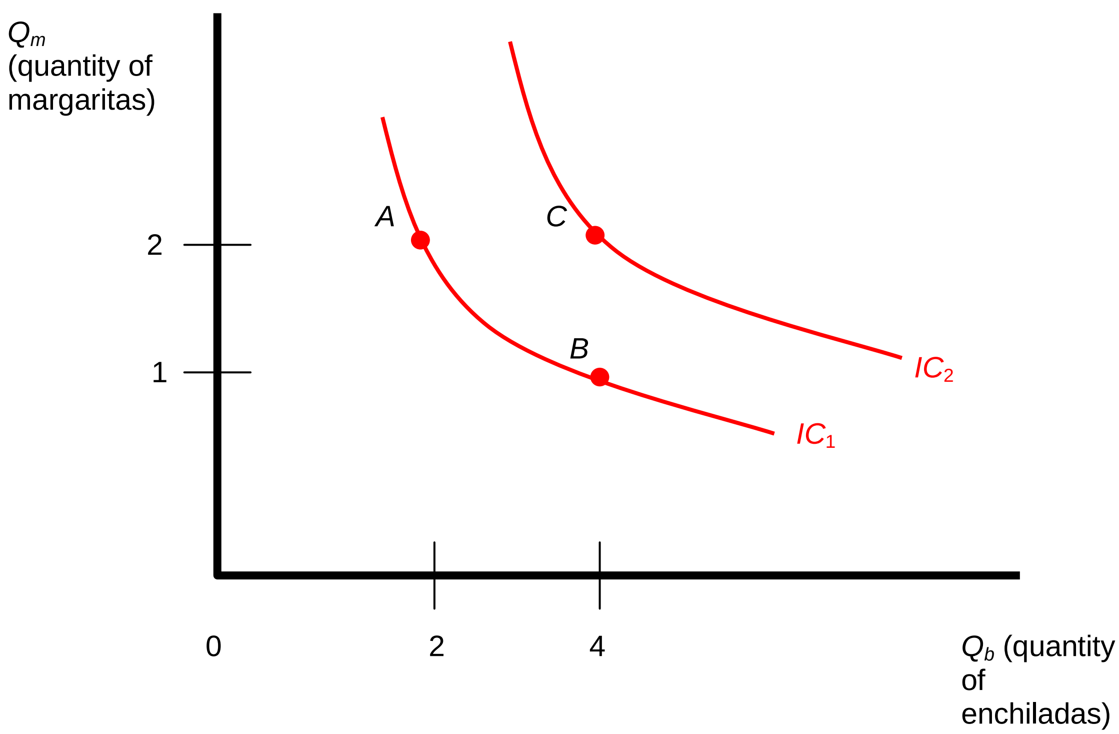
- Constrained Utility Maximization
  - General idea: given an individual's preferences and his budget constraint, what *bundle* of goods will he choose to buy
  - Assume: individuals have a well-defined utility function (i.e. a mathematical function translating consumption into units that can compared).
    $U = f(x_1, x_2, ..., x_n)$
  - **Utility**: that on which an individual's preferences are based (e.g. happiness, well-being, satisfaction).
  - **Budget constraint**: describes the choices available to an individual, given his resources, e.g. $p_1x_1 + p_2x_2 + ... + p_nx_n \le I$
  - We then assume that individuals seek to maximize their own utility subject to their budget constraint.
    $$\underset{x_1,x_2}{Max} U = f(x_1, x_2)$$
    $$s.t. \;\; p_1x_1 + p_2x_2 \le I$$
  - **Indifference Curves**: a graphical representation of bundles between which an individual is indifferent. Shows all bundles that make an individual equally well off.

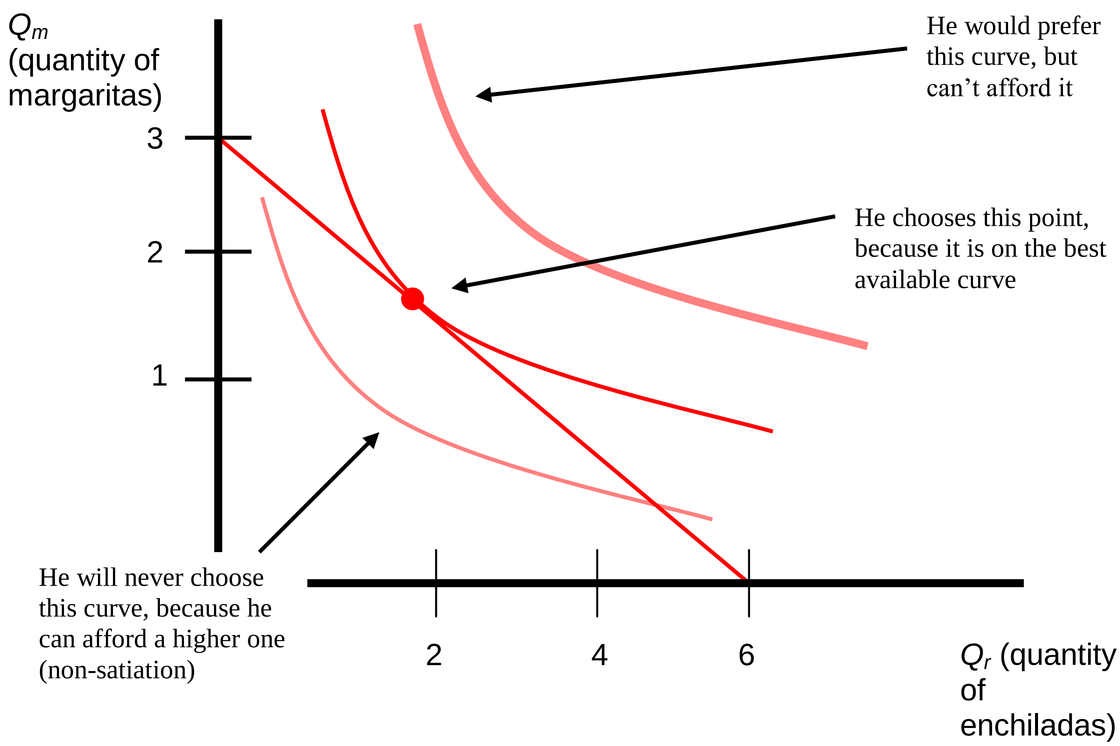


  - In this case, bundles A & B lie on the same IC, thus the individual is indifferent between a bundle containing 4 beers and 1 margarita and one containing 2 enchiladas and 2 margaritas.
  - Key assumption**: Non-satiation** (more is better).
  - Therefore, bundle C (4 enchiladas and 2 margaritas) is preferred to A or B and lies on a higher IC. Higher ICs are *always* preferred.
  - Non-satiation also means that IC's always slope downwards.

- Indifference curves are derived from the utility function by setting utility equal to an arbitrary value and the finding all combinations of goods that lead to that utility level.
  - Example: $U = x_b x_m$

    $4 = x_b x_m$

    Then we draw a line through all the points that satisfy the equality (e.g. [1,4] [2,2] [4,1])
- **Marginal Utility:** the additional utility gained by consuming one more unit of the good. Marginal Utility is almost always decreasing.
  - Derivatives show how the function changes in response to a change in an input. Show how to take partial derivative
  - We find the MU of a good by taking the *partial derivative* of the utility function with respect to that good. $MU_{Xm} = \frac{\partial U}{\partial x_m} = x_e$
- **Marginal Rate of Substitution:** describes how much of one good an individual is willing to give up in order to get more of another good, i.e. the marginal benefit of x in terms of y.
  - **MRS = -MUb / MUm**
  - MRS is the slope of the IC.
  - MRS changes from point to point and is diminishing in absolute value as you move to the right along the IC.
- Add **budget constraint** to graphical representation:
  - Assume that there is no borrowing or saving, only the current period's income.
  - The vertical intercept is the number of margaritas you could buy if you spent all your income on margaritas.
  - The slope of the budget constraint is the negative of the price ratio, ${}^{p_e}\!/_{p_m}$.

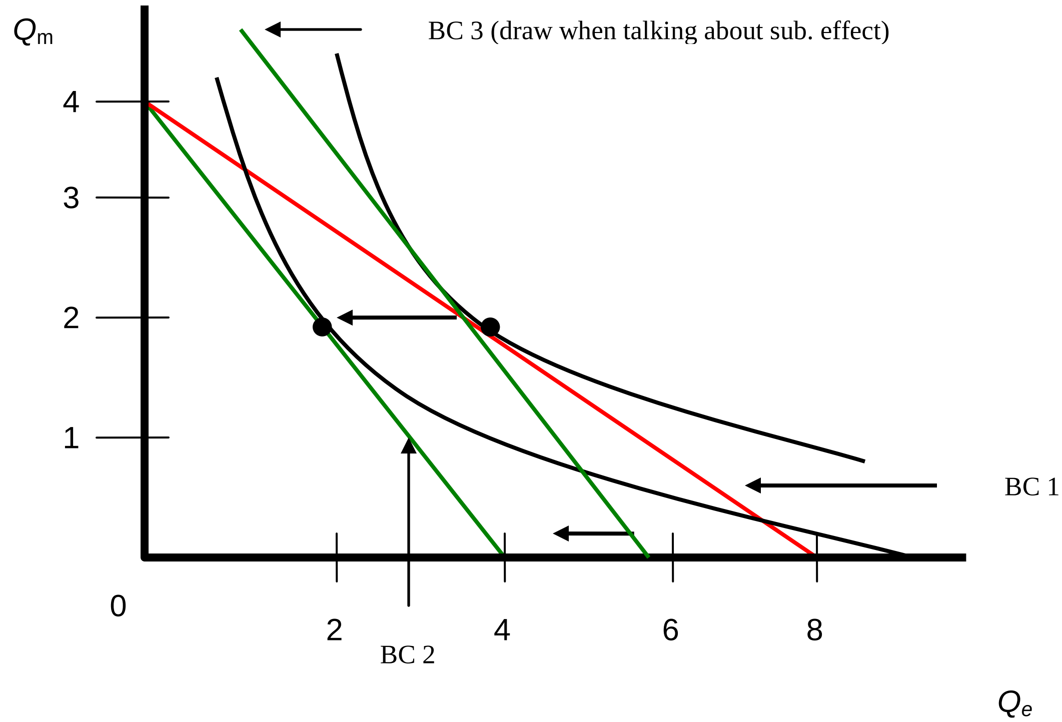


- The individual can afford anything on, or to the left of the budget constraint.
- Since he is trying to maximize his utility, he chooses the highest possible indifference curve. This curve will always be tangent to the budget constraint.
- Therefore, $MRS = {}^{-MU_e}\!/\!_{MU_m} = {}^{-p_e}\!/\!_{p_m}$

- Then, we know utility is maximized where: $MRS = {}^{-p_e}\!/\!_{p_m}$

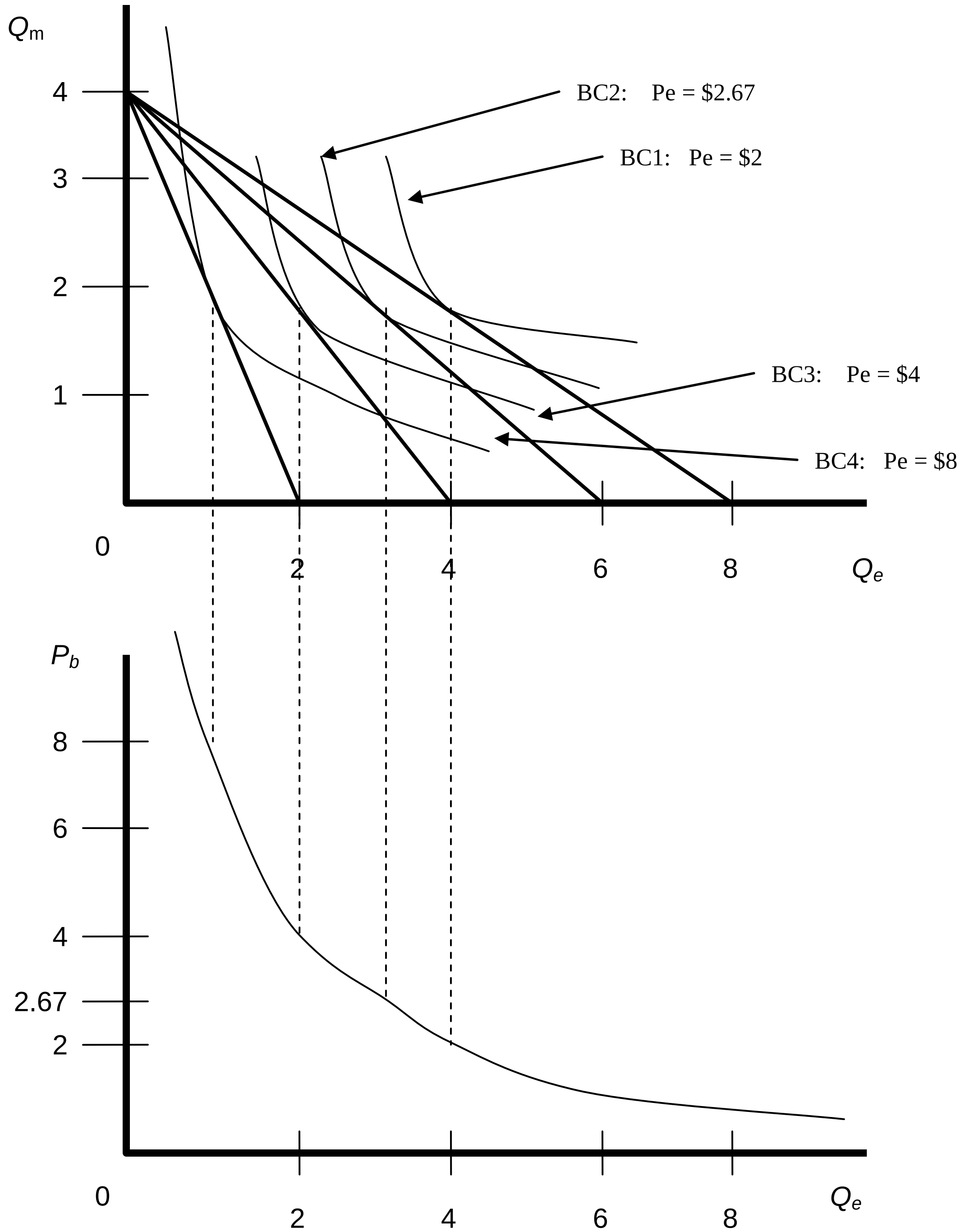
  and $p_e x_e + p_m x_m = I$. In English, this means that the negative of the price ratio is tangent to the indifference curve and the individual is spending all of his money.

- Price Changes: since $-MRS = {}^{p_e}\!/\!_{p_m}$, when p_e rises, the absolute value of MRS also rises. Recall that the absolute value of MRS rises as we move to the left along the IC. This implies that q_e falls relative to q_m.



- For example: with BC_1, $p_m = 4$, $p_e = 2$ and $I = 16$. With BC_2, $p_m = 4$, $p_e = 4$ and $I = 16$. q_b goes from 4 to 2.
- There are actually two parts to the effect of a price change:
    - **Substitution effect**: holding utility constant, the change in consumption due to the change in relative prices (movement along the IC).
    - **Income effect**: the change in consumption due to being "poorer" (if you can't buy as much you are poorer) after a price change (movement between ICs).
    - Look at book example on p. 35.

- **Demand Curves**: derived from utility maximization, they show the relationship between the price of a good and the quantity demanded.
    - Individual demand curves show an individual's willingness to pay, or marginal value.



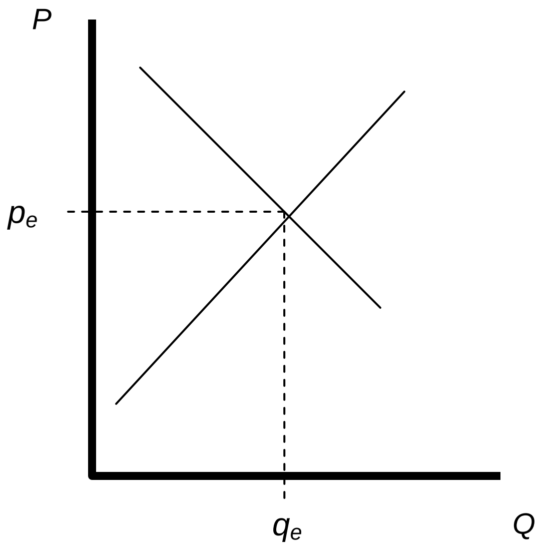
- The **market demand curve** is found by horizontally summing the individual demand curves.
- **Elasticity of demand**: the percentage change in quantity demanded for each percentage change in price, i.e. $\varepsilon = \frac{\Delta Q / Q}{\Delta P / P}$.

        - Elasticity of demand is usually negative because when prices rise, quantity falls.
        - Elasticity of demand usually is not constant along the demand curve.
        - A vertical demand curve is perfectly inelastic; a horizontal demand curve is perfectly elastic.
    - Cross price elasticity: the percentage change in quantity demanded for each percentage change in a *different* good, i.e. $\varepsilon = \dfrac{\Delta Q_m / Q_m}{\Delta P_e / P_e}$.

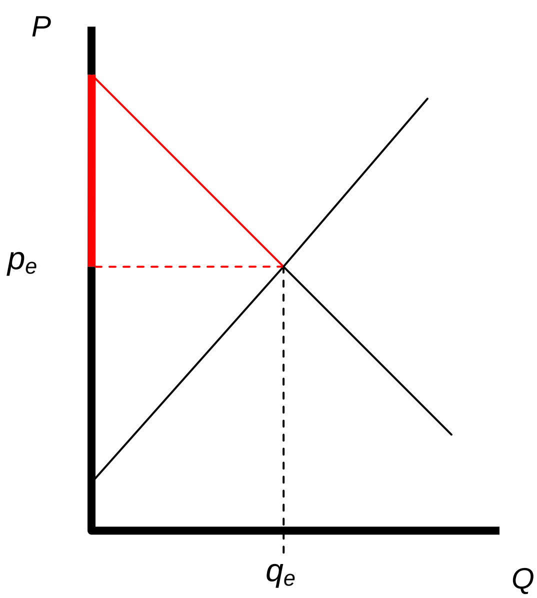
- Example from text
    - Suppose we have a single mother, Sarah, who can work up to 2,000 hours per year and earn $10 per hour.
        - She would prefer to stay home with her kids, but each hour at home costs her income that she could spend on other things (food, housing).
        - The trick to modeling her behavior is to realize she is trading off income and leisure (an hour spent not working is an hour of leisure)
    - We can then draw the budget constraint that shows her choices.
        - With consumption on the vertical axis and leisure on the horizontal, the vertical intercept is $20,000 and the horizontal intercept is 2,000 hours.
        - The slope of the budget constraint is the negative of the ratio of the price of consumption to the price of leisure (what is the price of leisure?).
        - She will choose to consume where her indifference curve is tangent to the budget constraint. How do we find that point?
            - We can write the budget constraint $C = 10(2000 - L)$
            - Assume her utility function is $U = 100\ln(C) + 175\ln(L)$
            - Solve
    - Suppose that we implement a cash welfare system where Sarah receives $5000 per year if she earns no money, but loses 50 cents of this for every dollar she earns.
        - Now her budget constraint has two sections.
            - If she doesn't work, she gets 2,000 hours of leisure and $5,000 of consumption.
            - If she works 2,000 hours she makes $20,000 and gets no cash welfare payments.
            - What is the slope of the budget constraint? The negative of the price ratio.
                - -1/10 for the upper section and -1/5 for the lower.
                - They meet at 1,000 hours of leisure and $10,000 of consumption.
        - We can write the new budget constraint for the lower section $C = 5000 + 10(2000 - L)*.5$ The upper section stays the same.

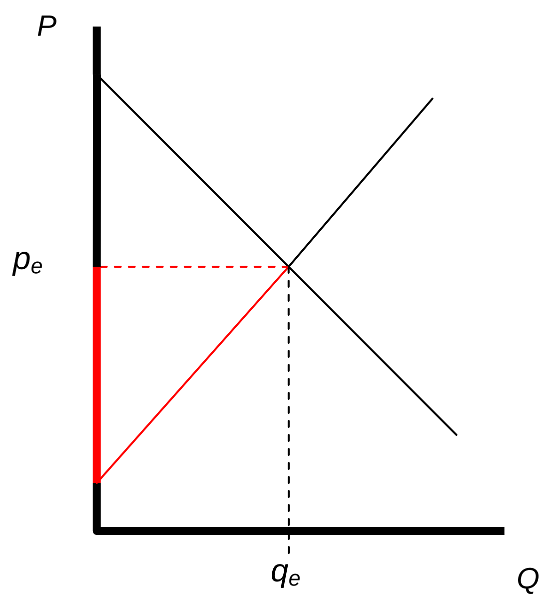
        - In order to see where she chooses to consume we have to maximize her utility with the new section of the budget constraint and see if that yields higher utility (it does).
        - Solve
        - As a result of this cash welfare program Sarah works less and consumes less, but is better off.

- **Supply Curve**: Shows the relationship between the price of a good and the quantity firms are willing to supply.
    - Firms act to maximize profits, given by $\pi = pq - TC$.
    - Firms are faced with a **production function** that shows the output produced by a given level of inputs, e.g. $q = K^{\frac{1}{2}}L^{\frac{1}{2}}$.
    - Firms usually face diminishing marginal returns to an input. That is, holding capital constant, each additional unit of labor increases the output by less than the last.
    - Since we usually assume that capital cannot be changed in the short-run and input prices are fixed, this implies that marginal cost increases with quantity.
    - In maximizing profit, firms choose to produce as long as marginal revenue is greater than or equal to marginal cost (in a competitive market).
        - Since marginal revenue is the price, firms produce as long as marginal cost is below or equal to price.
        - Therefore, a firm's marginal cost curve is its supply curve.
    - We obtain the market supply curve by horizontally summing individual firms' supply curves.
    - The **market supply curve** shows the quantity supplied at a given price.
- Putting the market supply and demand curve together shows how we get market equilibrium.
    - Remember, the market supply curve shows the quantity firms are willing to supply at a given price and the market demand curve shows the quantity consumers are willing to buy at a given price. Where these curves cross, consumers want to buy the exact same quantity producers want to sell, at the same price, and the market clears.



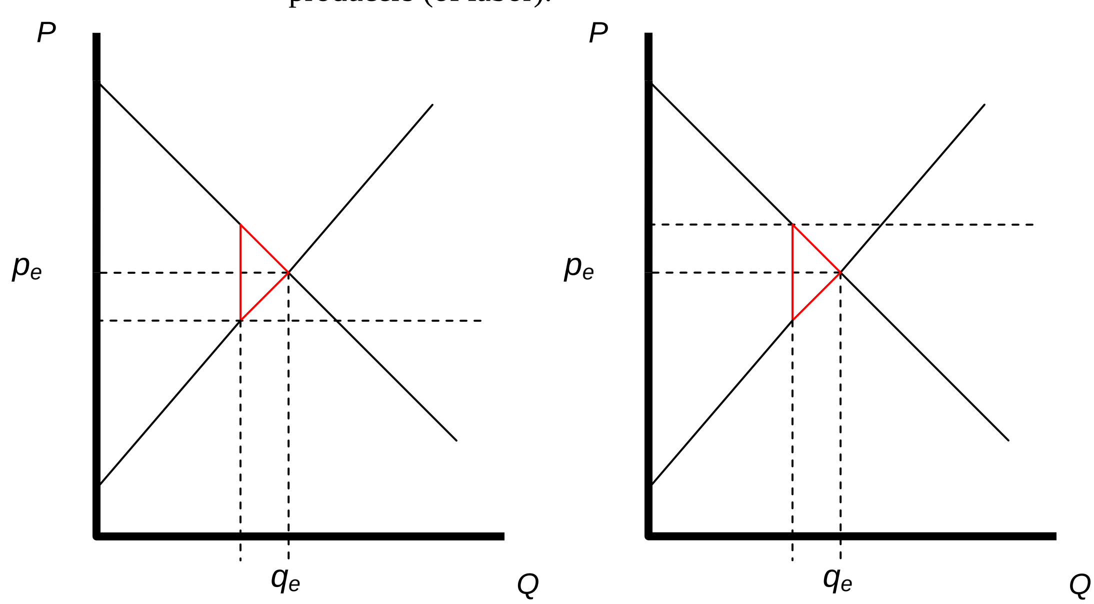
- **Social Efficiency**: the total gain to society from all trades made in a market.
    - **Consumer Surplus**: the benefit derived by consumers minus the price they paid for a good. Since the demand curve represents marginal benefit, this is the area between it and the price paid.
        - Note that CS is highest on the first unit purchased. This is because it represents the consumer who values the good the most.
        - CS is determined by the equilibrium price and the elasticity of demand. Recall that as demand becomes more elastic the demand curve becomes flatter, compressing the area of CS
    - **Producer Surplus**: the benefit derived by firms minus what it costs to produce a good. Since the supply curve represents marginal cost, this is the area between it and the price paid.





- **Total Surplus**: the sum of consumer and producer surplus. The same thing as social efficiency. Often referred to as the "size of the pie."

- **First Fundamental Theorem of Welfare Economics**: The competitive equilibrium, where supply equals demand is Pareto efficient.
  - **Pareto Efficient**: no one can be made better off without someone else being made worse off. Implies that social efficiency, the "size of the pie," is maximized.
    - Note that Pareto efficiency is unrelated to the distribution of the pie. There are multiple Pareto efficient distributions.
  - We can "prove" the 1st FWT by examining what happens when we impose other prices.
    - **Price Ceiling**
      - Firms only produce until price equals marginal cost. Thus the quantity supplied shifts to the left.
      - However, at these prices consumers want to buy more, thus there is a shortage.
      - The result is that the surplus in the triangle is lost. This is deadweight loss (the reduction in efficiency from denying trades for which the benefits outweigh the costs).
      - Example is rent control in New York. Lead to a shortage of apartments and long waiting lists. Also gasoline price controls in the 1970s led to shortages, long lines and rationing.
    - **Price Floor**
      - Consumers only buy as long as the marginal benefit exceeds the price. Thus the quantity demanded shifts left.
      - Firms would like to sell more at these prices; this can lead to a surplus.
      - The result is that the surplus in the triangle is lost. This is deadweight loss.
      - Example is minimum wage laws discussed in class. In this case employers are the consumers (of labor) and workers are the producers (of labor).



- Minimum wage – the economic analysis seems clear, yet polls say that over 80% of Americans support a $2 increase in the minimum wage. What gives?
  - People don't understand the economic theory?
  - People are willing to trade off the relatively small loss in employment for an increase in the wages of those who are employed?
    - Yet by increasing the minimum wage we make high skill workers relatively more valuable.
      - For example suppose I can hire four low skill workers or one high skill worker to do a job. The wage of the high-skill worker is $25 and the low skill workers make $6 each.
      - If the minimum wage is raised to $7, I should fire the low skill workers and hire the high skill worker
  - The debate is about morality?
    - The living wage is not about living standards or economics, but people object morally to the idea that wages, the value of someone's labor, is determined by amoral market mechanisms
    - Americans feel it is demeaning to pay someone too low of a wage?

- **Second Fundamental Theorem of Welfare Economics**: Society can obtain any Pareto efficient outcome through lump-sum transfers and competitive markets.
  - In theory there are many Pareto efficient outcomes. Pareto efficiency is unrelated to the "distribution of the pie."
  - In reality, lump-sum transfers are impractical and distortionary. Additionally government would need perfect knowledge of people's preferences.
  - In practice, reallocation generally takes place through taxes and subsidies. However, shifting resources in this way almost always affects individuals' behavior.
    - If you take 10% of Alex's income and give it to Anna, both Alex and Anna will work fewer hours.
    - Assuming that we started at a competitive equilibrium, this distortion moves us away from the competitive equilibrium and leads to deadweight loss.
  - Thus governments face an **equity-efficiency** tradeoff.
    - As a result they must choose how to value the welfare of all the different individuals in society.
    - **Social Welfare Function**: quantifies total social welfare as a function of individuals' welfare (not necessarily the same as social efficiency).
      - **Utilitarian SWF**: social welfare is the sum of individuals' welfare. $W = U_1 + U_2 + ... + U_n$
      - **Rawlsian SWF**: social welfare is defined as the welfare of the worst off individual. $W = \min(U_1, U_2, ..., U_n)$ This implies that maximizing social welfare consists of having everyone with the same level of individual welfare.

- Even with lump-sum transfers, maximizing these social welfare functions requires perfect knowledge of people's preferences.
- SWF's also require that we can compare the individual welfare of different people. How do we compare my happiness when I eat chocolate to yours when you eat chocolate?