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PROJECT REPORT: PART 2

The objective of second phase of the report is a multi-objective optimization problem which maximizes the fall time while minimizing the standard deviation of time. In order to do that we need to generate the Pareto front of the two objectives functions based on the surrogate. One objective function is the fall time (maximized) and the other objective is the standard deviation of fall time. This second objective will require a surrogate. Then four designs on the Pareto front needs to be built and compared to the surrogate prediction. Two of the points are the optima associated with a single objective, and the other two associated with the optimum associated with a weights (w) of 1/3 and 2/3 to the normalized objective. That is, each of these two designs minimizes the following compromise objective function

$$f_{compromise} = -w\frac{t_{fall}}{t_{best}} + (1-w)\frac{\sigma}{\sigma_{best}}$$

Where t_{fall} and σ are the fall time and its standard deviation, respectively, and they are normalized by their best values (optima of a single objective).

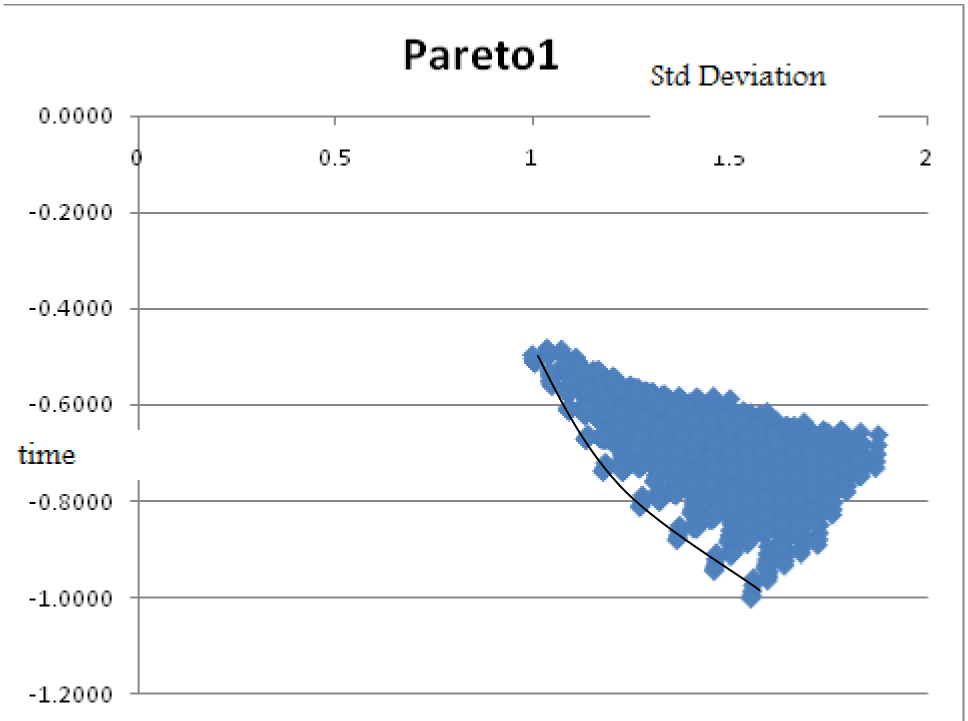
The first step is to build the surrogate for the time and the standard deviation on the entire design space, this is discussed in more detail in part 1 of the project. The beta coeeficients can be found in appendix 1 here. We use a quadratic fit for time but a linear fit for the standard deviation. If we use the linear fit for both time and standard deviation the front turns out to be straight but if we use a quadratic fit for the standard deviation it appears that we get negative values from the surrogate which is not possible.

In the following the table you can find the errors to justify that choice

	Time		Standard deviation	
	Linear	Quadratic	Linear	Quadratic
R^2	0.3244	0.7792	0.1754	0.445
R_a^2	0.2306	0.5690	0.0609	-0.1422
PRESSrms	0.5683	0.5804	0.1005	0.1352
Prediction Variance	0.252	0.1412	0.0089	0.0108

We are unable to justify the negative value of R_a^2.

The pareto front is as shown below.

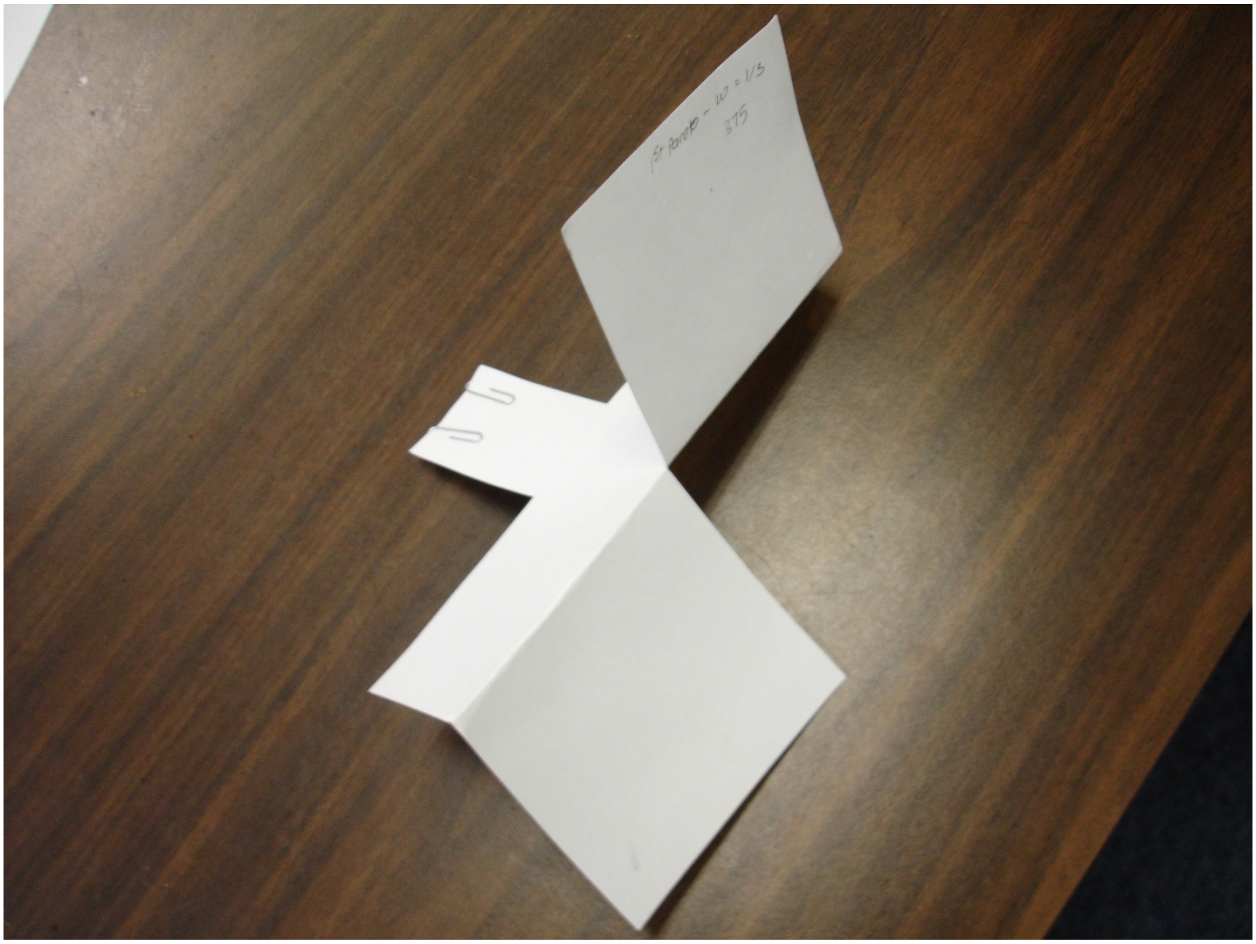


Now we use the compromise objective function and we get the following designs on the pareto front for the helicopters. Also it gives only the design and the expected fall time and standard deviation (std) from the surrogate and the experimental ones.

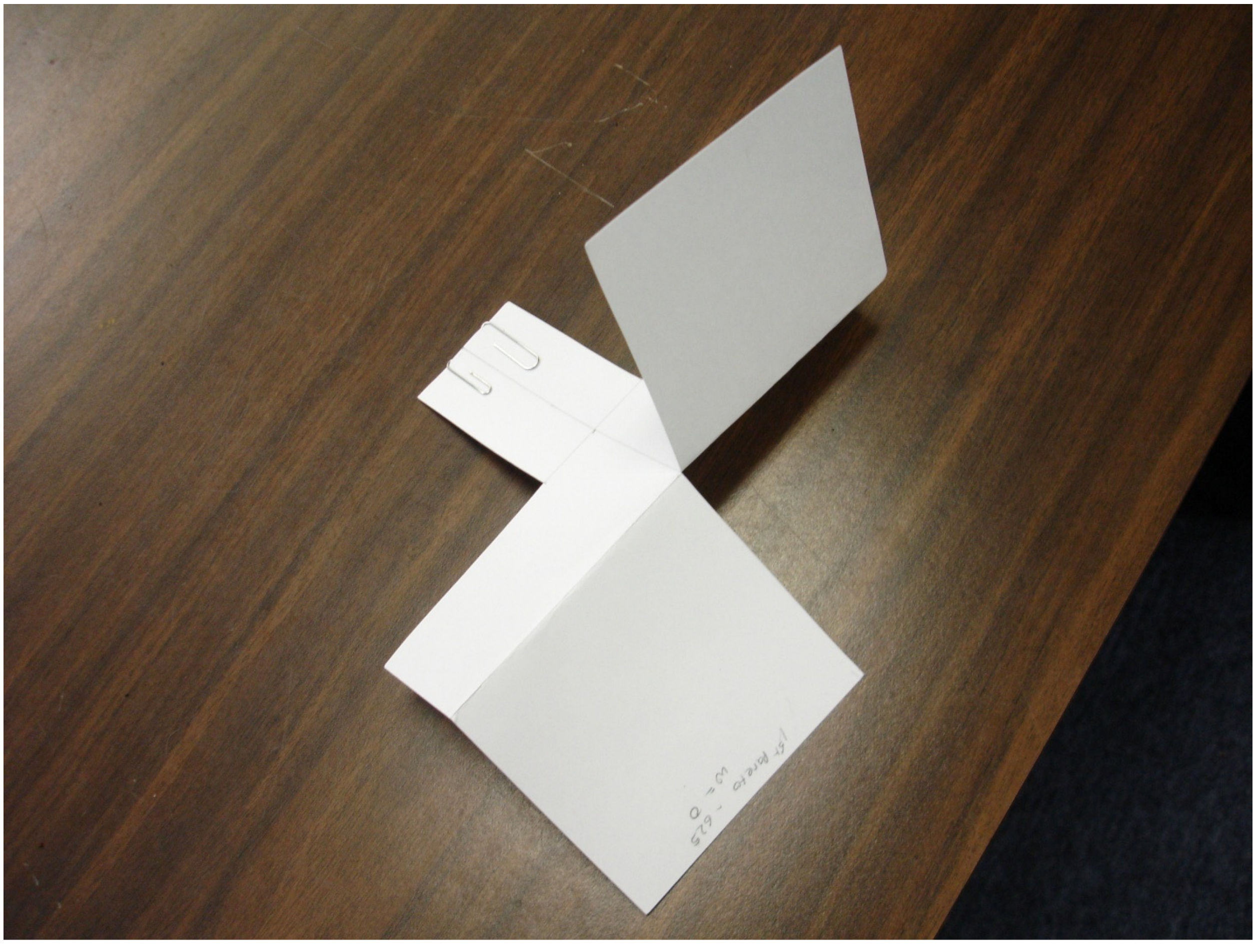
Weight	Design #	Rr	Tw	Tl	Rw	Bl	Expected		Actual	
							Time	Std	Time	Std
0	625	9.4	5.1	7	10.8	4.4	2.57	0.1649	2.81	0.1542
1/3	375	9.4	4.3	7	10.8	4.4	2.64	0.1654	2.65	0.1798
2/3,1	3105	16.5	5.1	7	4.9	4.4	5.17	0.2564	4.70	0.2687

In the above table we see that there are same design for two weights.

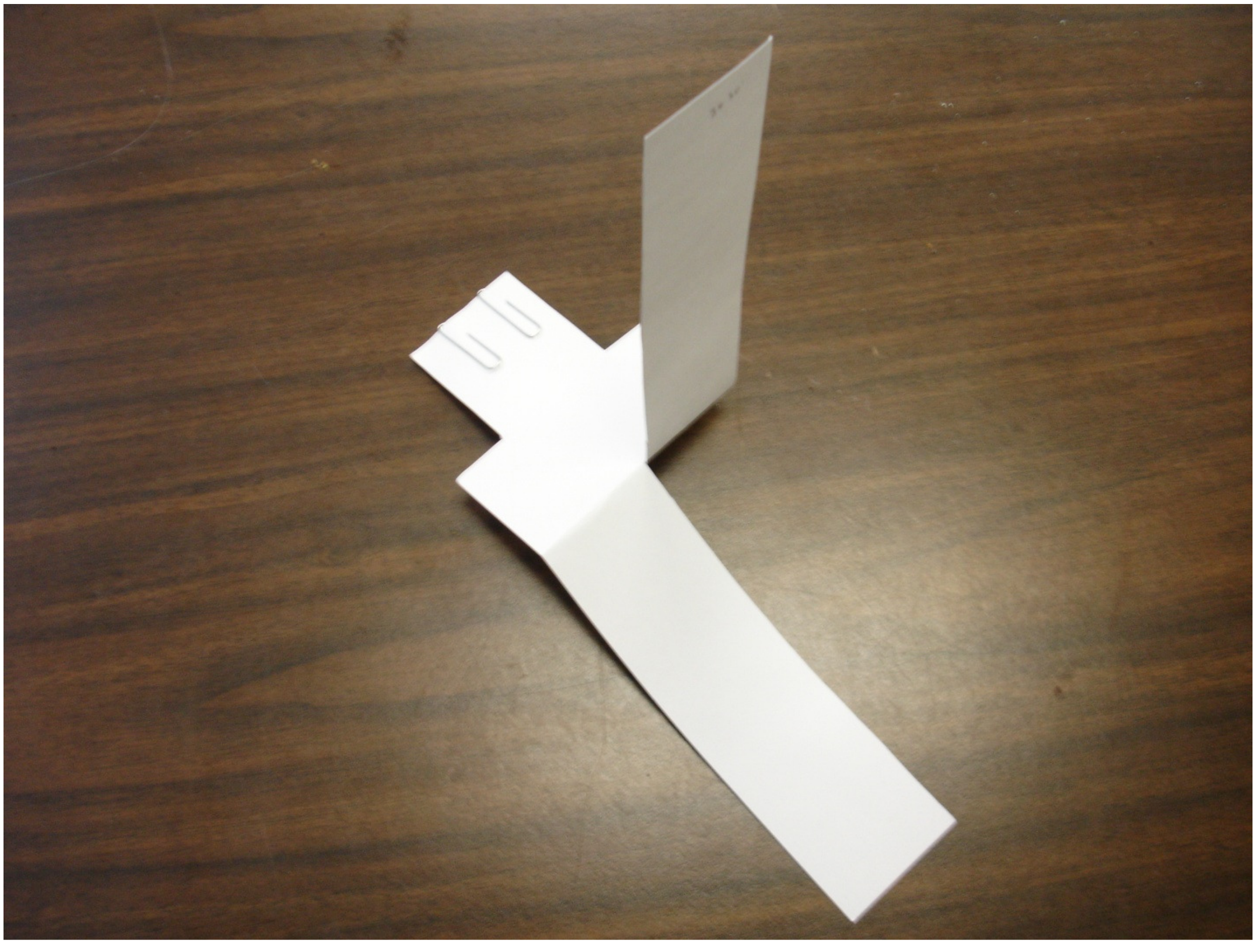
PICTURES FOR ABOVE PARETO HELICOPTER:
Weight 1/3, Design 375



Weight 0; design 625



Weight 2/3, 1 : Design 3105



Base on those designs we decided to reduce the design space for the rotor dimensions because they appear to be the one having the most influence in our experiments the new design space is then

	Rr	Tw	Tl	Rw	Bl
Minimum	9.7	3.2	5.0	4.9	1.8
Maximum	16.5	5.2	7.0	10.8	4.4

After this reduction we keep 18 designs from the DoE. We again generate other 23 designs in the above range to fill the entire space using the Latin Hypercube Sampling. So again we get a total of 41 points. This results are attached in the appendix 2. Using these we find new betas and trhat can be found in Appendix 4.

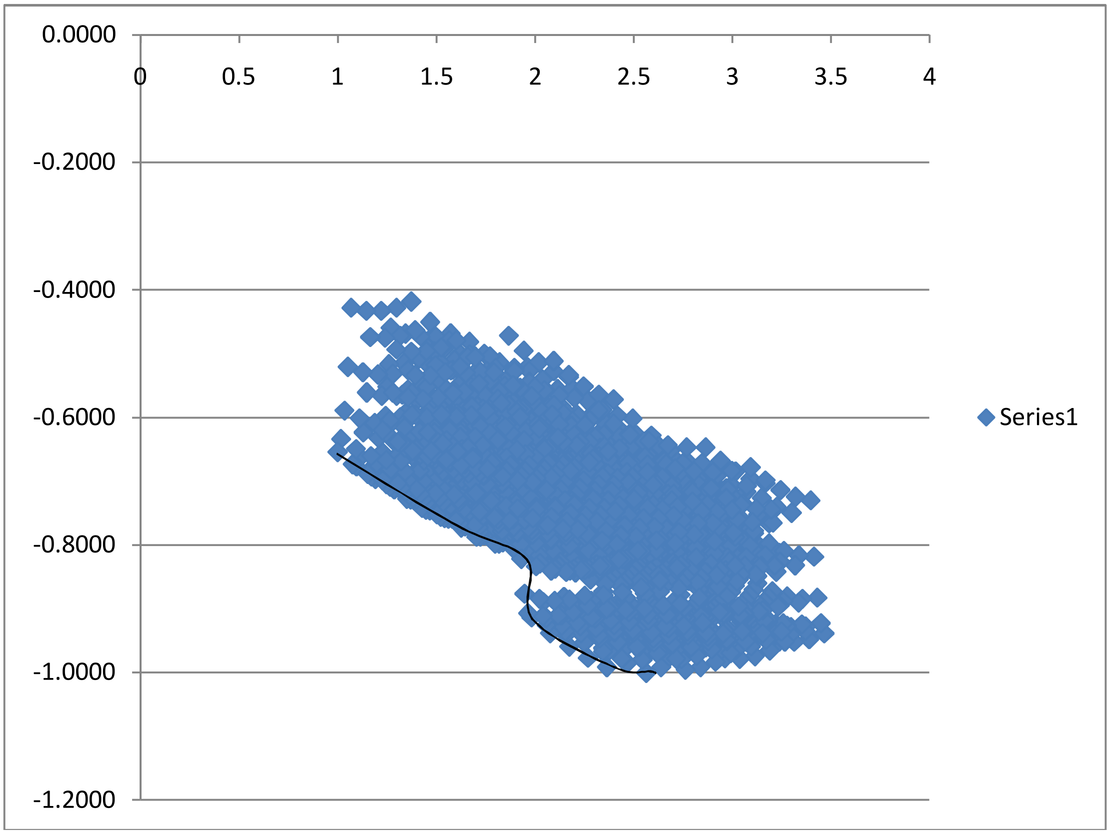
Again we use a quadratic fit for time and a linear fit for standard deviation. The error results are shown below;

	Time		Standard deviation	
	Linear	Quadratic	Linear	Quadratic
R^2	0.4207	0.7788	0.1151	0.5701
R_a^2	0.3379	0.5576	-0.0113	0.1401
PRESSrms	0.2944	0.4529	0.1351	0.1608

Prediction Variance	0.0734	0.0491	0.0151	0.0129

From the above error results we can see that the surrogate gives a much better result for time even if it is slightly less for standard deviation.

The pareto is generated as shown below:



Now we see that the pareto generated is not a convex. So we are getting the same design for the weights of 0, 1/3, 2/3. So we change the weights and take the weights as 0.8 and 0.9.

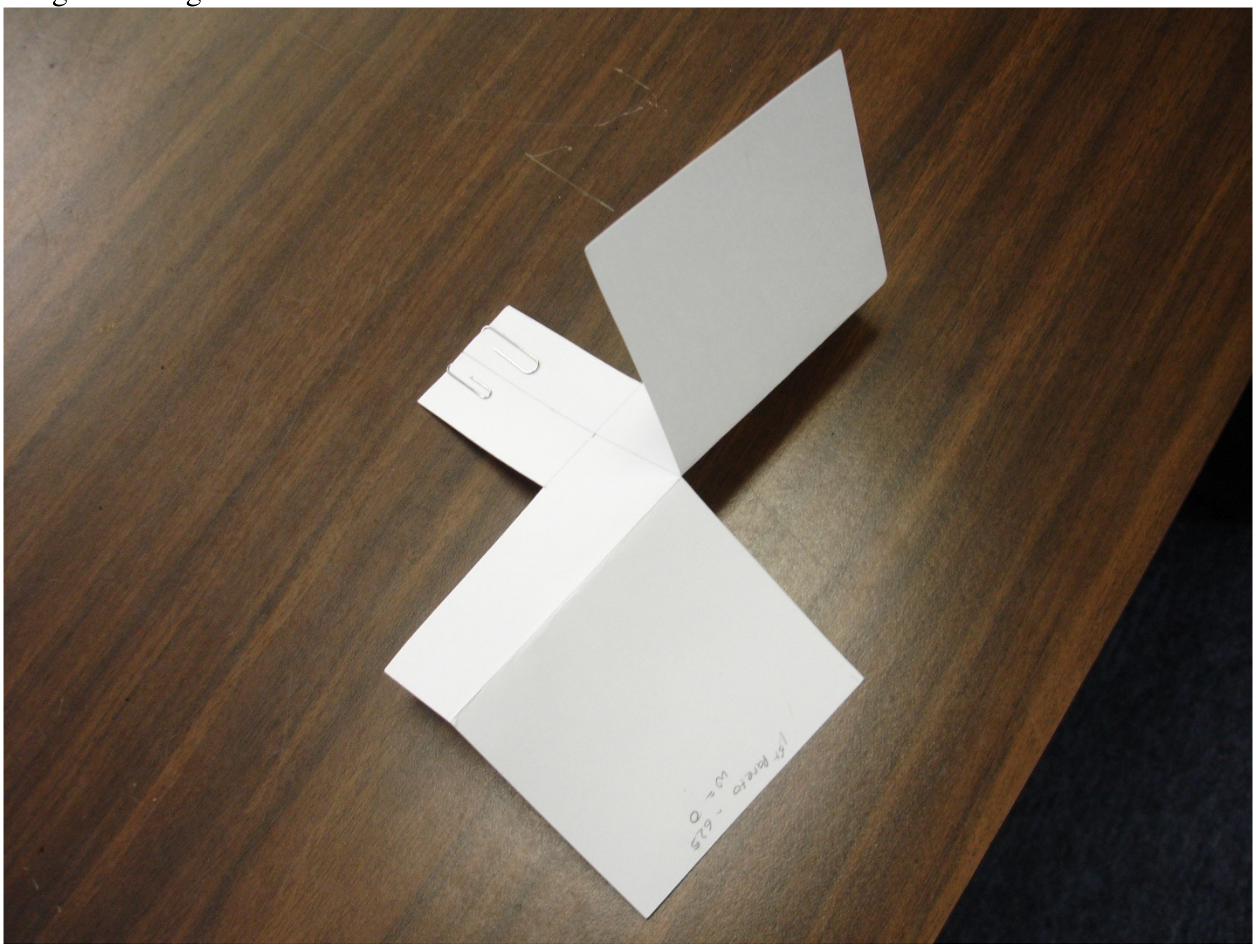
From that Pareto front we get the following 4 designs:

Weight	Design #	Rr	Tw	Tl	Rw	Bl	Expected		Actual	
							Time	Std	Time	Std
0	625	9.4	5.1	7	10.8	4.4	2.57	0.1649	2.81	0.1542
0.8	3080	16.5	5.1	6.6	4.9	4.4	4.40	0.2004	4.05	0.2556
0.9	3005	16.5	5.1	5.4	4.9	4.4	4.65	0.2280	4.15	0.1803
1	2880	16.5	4.7	5.4	4.9	4.4	4.69	0.2473	4.22	0.2212

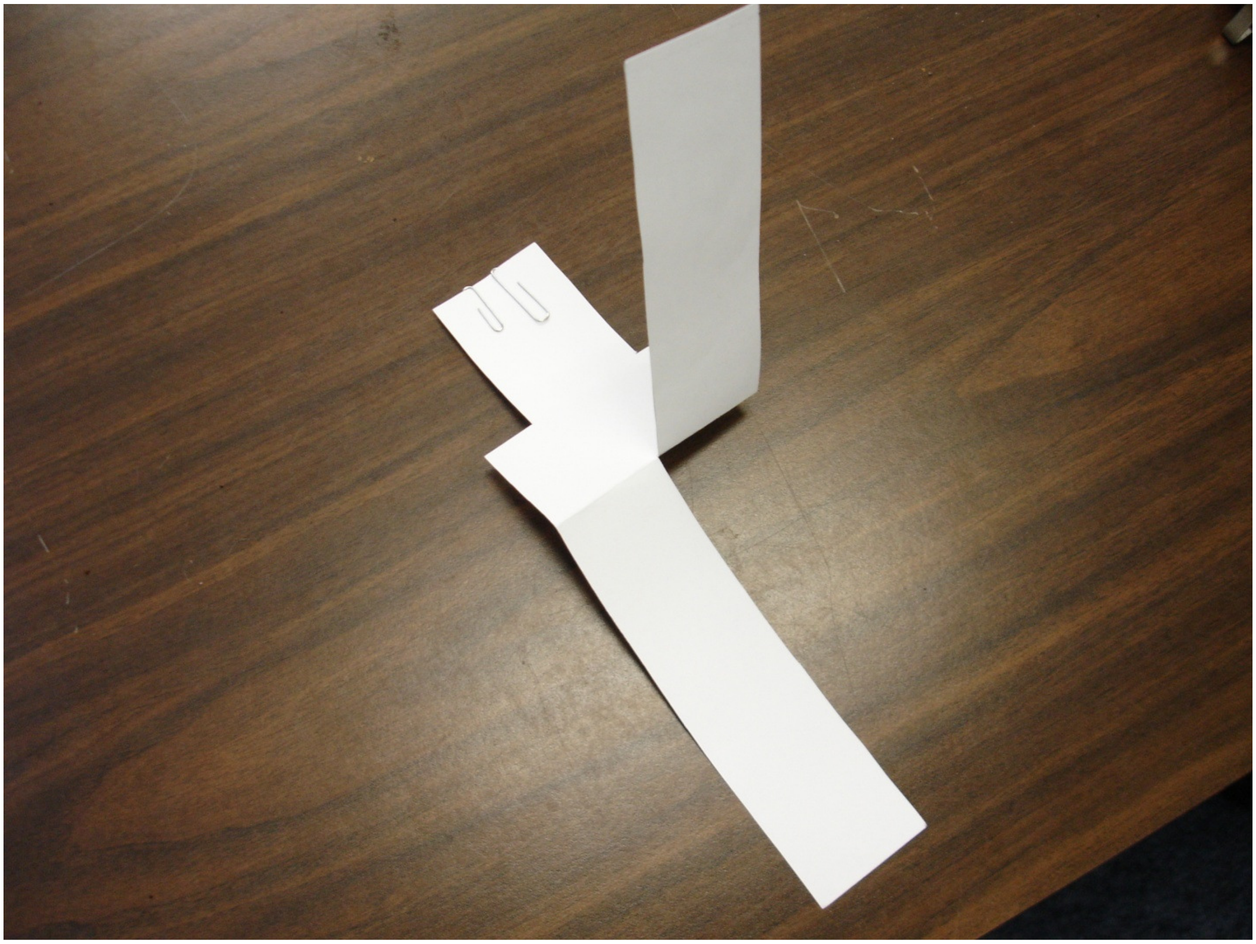
Here we see that the experimental values are in agreement with the experimental values.

The pictures of the above pareto helicopters are as shown below:

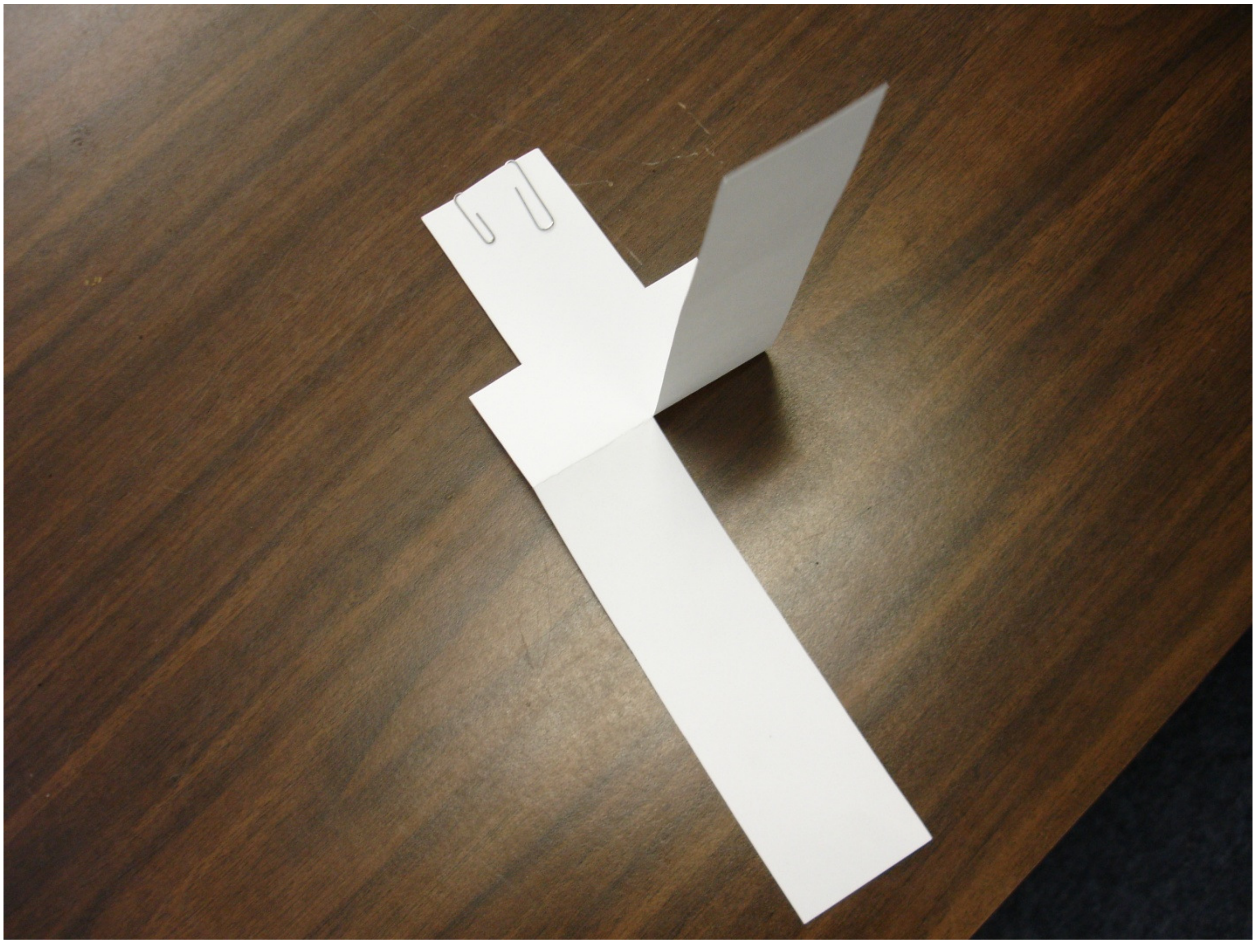
Weight 0: design 625

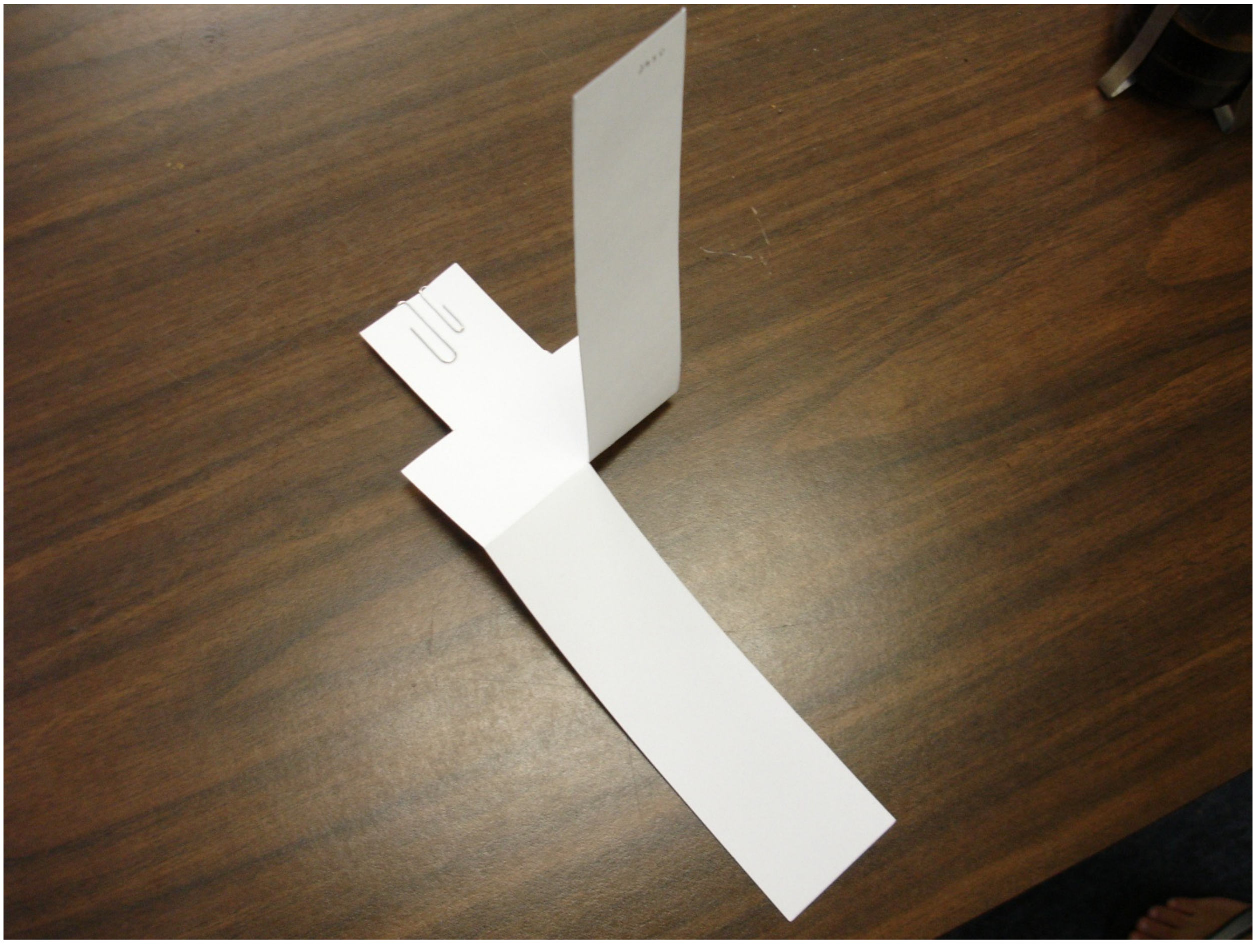


Weight 0.8: Design 3080



Weight 0.9: Design 3005


Weight 1 : Design 2880



CROSS VALIDATION

The four pareto designs and their dimensions are as shown in the above table. Now we find the distance of the 41 designs from the above four designs and we find the designs with the minimum distance from each pareto helicopter. The designs with minimum distance are shown below:

Design	Rr	Tw	Tl	Rw	Bl	Time
D 37	16.5	5.1	7.0	7.1	3.1	4.382
A15	9.4	4.4	6.9	8.6	4.2	3.038
A21	14.4	4.3	5.8	5.6	3.3	3.936
A21	14.4	4.3	5.8	5.6	3.3	3.936

Now we use the remaining 37 designs and find the prediction variance and PRESS by fitting a surrogate for them. The cross validation error is as shown below:

RMS error	Prediction Variance	Maximum Absolute error	PRESS Rms
0.1780	0.0522	0.4908	0.47

Now, $\hat{\sigma} = \sqrt{prediction\ variance} = \sqrt{0.0522} = 0.2284$

Here we see that the value of RMS error and PRESS Rms is small. Also we see that the standard error is 0.2284 which is good. So from the above error measures we can say that the pareto that we have chosen is good.

So now using these new points we again fit those using a PRS and

Appendix 1 : Beta coefficients of the surrogates

	Time		Standard deviation	
	Linear	Quadratic	Linear	Quadratic
1	3.2927	2.7212	0.2529	0.1205
Rr	0.5403	0.7705	0.0775	0.4536
Tw	0.1471	0.5634	-0.0014	0.1120
Tl	0.3268	-0.2760	-0.0303	0.0165
Rw	-0.3178	1.0142	-0.0368	0.1478
Bl	-0.2144	-0.2537	-0.0349	0.0464
Rr^2		-0.3633		-0.2585
Rr*Tw		0.1969		-0.0391
Rr*Tl		0.4442		0.0007
Rr*Rw		-0.7840		-0.1858
Rr*Bl		0.8779		-0.0298
Tw^2		-0.1837		-0.0301
Tw*Tl		-0.0649		-0.0257
Tw*Rw		-0.2740		-0.0049
Tw*Bl		-0.1490		-0.0339
Tl^2		0.9670		-0.0348
Tl*Rw		-1.1522		0.0712
Tl*Bl		0.0011		-0.0244
Rw^2		0.3158		-0.1352
Rw*Bl		-1.3738		0.0302
Bl^2		0.4241		-0.0376

Appendix 2 : Designs for the second Pareto

Design	Rr	Tw	Tl	Rw	Bl	Bw
D 20	12.1	3.2	5.0	7.1	1.8	14.2
D 21	12.1	4.2	5.0	7.1	3.1	14.2
D 22	12.1	3.2	6.0	7.1	3.1	14.2
D 23	12.1	4.2	6.0	7.1	4.4	14.2
D 24	12.1	5.1	5.0	10.8	4.4	21.6
D 25	12.1	3.2	6.0	10.8	4.4	21.6
D 26	12.1	4.2	6.0	10.8	1.8	21.6
D 27	12.1	3.2	7.0	10.8	1.8	21.6
D 28	12.1	4.2	7.0	10.8	3.1	21.6
D 34	16.5	4.2	5.0	7.1	1.8	14.2
D 35	16.5	3.2	6.0	7.1	4.4	14.2
D 36	16.5	5.1	7.0	7.1	1.8	14.2
D 37	16.5	5.1	7.0	7.1	3.1	14.2
D 28	16.5	4.2	5.0	10.8	4.4	21.6
D 29	16.5	5.1	6.0	10.8	1.8	21.6
D 40	16.5	5.1	6.0	10.8	3.1	21.6
D 41	16.5	4.2	7.0	10.8	4.4	21.6
D 42	16.5	5.1	7.0	10.8	3.1	21.6
A 1	13.4	3.3	5.9	6.5	3.8	13.0
A 2	14.7	3.9	5.6	7.0	4.1	14.1
A 3	16.3	4.2	5.8	10.4	3.6	20.8
A 4	10.0	4.7	6.2	8.8	2.3	17.6

A 5	9.6	3.3	6.8	7.8	2.9	15.6
A 6	13.2	4.0	6.5	5.0	2.9	10.0
A 7	13.9	4.7	6.9	5.4	2.4	10.9
A 8	12.3	3.6	6.7	6.7	3.3	13.4
A 9	16.2	3.8	5.5	10.2	2.2	20.4
A 10	15.6	5.0	5.3	8.3	2.0	16.6
A 11	13.0	4.1	5.1	7.4	2.0	14.7
A 12	11.9	4.9	6.3	7.5	4.1	15.0
A 13	15.0	3.5	6.1	10.6	2.6	21.3
A 14	10.4	4.1	5.7	6.3	1.8	12.6
A 15	9.4	4.4	6.9	8.6	4.2	17.2
A 16	15.5	4.6	6.6	9.7	3.6	19.4
A 17	10.7	3.4	6.0	8.1	2.7	16.2
A 18	12.5	3.7	5.4	9.8	3.9	19.6
A 19	10.9	4.5	5.0	5.1	3.4	10.2
A 20	14.3	5.1	5.3	9.1	3.1	18.2
A 21	14.4	4.3	5.8	5.6	3.3	11.3
A 22	11.5	4.8	6.5	5.9	2.6	11.9
A 23	11.6	3.6	6.1	9.3	4.4	18.5

Appendix 3 : Results for the new Pareto

Helicopter	Time 1	Time 2	Time 3	Time 4	Time 5	Time 6	Average time	st. deviation
D 20		3.12	3.4	3.5	3.66		3.420	0.227
D 21	3.6	4.13	3.32	3.38	4.12		3.710	0.393
D 22	3.53	3.38	3.09	2.96	3.5		3.292	0.254

D 23	3.22	3.62	3.06	3.31	2.97		3.236	0.252
D 24	3.22	2.82	3.37	3.37	3.41		3.238	0.245
D 25	2.5	3.1	2.62	3.03	3.25	3.37	2.978	0.347
D 26	3.53	3.28	3.29	3.72	3.87	3.91	3.600	0.278
D 27	3.16	3.22	3.4	3.22	3.31	3.75	3.343	0.216
D 28	3.28	3.44	3.4	3.18	2.97	3.29	3.260	0.170
D 34	4.56	4.13	4.68	4.25	4.35		4.394	0.225
D 35	3.59	3.12	3.9	3.46	4.75		3.764	0.618
D 36	3.78	4.41	4.22	3.75	4.22		4.076	0.294
D 37	4.84	4.69	3.97	4.25	4.16		4.382	0.368
D 38	3.19	3.6	3.35	3.28	3.25		3.334	0.159
D 39	3.18	3.53	3.81	3.41	3.53	4	3.577	0.291
D 40	3.35	3.25	3.4	3.22	3.25		3.294	0.077
D 41	3.31	3.72	3.65	3.46	3.41		3.510	0.170
D 42	3.5	3.41	3.5	3.96	3.88	3.78	3.672	0.231
A1	3.48	3.29	3.53	3.73	3.42		3.490	0.161
A2	3.87	4.01	3.91	4.430	4.01		4.046	0.223
A3	3.63	3.96	3.87	3.13	3.680	3.35	3.603	0.314
A4	3.46	2.93	3.87	2.88	3.13		3.254	0.413
A5	3.48	3.90	3.63	3.21			3.555	0.288
A6	2.82	2.80	3.20	3.00	3.51		3.066	0.296
A7	2.93	2.74	3.30	3.06	3.35		3.076	0.255
A8	3.33	3.33	3.21	3.20			3.268	0.072
A9	3.33	3.20	3.74	3.760			3.508	0.285
A10	3.25	3.56	3.17	3.38			3.340	0.170

A11	3.06	3.92	3.19	3.41	4.33		3.582	0.531
A12	3.28	3.19	3.23	3.15	3.03		3.176	0.095
A13	3.19	3.87	3.50	4.20	3.28	3.620	3.610	0.378
A14	3.32	3.03	3.16	3.07	3.28	3.400	3.210	0.147
A15	3.18	2.98	3.05	3.01	2.970		3.038	0.085
A16	3.56	3.52	3.69	3.67	3.42		3.572	0.111
A17	3.40	3.43	3.52	3.37	3.48		3.440	0.060
A18	3.60	3.32	2.87	3.04	2.94	3.06	3.138	0.273
A19	3.53	3.00	2.87	3.02	3.08		3.100	0.252
A20	3.45	3.48	3.19	3.420	3.57		3.422	0.141
A21	3.93	3.98	3.87	4.05	3.85		3.936	0.082
A22	3.29	3.42	3.38	3.35	3.24		3.336	0.072
A23	3.61	3.42	3.18	3.05	3.32		3.316	0.216

Appendix 4 : second pareto new design space

	time quad	time lin	std quad	std lin
1	3.155184	3.346909	0.149723	0.282261
Rr	-0.32825	0.669093	-0.43663	0.087778
Tw	0.054578	-0.08224	0.345407	-0.09616
Tl	-0.41821	0.015866	-0.67658	-0.04609
Rw	1.72437	-0.23874	0.676085	-0.00665
Bl	0.252043	-0.182	0.381669	-0.03664
Rr^2	1.399375		0.43969	
Rr*Tw	0.59489		-0.4329	
Rr*Tl	0.5049		0.611975	
Rr*Rw	-1.84965		-0.32934	
Rr*Bl	0.097867		0.096068	
Tw^2	-0.7883		0.036712	
Tw*Tl	0.092279		-0.06478	
Tw*Rw	0.027052		-0.06694	
Tw*Bl	0.544618		-0.43897	
Tl^2	-0.19026		0.412788	
Tl*Rw	0.538471		-0.38331	
Tl*Bl	-0.40427		0.13293	
Rw^2	-0.89592		0.021474	
Rw*Bl	-0.34607		-0.38693	
Bl^2	-0.28011		-0.11843	

R2	0.7788	0.4207	0.5701	0.1151
Ra2	0.5576	0.3379	0.1401	-0.0113
PredVar	0.0491	0.0734	0.0129	0.0151
PRESSrms	0.4529	0.2944	0.1608	0.1351