Jainil N Desai

Project Report – Phase 1

UFID: 96950890

Approximation and Optimum Design

1. **Description of how the weights of the different components were measured and the magnitude of the error in these measurements:**

We measured the weight with the help of a scale with a least count of 0.005g. At a time we measured single paper and a paper clip. As we were having a sensitive scale we measured the weight of a single paper and a clip at a time. The table summarizes the measurements, average and standard deviation.

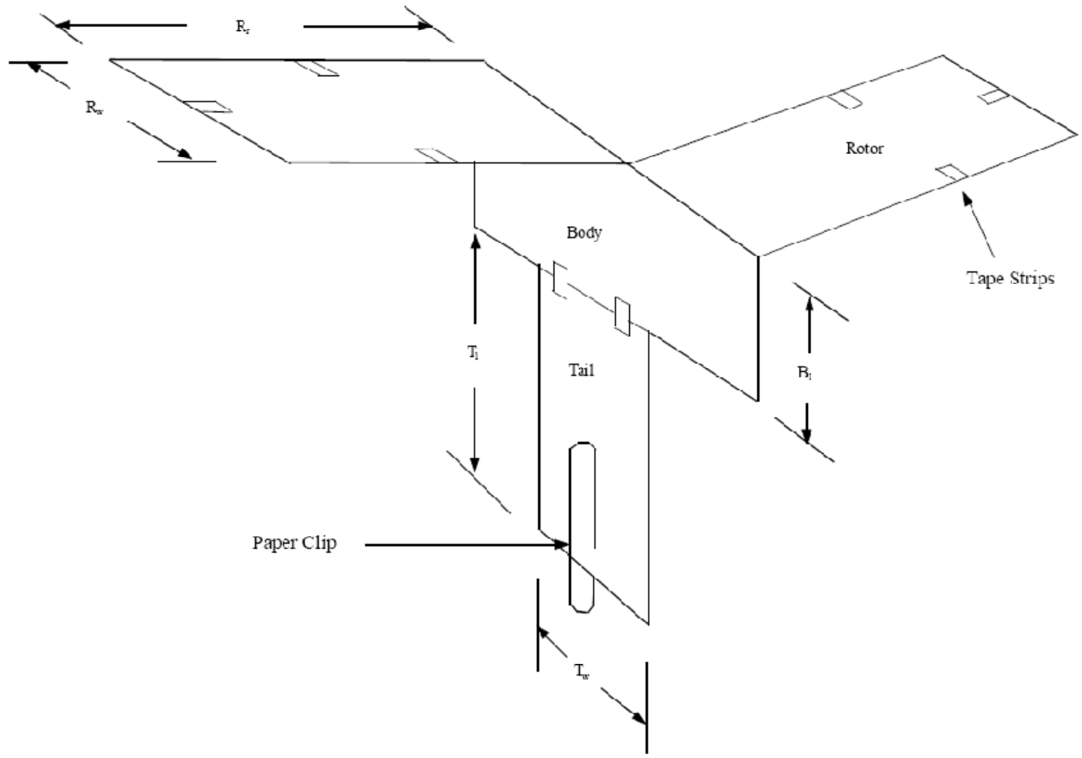
	PP clip (g)	Paper (g)
1	0.39	11.24
2	0.39	11.28
3	0.4	11.19
4	0.4	11.22
5	0.4	11.2
6	0.4	
Average	0.396667	11.226
St. Dev	0.004714	0.032

2. **Description of how repeatable the times measured was for a given helicopter to reach the ground:**

We have measured the time for each helicopter 5 times. The data points are shown in the Appendix 1.

The helicopter is as shown below:

s



Also the constraints on the design are as follows,

Parameter	Description	Min	Nom	Max
Rr	Rotor Radius	7.62 cm	---	16.50 cm
Tw	Tail Width	3.20 cm	---	5.10 cm
Tl	Tail Length	5.00 cm	---	7.00 cm
Rr0	Nominal Rotor Radius	---	7.62 cm	---
Rw	Rotor Width	---	5.40 cm	---
Bl	Body Length	---	3.80 cm	---
Bw	Body Width	---	10.8 cm	---

3. Discussion of the estimate of the drag coefficient and the expected magnitude of error:

The drag force on the helicopter is ;

$D = \frac{1}{2}\rho_{air}V^2SC_D$

Where,
$S = area\ spanned\ by\ the\ rotor = \pi R^2$

The helicopter reaches the steady state velocity fairly faster and we can use that to calculate drag coefficient. So,

$C_d = \frac{2W}{\rho_{air}\pi R_r^2 V_{SS}2}$

Here we get Vss from the relation V=d/t, d=height=5.53m and t= the average time for that helicopter.

We differentiate the above equation of Cd with respect to each variable, and then multiply the derivatives with the uncertainties of each variable.

Variable	Value
h	5.53 m
t	4.724 s
ρair	1.169 kg/m^3
m	0.00321 kg
g	9.81 m/s2

These are the derivatives of the CD equation

$$Derivative\ of\ C_D\ w.r.t\ \rho_{air}, \frac{\partial C_D}{\partial \rho_{air}} = -\frac{2mg}{\rho_{air}^2 V_{SS}^2 \pi R_r^2}$$

$$Derivative\ of\ C_D\ w.r.t\ R_r, \frac{\partial C_D}{\partial R_r} = -\frac{4mg}{\rho_{air} V_{SS}^2 \pi R_r^3}$$

$$Derivative\ of\ C_D\ w.r.t\ V_{SS}, \frac{\partial C_D}{\partial V_{SS}} = -\frac{4mg}{\rho_{air} V_{SS}^3 \pi R_r^2}$$

$$Derivative\ of\ C_D\ w.r.t\ m, \frac{\partial C_D}{\partial m} = \frac{2g}{\rho_{air} V_{SS}^2 \pi R_r^2}$$

$$Derivative\ of\ C_D\ w.r.t\ g\ , \frac{\partial C_D}{\partial g} = \frac{2m}{\rho_{air}\ V_{SS}^2 \pi R_r^2}$$

$$Derivative\ of\ V\ w.r.t.\ h, \frac{\partial V}{\partial h} = -\frac{h}{t^2}$$

$$Derivative\ of\ V\ w.r.t\ t, \frac{\partial V}{\partial t} = \frac{1}{t}$$

The uncertainties used for this analysis are tabulated below:

In the table, the sources for uncertainty in height are due to two reasons. Firstly, due to the reason that we measured the height with the thread and measured that thread length using a scale. So we take an uncertainty of 0.025m . Secondly due to the difference in the drop heights that may have taken place. So we take the uncertainty due to that as 0.025m.

Similarly for uncertainty in time, there are two causes, firstly due to the error in the time measured due to human error and also due to the accuracy of the stop watch used. So we take uncertainty as 0.1 seconds due to human errors and the uncertainty as 0.005 as measurement accuracy.

Variable	Sources of Uncertainty	Uncertainty(Ui)
h	height measurement, different drop heights	sqrt((0.025m)^2+(0.025m)^2)
t	reaction time, measurement accuracy	sqrt((0.1s)^2+(0.005s)^2)
Vss	h, t	*see appendix*
ρ_air	natural fluxuations	0.1% = 0.001169 kg/m^3
Rr	Rr measurement, human cutting error	sqrt((0.0005s)^2+(0.0005s)^2)
m	scale accuracy	0.005 g = 0.000005 kg
C_D	Vss, ρ_air, Rr, m, g	*see appendix below*

$$\emptyset = \frac{\partial C_D}{\partial x_i}$$

U_i = uncertainty of the ith variable

The uncertainty is then calculated by taking the square root of the sum of the squares of products of derivatives and their uncertainties.

$$U_{CD} = \sqrt{\sum(\emptyset_i U_i)}$$

The total uncertainities for all the results are shown in the Appendix 2

So the final uncertainty is 1.87%.

4. Formulation of the approximation and the various error measures used to test its accuracy:

The data points are given in the appendix1 along with the average time values for each dimensions of the 42 helicopters used in experiment.

The approximation is based on the Design of experiments that is created using the D-Optimal design. First we used the LHD design for DoE but the values it generated were random and did not cover the whole design space. The combinations were obtained on Matlab. Then we did the D-optimal design to get the final values of the five design variables, namely the rotor radius Rr, the tail width Tw, the tail length, Tl, rotor width Rw and body length Bl, as that gets the data points that are on the vertices and as well covering the whole design space. For those combinations of dimensions, we prepared the models and tested them for the time that it took to fall. The PRS for time is generated using the surrogate toolbox in Matlab. The values related to PRS and the polynomial function that was fitted for time as well as coefficient of drag using the data points generated using Matlab are shown below.

The Values of coefficients that the PRS generated for the fit for time are given below for a linear and a quadratic fit.

	Time Linear fit	**Time quadratic fit**
1	3.2927	2.7212
Rr	0.54033	0.77054
Tw	0.14713	0.56338
Tl	0.32679	-0.27603
Rw	-0.31777	1.0142
Bl	-0.21439	-0.25368
Rr^2		-0.36332
Rr*Tw		0.19686
Rr*Tl		0.44416
Rr*Rw		-0.78399
Rr*Bl		0.87793
Tw^2		-0.18367
Tw*Tl		-0.064931
Tw*Rw		-0.27398

Tw*Bl		-0.14904
Tl^2		0.96705
Tl*Rw		-1.1522
Tl*Bl		0.0011442
Rw^2		0.3158
Rw*Bl		-1.3738
Bl^2		0.42413

The error measures for the linear and the quadratic fit are as shown. They can be easily compared from the table.

Quantity	Type of fit	R2	Ra2	PRESS rms	PRESS	Prediction Variance
Time	Linear	0.3244	0.2306	0.5683	13.5626	0.252
Time	Quadratic	0.7792	0.569	0.5805	14.1533	0.1412

For a good fit, we need to have R^2 as close to 1 as possible. $R^2 = 1$ is an ideal fit and it occurs only when the PRS exactly passes through the data points. As we see that value of R^2 for the linear fit is really poor as compared to that of the quadratic fit from the above table. There is not much difference between the PRESS rms for both the fits.

5. Description of how group selected best approximation to be tested:

We used two designs,
1) A linear fit
2) A quadratic fit.

From the above error measures, we can see that that value of R^2 for the linear fit is 0.3244 while that for the quadratic fit is 0.7792. This shows that in the linear fit, the time does not change itself much with the change of values of the dimensions of the helicopters, while that for the quadratic fit, the PRS accommodates itself to fit the data and hence gives a better estimation of the time.

Also we see that the value of prediction variance is 0.252 for linear fit and 0.1412 for quadratic fit. This shows that for the quadratic fit, the loss of prediction accuracy is less than that for linear fit. Also we could see there is no much difference between PRESS rms of the linear fit and the quadratic fit.

Hence we choose the quadratic fit for the PRS.

6. Discussion of how well the approximation was validated by the additional helicopters:

We prepared 10 additional helicopters and found the time and dag coefficient for each. Then we predicted how well our fit is by fitting this additional readings and getting the prediction variance. The data points that were selected are as shown below:

Helicopter	Average time	st. deviation	**Rr**	**Tw**	**Tl**	**Rw**	**Bl**	**Bw**
6	3.096	0.145	8.5	4	5.5	5.5	3.5	11
8	3.015	0.165	9	4.5	6.5	6	4.5	12
10	4.747	0.403	16.5	5.1	5	5.37845	3.8	10.7569
12	4.940	0.291	16.5	3.2	7	5.4	3.8	10.8
13	4.040	0.676	16.5	5.1	7	5.4	3.8	10.8
15	3.004	0.097	8.5	4	6	5.4	3.8	10.8
19	4.445	0.991	16.5	4	5	5.4	3.8	10.8
22	4.074	0.304	14	3	7	5.4	3	10.8
24	3.106	0.185	12	5	7	5.4	6	10.8
E1	3.540	0.135	9.00	3.50	5.50	3.40	4.00	6.8

Cdtest	Cdhat	Rms	Max Abs Error	Prediction variance
0.86603	0.79164	0.1898	0.7761	0.1412
0.860021	0.98248			
0.767213	0.55894			
0.848209	0.69833			
0.579866	0.82068			
0.889512	0.88764			
0.677865	0.51361			
0.691842	0.65751			
0.598842	1.3751			
0.7856	0.60807			

In above we calculated the values of the prediction variance, e_{rms} and maximum absolute error with the help of matlab. The value of standard error is ,

$$\hat{\sigma} = \sqrt{Prediction\ variance} = \sqrt{0.1412} = 0.3757$$

So we see that erms is 0.1898 which is reasonable and the standard error is 0.3757, which again is good. Also the maximum absolute error is 0.776. So we see that by using the additional helicopters the surrogate predicts values in a better way outside the data as well.

APPENDIX 1

Helicopter	Time 1	Time 2	Time 3	Time 4	Time 5	Time 6	Time 7	Time 8	Average time	st. deviation	Range/Mean
D 1	3.06	3.34	3.38	3.10	3.19				3.214	0.142	2.428
D 2	3.25	3.47	3.28	3.35	3.25				3.320	0.093	2.491
D 3	3.78	3.34	3.50	3.25	3.22				3.418	0.230	2.838
D 4	3.40	3.22	3.35	3.28	3.40				3.330	0.079	2.433
D 5	3.50	3.18	3.25	3.28	3.16				3.274	0.136	2.535
D 6	3.03	3.09	3.20	3.00	2.63				2.990	0.215	2.320
D 7	3.06	2.90	2.87	2.75	2.44				2.804	0.232	2.190
D 8	2.81	3.03	3.06	2.94	2.56	2.71	3.00		2.873	0.187	2.169
D 9	3.47	3.28	3.00	3.21	2.93				3.178	0.218	2.548
D 10	2.94	3.06	2.90	2.97	2.97	2.75	2.50		2.870	0.189	2.189
D 11	3.44	3.28		3.44	3.00	3.40			3.312	0.186	2.534
D 12	2.75	2.69	2.69	2.97	2.75	2.68	2.87		2.771	0.109	2.003
D 13	4.19	4.31	4.25	4.19	4.69				4.326	0.209	3.721
D 14	2.57	3.21	2.9	3.31	2.69				2.936	0.320	2.435
D 15	4.09	3.97	3.87	3.94	4.32				4.038	0.177	3.362
D 16	3.29	3.81	3.81	3.28	2.94				3.426	0.378	2.952
D 17	4	4.35	4.6	4.69					4.410	0.309	3.783
D 18	3.31	4.35		3.72	4.43				3.953	0.533	3.593
D 19	4.84	4.78	4.53	4.75	4.72				4.724	0.117	3.881
D 20		3.12	3.4	3.5	3.66				3.420	0.227	2.748
D 21	3.6	4.13	3.32	3.38	4.12				3.710	0.393	3.235
D 22	3.53	3.38	3.09	2.96	3.5				3.292	0.254	2.631
D 23	3.22	3.62	3.06	3.31	2.97				3.236	0.252	2.702
D 24	3.22	2.82	3.37	3.37	3.41				3.238	0.245	2.539
D 25	2.5	3.1	2.62	3.03	3.25	3.37			2.978	0.347	2.531
D 26	3.53	3.28	3.29	3.72	3.87	3.91			3.600	0.278	2.999
D 27	3.16	3.22	3.4	3.22	3.31	3.75			3.343	0.216	2.805
D 28	3.28	3.44	3.4	3.18	2.97	3.29			3.260	0.170	2.529
D 29	2.81	3.56	3.5	3.68	3.41	3.78			3.457	0.343	2.967
D 30	3.18	2.63	3.59	3.37					3.193	0.411	2.766
D 31	3.47	3.78	3.28	3.4	4.03				3.592	0.307	3.117
D 32	3.87	3.79	3.57	3.44	3.68				3.670	0.171	2.933
D 33	5.53	5.56		5.44	5.96				5.623	0.231	4.992

D 34	4.56	4.13	4.68	4.25	4.35				4.394	0.225	3.740
D 35	3.59	3.12	3.9	3.46					3.518	0.323	3.013
D 36	3.78	4.41	4.22	3.75	4.22				4.076	0.294	3.490
D 37	4.84	4.69	3.97	4.25	4.16				4.382	0.368	3.934
D 38	3.19	3.6	3.35	3.28	3.25				3.334	0.159	2.643
D 39	3.18	3.53	3.81	3.41	3.53	4			3.577	0.291	3.111
D 40	3.35		3.25	3.4	3.22	3.25			3.294	0.077	2.422
D 41	3.31	3.72	3.65	3.46	3.41				3.510	0.170	2.777
D 42	3.5	3.41	3.5	3.96	3.88	3.78			3.672	0.231	3.031

APENDIX 2

Helicopter	uncertainity Vss	uncertainity wrt V	ucertainty in row air	uncertainity in Rr	Uncertainty in m	**total uncertainity**	CD (SS)	**% uncertainity**
D 1	0.002985	2.64E-05	7.39E-07	0.000256	6.1E-05	0.018544	0.85917	2.158344
D 2	0.002628	1.92E-05	6.46E-07	0.000224	6.94E-05	0.01769	0.80343	2.20174
D 3	0.002345	1.44E-05	9.31E-07	0.000322	7.8E-05	0.02039	0.96464	2.113793
D 4	0.002597	1.86E-05	7.49E-07	0.000259	7.03E-05	0.018684	0.86530	2.159208
D 5	0.002776	2.2E-05	9.6E-07	0.000332	6.57E-05	0.020513	0.97909	2.095095
D 6	0.003963	5.37E-05	1.06E-06	0.000366	2.4E-06	0.020567	1.02750	2.001683
D 7	0.005103	0.000101	7.5E-07	0.00026	1.86E-06	0.019066	0.86559	2.202627
D 8	0.004638	7.97E-05	1.31E-06	0.000453	2.05E-06	0.023151	1.14328	2.024971
D 9	0.003119	2.95E-05	1.64E-06	0.000568	3.07E-06	0.024532	1.27991	1.916675
D 10	0.004656	8.05E-05	1.26E-06	0.000437	2.04E-06	0.02283	1.12356	2.031948
D 11	0.002653	1.96E-05	3.25E-06	0.001125	6.75E-07	0.033884	1.80151	1.880859
D 12	0.005343	0.000114	1.91E-06	0.000662	3.31E-07	0.027892	1.38223	2.017859
D 13	0.000939	1.44E-06	8.08E-06	0.002798	1.97E-06	0.053	2.84141	1.865288
D 14	0.004258	6.43E-05	1.87E-06	0.000647	4.17E-07	0.026708	1.36616	1.954964
D 15	0.001225	2.82E-06	6.21E-06	0.002149	1.49E-06	0.046473	2.49047	1.866019
D 16	0.002324	1.41E-05	2.04E-07	2.81E-05	7.87E-05	0.011007	0.45189	2.43584
D 17	0.000872	1.19E-06	5.4E-07	7.44E-05	0.000216	0.017097	0.73471	2.327027
D 18	0.001331	3.47E-06	3.49E-07	4.8E-05	0.000139	0.013832	0.59018	2.343621
D 19	0.000669	6.13E-07	6.58E-07	9.06E-05	0.000285	0.019404	0.81084	2.393078
D 20	0.00234	1.43E-05	4.98E-07	6.85E-05	4.11E-06	0.009351	0.70502	1.326328
D 21	0.001703	6.45E-06	7.95E-07	0.000109	5.69E-06	0.011061	0.89092	1.241496

D 22	0.002717	2.08E-05	4.91E-07	6.76E-05	3.53E-06	0.009614	0.70017	1.373044
D 23	0.002906	2.47E-05	5.38E-07	7.41E-05	3.3E-06	0.010129	0.73325	1.381412
D 24	0.002899	2.45E-05	1.01E-06	0.000139	6.17E-07	0.012849	1.00410	1.279682
D 25	0.004025	5.58E-05	7.03E-07	9.67E-05	4.42E-07	0.012399	0.83777	1.479955
D 26	0.001915	8.66E-06	1.15E-06	0.000158	9.43E-07	0.012979	1.06964	1.213371
D 27	0.002557	1.79E-05	8.45E-07	0.000116	7.01E-07	0.01165	0.91852	1.268358
D 28	0.002823	2.29E-05	9.38E-07	0.000129	6.34E-07	0.012395	0.96792	1.28058
D 29	0.002245	1.29E-05	7.88E-08	5.79E-06	8.16E-05	0.010018	0.28062	3.569937
D 30	0.003064	2.82E-05	5.34E-08	3.92E-06	5.94E-05	0.009566	0.23086	4.143653
D 31	0.001932	8.85E-06	8.31E-08	6.1E-06	9.51E-05	0.010496	0.28812	3.643122
D 32	0.001777	7.17E-06	1.01E-07	7.44E-06	0.000104	0.01088	0.31805	3.420995
D 33	0.000345	1.15E-07	6.43E-07	4.72E-05	0.000571	0.024882	0.80126	3.105347
D 34	0.000884	1.24E-06	5.93E-07	4.35E-05	1.12E-05	0.00752	0.76936	0.977411
D 35	0.002097	1.09E-05	2.95E-07	2.17E-05	4.6E-06	0.006119	0.54305	1.126761
D 36	0.001182	2.57E-06	4.67E-07	3.43E-05	8.3E-06	0.006757	0.68335	0.98886
D 37	0.000893	1.27E-06	6.9E-07	5.07E-05	1.11E-05	0.007984	0.83046	0.961382
D 38	0.002585	1.84E-05	4.66E-07	3.43E-05	6.94E-07	0.007335	0.68259	1.074566
D 39	0.001964	9.23E-06	5.13E-07	3.77E-05	9.19E-07	0.006951	0.71580	0.97115
D 40	0.00271	2.07E-05	4.18E-07	3.07E-05	6.61E-07	0.007244	0.64612	1.121135
D 41	0.002114	1.11E-05	5.49E-07	4.03E-05	8.52E-07	0.007269	0.74075	0.981265
D 42	0.001774	7.14E-06	7.14E-07	5.24E-05	1.02E-06	0.007828	0.84429	0.927125
							total	78.84757
							average uncdertainity	1.877323