Construction of Unimodular Sequence Sets for Periodic Correlations

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***Abstract*—Sequence sets with good periodic correlation properties can be used in many areas, including communications, medical imaging, radar (such as over-the-horizon radar) and sonar. Practical hardware constraints, such as power amplifiers, usually require the transmitted waveforms be unimodular. We present herein new computationally efficient algorithms that can be used for the design of unimodular sequence sets with essentially zero auto-correlation sidelobes and cross-correlations in a specified time lag zone, as well as of sequence sets with good correlations over all time lags. The proposed algorithms start from random phase initializations and can generate many different sequence sets (including very long sequence sets) possessing similarly good correlation properties.**

I. Introduction

Let $\{x_m(n)\}$ ($m = 1, \ldots, M$ and $n = 1, \ldots, N$) denote a set of M sequences, each of which is of length N. Every element of the sequence set is a complex-valued number that is unimodular: $|x_m(n)| = 1$. The periodic cross-correlation between the m_1^{th} and m_2^{th} sequence at time lag k is defined by

$$\begin{aligned} r_{m_1 m_2}(k) = &\sum_{n=1}^{N} x_{m_1}(n) x^*_{m_2}(n - k \text{ mod } N) \quad (1)\\ =& r^*_{m_2 m_1}(-k) = r^*_{m_2 m_1}(N-k) \\ & m_1, m_2 = 1, \ldots, M \text{ and } k = 0, \ldots, N-1. \end{aligned}$$

When $m_1 = m_2$, the correlation above becomes the auto-correlation.

Sequence sets having low (or even zero) auto- and cross-correlations are useful in many application areas. In radar range compression, low auto-correlations improve the detection of weak targets [1]; in code division multiple access (CDMA) systems, low auto-correlation helps with synchronization and low cross-correlation reduces interference from other users [2]; and the situation is similar in many other applications like ultra-sonic imaging [3].

Since the early research on maximum-length sequences (m-sequence) in the 1950s, many sequence families with good correlations have been proposed, such as the Gold sequences [4], the Kasami sequences [5] and many others [6][7][8][9]. Some of the sequence sets asymptotically meet the Welch bound, which is a theoretical lower bound of correlations derived in [10]. [11] extends the Welch bound to the situation that correlation sidelobes can be made zero within a time lag zone (here and after, correlation sidelobes denote both auto-correlation sidelobes and cross-correlations); these sequences are referred to as the zero-correlation zone (ZCZ) sequences. Again, plenty of literature is available on ZCZ sequences, see, e.g. [12][13][14][15] and the references therein.

This work was supported in part by the Army Research Office (ARO) under Grant W911NF-07-1-0450, the Office of Naval Research (ONR) under Grants N00014-09-1-0211 and N00014-07-1-0193, the National Science Foundation (NSF) under Grant CCF-0634786, the Swedish Research Council (VR) and the European Research Council (ERC).

In this paper we extend our earlier works [16][17][18] and propose two new cyclic algorithms for the design of unimodular sequence sets with low periodic correlations. The first algorithm can be used to generate sequence sets with almost zero correlation sidelobes in a specified time lag interval; these designed sequence sets are essentially ZCZ sequence sets. The second algorithm aims at achieving good correlations for all time lags; it is based on FFT computations and thus can efficiently generate sequence sets of large sizes. Unlike most existing sequence construction methods which are algebraic and deterministic in nature, both proposed algorithms start from random initializations and then proceed to cyclically minimize the desired metrics. In this way we can generate many different sequence sets bearing similarly good correlation properties; these randomly distributed sequences are especially useful in applications such as to counter coherent repeater jamming in radar systems (see, e.g., [19][20]).

Notations: We use bold lowercase and uppercase letters to denote vectors and matrices, respectively. $(\cdot)^H$ denotes the complex conjugate transpose, $\|\cdot\|_F$ the Frobenius matrix norm and $\mathbf{I}_n$ the $n \times n$ identity matrix.

II. PeCA

Let $\mathbf{X}_m$ denote the right circulant matrix related to the m^{th} sequence:

$$\mathbf{X}_m \quad (2)$$
$$= \begin{bmatrix} x_m(1) & x_m(2) & \cdots & x_m(N) \\ x_m(N) & x_m(1) & \cdots & x_m(N-1) \\ \vdots & & & \vdots \\ x_m(N-P+2) & & \cdots & x_m(N-P+1) \end{bmatrix}_{P \times N},$$

that is, every row of $\mathbf{X}_m$ is a cyclically-shifted version of the sequence $\{x_m(n)\}_{n=1}^N$ and $\mathbf{X}_m$ has P rows ($0 < P \leq N$). We stack all $\{\mathbf{X}_m\}$ together:

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \vdots \\ \mathbf{X}_M \end{bmatrix}_{MP \times N} \tag{3}$$

and the outer-product of $\mathbf{X}$ brings in the correlations:

$$\mathbf{X}\mathbf{X}^H = \begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} & \cdots & \mathbf{R}_{1M} \\ \mathbf{R}_{21} & \mathbf{R}_{22} & \cdots & \mathbf{R}_{2M} \\ \vdots & & & \vdots \\ \mathbf{R}_{M1} & \mathbf{R}_{M2} & \cdots & \mathbf{R}_{MM} \end{bmatrix}_{MP \times MP} \tag{4}$$

where

$$\begin{aligned} &\mathbf{R}_{m_1 m_2} \\ &= \begin{bmatrix} r_{m_1m_2}(0) & r_{m_1m_2}(1) & \cdots & r_{m_1m_2}(P-1) \\ r_{m_1m_2}(-1) & \ddots & \ddots & \vdots \\ \vdots & & & r_{m_1m_2}(1) \\ r_{m_1m_2}(-P+1) & \cdots & & r_{m_1m_2}(0) \end{bmatrix}. \end{aligned} \tag{5}$$

Note that $r_{m_1m_2}(k)$ ($k = -P+1, \ldots, P-1$) appears $P - |k|$ times in the above Toeplitz matrix, so more emphasis is given to the correlations of smaller time lags. Moreover, all diagonal elements in the $MP \times MP$ matrix in (4) are the in-phase auto-correlations and are equal to N (a constant). Therefore we can minimize all cross-correlations and out-of-phase auto-correlations within the P time lags by minimizing the following criterion:

$$C_P = \left\| \mathbf{X}\mathbf{X}^H - N\mathbf{I}_{MP} \right\|_F^2. \tag{6}$$

(Although not explicit in notation, the size of the matrix $\mathbf{X}$ is also dependent on P).

A close scrutiny of the criterion C_P leads to the fact that only for $P \leq N/M$ is it possible to minimize C_P to zero. If $MP > N$, the matrix $\mathbf{X}$ in (3) will be “tall” and the rank of $\mathbf{X}\mathbf{X}^H$ will be at most N. In this case the rank of $\mathbf{X}\mathbf{X}^H$ is always smaller than the rank of $\mathbf{I}_{MP}$ no matter what sequences we choose, which means that C_P cannot be minimized to zero. As a matter of fact, the condition $P \leq N/M$ coincides with the theoretical bound shown in [11].

When $P \leq N/M$, it is easy to observe that C_P becomes zero if $\mathbf{X}$ is a semi-unitary matrix. Therefore, instead of minimizing C_P directly, we consider the following minimization problem:

$$\begin{aligned} &\min_{\{x_m(n)\}, \mathbf{U}} \left\| \mathbf{X} - \sqrt{N}\mathbf{U} \right\|_F^2 \\ &\text{s.t. } |x_m(n)| = 1, \quad m = 1, \ldots, M \text{ and } n = 1, \ldots, N \\ &\qquad \mathbf{U}\mathbf{U}^H = \mathbf{I} \end{aligned} \tag{7}$$

where $\mathbf{U}$ is an $MP \times N$ semi-unitary matrix that serves as an auxiliary matrix.

Eq. (7) can be solved in the following cyclic way. $\mathbf{X}$ is first initialized by a set of randomly generated unimodular sequences. Then (7) is iteratively minimized by fixing $\mathbf{X}$ to compute $\mathbf{U}$, then fixing $\mathbf{U}$ to compute $\mathbf{X}$ and so on, until a given stop criterion is satisfied. During this iterative process, both $\mathbf{U}$ and $\mathbf{X}$ have closed-form updating formulae. We refer the readers to the cyclic algorithm (CA) proposed in [16] for details. Because we consider the periodic correlation in this paper rather than the aperiodic one in [16], the algorithm in this section is named PeCA (periodic CA).

III. PeCAN

Although it is not possible to make all off-diagonal elements of $\mathbf{X}\mathbf{X}^H$ zero for $P = N$, we can still try to make them small without any emphasis on certain time lags, as required by many applications such as radar range compression. To be clear, we try to minimize the criterion in (6) with P replaced by N:

$$C_N = \left\| \mathbf{X}\mathbf{X}^H - N\mathbf{I}_{MN} \right\|_F^2. \tag{8}$$

Note that here and from now on in this section, the matrix $\mathbf{X}_m$ in (2) becomes $N \times N$, $\mathbf{X}$ becomes $MN \times N$ and thus the correlations of all time lags are incorporated in $\mathbf{X}\mathbf{X}^H$.

It is well-known that the right circulant matrix $\mathbf{X}_m$ can be diagonalized by the FFT matrix (see, e.g., a proof in [18]):

$$\mathbf{X}_m = \mathbf{F}^H \mathbf{D}_m \mathbf{F} \tag{9}$$

where

$$\begin{aligned} [\mathbf{F}]_{kl} &= \frac{1}{\sqrt{N}} e^{j\frac{2\pi}{N}(k-1)(l-1)}, \quad k, l = 1, \ldots, N \\ \mathbf{D}_m &= \begin{bmatrix} y_m(1) & & \\ & \ddots & \\ & & y_m(N) \end{bmatrix} \\ y_m(k) &= \sum_{n=1}^N x_m(n) e^{-j\frac{2\pi}{N}(k-1)(n-1)}, \quad k = 1, \ldots, N. \end{aligned} \tag{10}$$

To simplify notations, define

$$\begin{aligned} &\tilde{\mathbf{F}} = \begin{bmatrix} \mathbf{F} & & \\ & \ddots & \\ & & \mathbf{F} \end{bmatrix}_{MN \times MN}, \quad \mathbf{D} = \begin{bmatrix} \mathbf{D}_1 \\ \vdots \\ \mathbf{D}_M \end{bmatrix}_{MN \times N} \\ &\mathbf{y}_p = \begin{bmatrix} y_1(p) \\ \vdots \\ y_M(p) \end{bmatrix}_{M \times 1}, \quad p = 1, \ldots, N. \end{aligned} \tag{11}$$

Then we proceed to write the criterion C_N in (8) as follows:

$$\begin{aligned} C_N &= \left\| (\tilde{\mathbf{F}}^H \mathbf{D}\mathbf{F})(\tilde{\mathbf{F}}^H \mathbf{D}\mathbf{F})^H - N\mathbf{I}_{MN} \right\|^2 \\ &= \left\| \tilde{\mathbf{F}}^H \mathbf{D}\mathbf{D}^H \tilde{\mathbf{F}} - N\mathbf{I} \right\|^2 = \left\| \mathbf{D}\mathbf{D}^H - N\mathbf{I} \right\|^2 \\ &= \sum_{p=1}^N \left\| \mathbf{y}_p \mathbf{y}_p^H - N\mathbf{I}_M \right\|^2 \end{aligned} \tag{12}$$

which can be further written as

$$C_N = \sum_{p=1}^{N} \mathrm{tr}\left\{(\mathbf{y}_p\mathbf{y}_p^H - N\mathbf{I})(\mathbf{y}_p\mathbf{y}_p^H - N\mathbf{I})^H\right\} \quad (13)$$
$$= \sum_{p=1}^{N} \left(\|\mathbf{y}_p\|^4 - 2N\|\mathbf{y}_p\|^2 + N^2 M\right)$$
$$= N^2 \sum_{p=1}^{N} \left(\left\| \frac{\mathbf{y}_p}{\sqrt{N}} \right\|^2 - 1 \right)^2 + N^3(M-1).$$

Similarly to the relation between (6) and (7), we consider the following related problem instead of minimizing C_N directly:

$$\min_{\{x_m(n)\},\{\boldsymbol{\alpha}_p\}} \sum_{p=1}^{N} \left\| \frac{1}{\sqrt{N}}\mathbf{y}_p - \boldsymbol{\alpha}_p \right\|^2 \quad (14)$$
$$\text{s.t. } |x_m(n)| = 1, \quad m = 1,\ldots,M \text{ and } n = 1,\ldots,N$$
$$\|\boldsymbol{\alpha}_p\|^2 = 1, \quad p = 1,\ldots,N \quad (\boldsymbol{\alpha}_p \text{ is } M \times 1)$$

where $\{\boldsymbol{\alpha}_p\}$ are auxiliary vectors.

Although derived in the case of periodic correlations, Eq. (14) bears the same structure as the minimization problem of aperiodic correlations in [21] and thus can be solved by the CAN (CA new) algorithm there. The CAN algorithm also starts from a random phase initialization, minimizes the criterion in (14) cyclically and has closed-form updating formulae during iteration. We call the algorithm in this section PeCAN (periodic CAN). As shown in [21], the computation is based on FFT and thus is very efficient. Indeed, the PeCAN algorithm can be used to design sequence sets with the size $NM \sim 10^5$. Also note that the PeCAN algorithm is an extension of the one proposed in [18], which only deals with the auto-correlation of a single sequence.

IV. Numerical Examples

We first introduce more definitions to facilitate our discussions. Making use of

$$\tilde{\mathbf{X}} = \begin{bmatrix} x_1(1) & \cdots & x_M(1) \\ x_1(2) & & x_M(2) \\ \vdots & & \vdots \\ x_1(N) & \cdots & x_M(N) \end{bmatrix}_{N\times M}, \mathbf{J} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{N-1} \\ 1 & \mathbf{0} \end{bmatrix}_{N\times N} \quad (15)$$

the auto- and cross-correlations of $\{x_m(n)\}_{m,n=1}^{M,N}$ at lag k can be expressed in a matrix form as

$$\mathbf{P}(k) \triangleq \tilde{\mathbf{X}}^H \mathbf{J}^k \tilde{\mathbf{X}} = \begin{bmatrix} r_{11}(k) & \cdots & r_{M1}(k) \\ \vdots & & \vdots \\ r_{1M}(k) & \cdots & r_{MM}(k) \end{bmatrix} = \mathbf{P}^H(-k) \quad (16)$$
$$k = 0,\ldots,N-1. \quad (\text{assuming that } \mathbf{J}^0 = \mathbf{I}_N)$$

Correspondingly the "correlation level" at lag k is defined to be:

$$\text{correlation level } = 20\log_{10} \frac{\|\mathbf{P}(k)\|_F}{\sqrt{MN^2}} \quad (17)$$
$$k = -N+1,\ldots,0,\ldots,N-1.$$

The above normalization factor $\sqrt{MN^2}$ is the Frobenius norm of $\mathbf{P}(0)$ when all sequences in the set are orthogonal to each other (i.e., $\mathbf{P}(0) = N\mathbf{I}_M$).

A. Minimization of C_P in (6)

Suppose that there are $M = 4$ transmitters (users), each transmit sequence is of length $N = 512$ and we are mainly interested in the correlations of the first $P = 60$ time lags. The PeCA algorithm is used to design such a set of sequences. Fig. 1 shows the correlation level of the so-generated sequence set, from which we observe that the correlation levels within the P time lags are all below -80 dB. The corresponding minimization criterion C_P is only 0.23, while the C_P of a randomly generated unimodular sequence set of the same size is usually on the order of 10^3.

Note that in Fig. 1, the correlations within the region of interest go up as the time lag (its absolute value) increases. This coincides with the "implicit weighting" discussion right after (5).

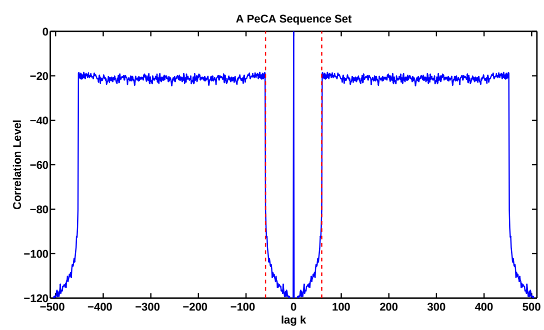


Fig. 1. The correlation level of a PeCA sequence set. $N = 512$, $M = 4$ and the goal is to minimize correlations of the first $P = 60$ time lags. (The dotted vertical lines signify the boundaries of the time lag zone under consideration.)

An alternative way to minimize C_P is as follows. We first generate a single sequence of length N whose auto-correlations are zero for all time lags. Such a sequence is called a perfect sequence, such as the Frank or Chu sequence [22]. Actually the PeCAN algorithm in Section III can be used to generate many perfect sequences of the same length (see [18]). Denote this length-N perfect sequence as $\mathbf{x}$. Then the desired sequence set is constructed as $\mathcal{K} = \{\mathbf{x}, T^P(\mathbf{x}),\ldots,T^{(M-1)P}(\mathbf{x})\}$, where the operator $T^k(\mathbf{x})$ denotes the k-element right cyclic-shift of the sequence $\mathbf{x}$ (considered as a row vector). In this way, the correlations of the sequences in $\mathcal{K}$ within P time lags can only take values from the auto-correlations of $\mathbf{x}$ within MP time lags, which are all zero. As an example, we still choose $N = 512$, $M = 4$ and $P = 60$ which are used in Fig. 1. We use the PeCAN algorithm to generate a perfect sequence $\mathbf{x}$ and construct the sequence set $\mathcal{K}$, whose correlation levels are shown in Fig. 2. We observe that the correlations within the region of interest (time lags less than P) are all zero. The tradeoff is the high sidelobes outside the region of interest, because the cross-correlation at a certain time lag between any two sequences in $\mathcal{K}$ can be as large as the in-phase auto-correlation.

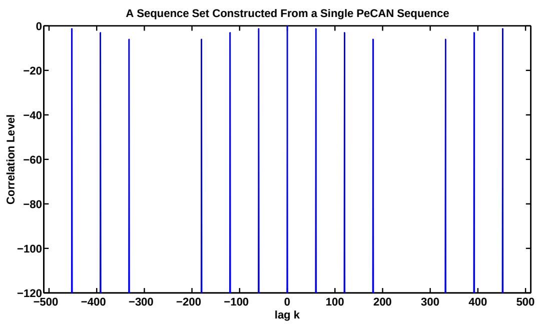


Fig. 2. The correlation level of an $N = 512, M = 4$ sequence set, constructed from a single perfect PeCAN sequence.

For the "shift and construct" approach outlined in the last paragraph, an extreme situation is the optimal ZCZ sequence set, whose correlation sidelobes are zero within the first N/M lags; see e.g. [13][15] (here N/M is assumed to be an integer). More specifically, we construct the sequence set $\mathcal{K}$ with $P = N/M$. Fig. 3 shows the correlation level of an optimal ZCZ sequence set of $N = 512$ and $M = 4$. It is interesting to observe that the correlations are either zero or as high as the in-phase auto-correlation.

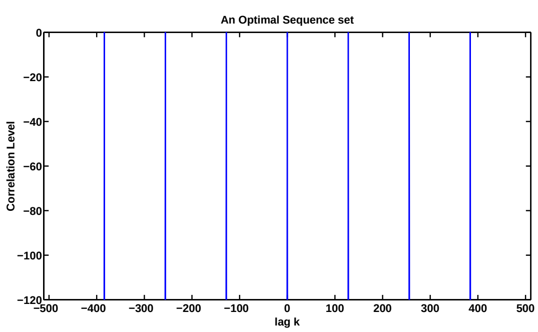


Fig. 3. The correlation level of an optimal ZCZ sequence set of $N = 512$ and $M = 4$.

B. *Minimization of C_N in* (8)

Following the criterion C_N in (8), we define the integrated sidelobe level (ISL) as

$$\text{ISL} = \frac{1}{N} C_N \tag{18}$$
$$= \sum_{m=1}^{M} \sum_{k=1}^{N-1} |r_{mm}(k)|^2 + \sum_{m_1=1}^{M} \sum_{\substack{m_2=1 \\ m_2 \neq m_1}}^{M} \sum_{k=0}^{N-1} |r_{m_1 m_2}(k)|^2.$$

The maximum correlation sidelobe is defined as

$$\begin{aligned} r_{\max} &= \max\{|r_{mm}(k)|,\ |r_{m_1 m_2}(l)|\} \\ m &= 1, \ldots, M \quad k = 1, \ldots, N-1 \\ m_1, m_2 &= 1, \ldots, M\ (m_1 \neq m_2) \quad l = 0, \ldots, N-1. \end{aligned} \tag{19}$$

There have been many discussions on $r_{\max}$ in the literature. The well-known Welch bound [10] states that for a set of sequences $\{x_m(n)\}_{m,n=1}^{M,N}$, the maximum correlation sidelobe satisfies the following lower bound:

$$r_{\max} \geq B = N\sqrt{\frac{M-1}{NM-1}}. \tag{20}$$

Usually people consider the asymptotic bound: $B_{\text{asymp}} = \sqrt{N}$, which is close to B when $N \gg 1$. A set of sequences is called optimal if its $r_{\max}$ asymptotically meets the B_{asymp}, i.e. $\lim_{N\to\infty} r_{\max} = \sqrt{N}$. Interestingly, there exist many optimal sequence sets (see e.g. [7][9]) and we choose the Kasami sequence set [5] as an example to compare with the PeCAN sequence set proposed in this paper. A Kasami sequence set is constructed from the exclusive-or of m-sequences. It has $M = \sqrt{N+1}$ sequences of length N, where N is restricted to be $2^k - 1$ and k is even (note that the PeCAN sequence set can be of any length). Its maximum correlation sidelobe $r_{\max}$ equals $1 + \sqrt{N+1}$.

We choose $N = 1023$ and $M = 4$. We generate the Kasami sequence set and the PeCAN sequence set. (The full Kasami sequence set has 32 sequences and we choose the first four sequences; other four-sequence combinations lead to very similar results.) Fig. 4 shows their correlation levels and Table I shows their ISL and $r_{\max}$ values. Compared to the optimal Kasami sequence set, the PeCAN sequence set gives higher $r_{\max}$ but lower ISL; this can be expected since ISL is precisely the criterion that the PeCAN algorithm aims to minimize.

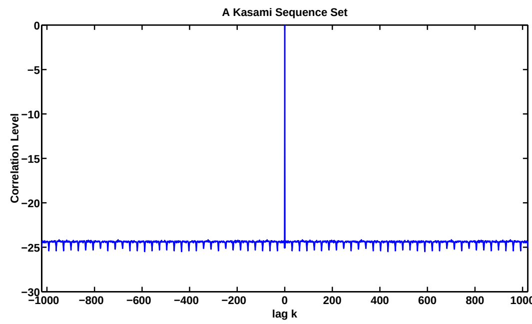
Another point worth mentioning is that different initializations lead to different PeCAN sequence sets. As far as we have tested, all these PeCAN sequence sets have the same ISL and similar $r_{\max}$ values. Therefore the PeCAN algorithm is able to generate many sequence sets that have similarly low correlation sidelobes.

TABLE I
KASAMI AND PECAN, $N = 1023, M = 4$

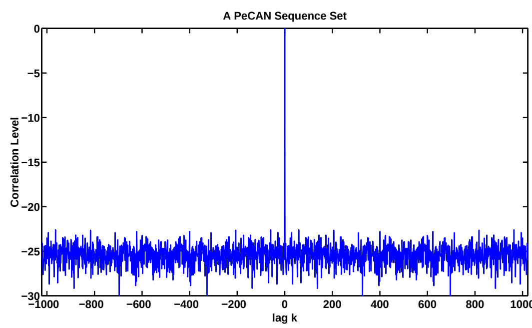
	ISL/N	$r_{\max}$
Kasami	15035.2	33.0
PeCAN	12276.0	81.3

C. *Phase Quantization Effect*

We do not constrain the phases of sequence sets we design to be on a finite constellation grid. The sidelobes will increase when the phases are quantized. As an example, we quantize the phases of the PeCA sequence set shown in Fig. 1 into 64 levels and then show its correlations again in Fig. 5, along with the correlations of a unimodular sequence set whose phases are randomly generated. Compared to Fig. 1, the PeCA correlation



(a)



(b)

Fig. 4. The correlation level of a sequence set of $M = 4$ sequences, each with length $N = 1023$. (a) The Kasami sequence set and (b) the PeCAN sequence set.

sidelobes in Fig. 5 are higher but the sidelobes inside the region of interest are still much lower than those of a random sequence set. A similar situation happens when the PeCAN sequence set is quantized.

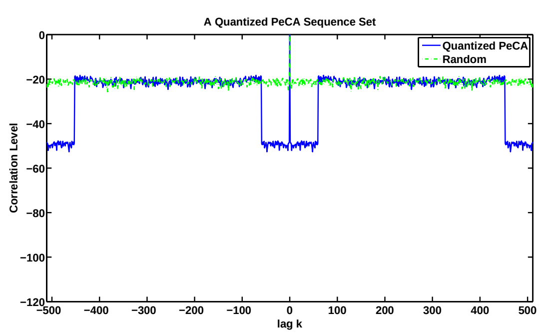


Fig. 5. The solid line is the correlation level of the PeCA sequence set shown in Figure 1, with the phases quantized into 64 levels. The dotted line is that of a randomly generated sequence set.

V. CONCLUSIONS

In this paper we have presented two new cyclic algorithms that can be used to generate a set of unimodular sequences with low periodic correlation sidelobes (i.e. auto-correlation sidelobes and cross-correlations). The PeCA algorithm is able to generate sequence sets that have almost zero correlation sidelobes within a time lag interval, while the PeCAN algorithm aims at minimizing correlation sidelobes of all time lags. The PeCAN algorithm is computationally very efficient and can be used to design large sequence sets. The so-generated sequence sets can be widely used in many areas such as communications and radar.

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